chapter seven

Solid Friction. Viscosity

SOLID FRICTION

Static Friction

WHEN a person walks along a road, he or she is prevented from slipping by the force of friction at the ground. In the absence of friction, for example on an icy surface, the person's shoe would slip when placed on the ground. The frictional force always *opposes* the motion of the shoe.

The frictional force between the surface of a table and a block of wood A can be investigated by attaching one end of a string to A and the other to a scale-pan S, Fig. 7.1. The string passes over a fixed

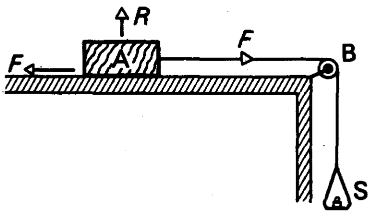

FIG. 7.1 Solid friction

grooved wheel B. When small weights are added to S, the block does not move. The frictional force between the block and table is thus equal to the total weight on S together with the weight of S. When more weights are added, A does not move, showing that the frictional force has increased, but as the weight is increased further, A suddenly begins to slip. The frictional force now present between the surfaces is called the *limiting frictional force*, and we are said to have reached *limiting friction*. The limiting frictional force is the maximum frictional force between the surfaces.

Coefficient of Static Friction

The normal reaction, R, of the table on A is equal to the weight of A. By placing various weights on A to alter the magnitude of R, we can find how the limiting frictional force F varies with R by the experiment just described. The results show that, approximately,

$$\frac{\text{limiting frictional force } (F)}{\text{normal reaction } (R)} = \mu, \text{ a constant,}$$

and μ is known as the *coefficient of static friction* between the two surfaces. The magnitude of μ depends on the nature of the two surfaces; for example, it is about 0·2 to 0·5 for wood on wood, and about 0·2 to 0·6 for wood on metals. Experiment also shows that the limiting frictional force is the same if the block A in Fig. 7.1 is turned on one side so that its surface area of contact with the table decreases, and thus the limiting frictional force is independent of the area of contact when the normal reaction is the same.

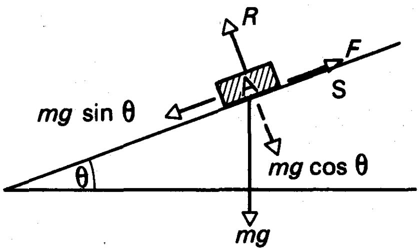

FIG. 7.2 Coefficient by inclined plane

The coefficient of static friction, μ, can also be found by placing the block A on the surface S, and then gently tilting S until A is on the point of slipping down the plane, Fig. 7.2. The static frictional force F is then equal to $mg \sin \theta$, where θ is the angle of inclination of the plane to the horizontal; the normal reaction R is equal to $mg \cos \theta$.

$$\therefore \mu = \frac{F}{R} = \frac{mg \sin \theta}{mg \cos \theta} = \tan \theta,$$

and hence μ can be found by measuring θ.

Kinetic Friction. Coefficient of Kinetic (Dynamic) Friction

When brakes are applied to a bicycle, a frictional force is exerted between the moving wheels and brake blocks. In contrast to the case of static friction, when one of the objects is just on the point of slipping, the frictional force between the moving wheel and brake blocks is called a *kinetic* (*or dynamic*) *frictional force.* Kinetic friction thus occurs between two surfaces which have relative motion.

The *coefficient of kinetic* (*dynamic*) *friction*, μ', between two surfaces is defined by the relation

$$\mu' = \frac{F'}{R},$$

where F' is the frictional force when the object moves with a uniform velocity and R is the normal reaction between the surfaces. The coefficient of kinetic friction between a block A and a table can be found by the apparatus shown in Fig. 7.1. Weights are added to the scale-pan, and each time A is given a slight push. At one stage A continues to move with a constant velocity, and the kinetic frictional force F' is then equal to the total weight in the scale-pan together with the latter's weight. On dividing F' by the weight of A, the coefficient can be calculated. Experiment shows that, when weights are placed on A to vary the normal reaction R, the magnitude of the ratio F'/R is approximately constant. Results also show that the coefficient of kinetic friction between two given surfaces is less than the coefficient of static friction between the same surfaces, and than the coefficient of kinetic friction between two given surfaces is approximately independent of their relative velocity.

Laws of Solid Friction

Experimental results on solid friction are summarised in the *laws of friction*, which state:

(1) The frictional force between two surfaces opposes their relative motion.

(2) The frictional force is independent of the area of contact of the given surfaces when the normal reaction is constant.

(3) The limiting frictional force is proportional to the normal reaction for the case of static friction. The frictional force is proportional to the normal reaction for the case of kinetic (dynamic) friction, and is independent of the relative velocity of the surfaces.

Theory of Solid Friction

The laws of solid friction were known hundreds of years ago, but they have been explained only in comparatively recent years, mainly by F. P. Bowden and collaborators. Sensitive methods, based on electrical conductivity measurements, reveal that the true area of contact between two surfaces is extremely small, perhaps one ten-thousandth of the area actually placed together for steel surfaces. This is explained by photographs which show that some of the atoms of a metal project slightly above the surface, making a number of crests or 'humps'. As Bowden has stated: 'The finest mirror, which is flat to a millionth of a centimetre, would to anyone of atomic size look rather like the South Downs—valley and rolling hills a hundred or more atoms high.' Two metal surfaces thus rest on each others projections when placed one on the other.

Since the area of actual contact is extremely small, the pressures at the points of contact are very high, perhaps 1000 million kgf per m^2 for steel surfaces. The projections merge a little under the high pressure, producing adhesion or 'welding' at the points, and a force which opposes motion is therefore obtained. This explains Law 1 of the laws of solid friction. When one of the objects is turned over, so that a smaller or larger surface is presented to the other object, measurements show that the small area of actual contact remains constant. Thus the frictional force is independent of the area of the surfaces, which explains Law 2. When the load increases the tiny projections are further squeezed by the enormous pressures until the new area of contact becomes big enough to support the load. The greater the load, the greater is the area of actual contact, and the frictional force is thus approximately proportional to the load, which explains Law 3.

VISCOSITY

If we move through a pool of water we experience a resistance to our motion. This shows that there is a *frictional force* in liquids. We say this is due to the **viscosity** of the liquid. If the frictional force is comparatively low, as in water, the viscosity of the liquid is low; if the frictional force is large, as in glue or glycerine, the viscosity of the liquid is high. We can compare roughly the viscosity of two liquids by filling two measuring cylinders with each of them, and allowing identical small steel ball-bearings to fall through each liquid. The sphere falls more slowly through the liquid of higher viscosity.

As we shall see later, the viscosity of a lubricating oil is one of the factors which decide whether it is suitable for use in an engine. The Ministry of Aircraft Production, for example, listed viscosity values to which lubricating oils for aero-engines must conform. The subject of viscosity has thus considerable practical importance.

Newton's Formula. Coefficient of Viscosity

When water flows slowly and steadily through a pipe, the layer A of the liquid in contact with the pipe is practically stationary, but the central part C of the water is moving relatively fast, Fig. 7.3. At other layers between A and C, such as B, the water has a velocity less than at C, the magnitude of the velocities being represented by the length of the arrowed lines in Fig. 7.3. Now as in the case of two solid surfaces

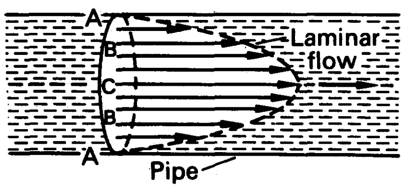

FIG. 7.3 Laminar (uniform) flow through pipe

moving over each other, a frictional force is exerted between two liquid layers when they move over each other. Thus because the velocities of neighbouring layers are different, as shown in Fig. 7.3, a frictional force occurs between the various layers of a liquid when flowing through a pipe.

The basic formula for the frictional force, F, in a liquid was first suggested by NEWTON. He saw that the larger the *area* of the surface of liquid considered, the greater was the frictional force F. He also stated that F was directly proportional to the *velocity gradient* at the part of the liquid considered. This is the case for most common liquids, called *Newtonian liquids.* If v_1, v_2 are the velocities of C, B respectively in Fig. 7.3, and h is their distance apart, the velocity gradient between the liquids is defined as $(v_1 - v_2)/h$. The velocity gradient can thus be expressed in (m/s)/m, or as 's^{-1}'.

Thus if A is the area of the liquid surface considered, the frictional force F on the surface is given by

$$F \propto A \times \text{velocity gradient},$$

or

$$F = \eta A \times \text{velocity gradient}, \quad . \quad . \quad . \quad (1)$$

where η is a constant of the liquid known as the *coefficient of viscosity.* This expression for the frictional force in a liquid should be contrasted with the case of solid friction, in which the frictional force is independent of the area of contact and of the relative velocity between the solid surfaces concerned (p. 173).

Definition, Units, and Dimensions of Coefficient of Viscosity

The magnitude of η is given by

$$\eta = \frac{F}{A \times \text{velocity gradient}}.$$

The unit of F is a newton, the unit of A is m^2, and the unit of velocity gradient is 1 m/s per m. Thus η may be defined as *the frictional force per unit area of a liquid when it is in a region of unit velocity gradient.*

The 'unit velocity gradient' = 1 m s^{-1} change per m. Since the 'm' cancels, the 'unit velocity gradient' = 1 per second. From $\eta = F/(A \times$ velocity gradient), it follows that η may be expressed in units of *newton s m*$^{-2}$ (N s m^{-2}), or 'dekapoise'.

The coefficient of viscosity of water at 10°C is $1{\cdot}3 \times 10^{-3}$ N s m^{-2}. Since $F = \eta A \times$ velocity gradient, the frictional force on an area of 10 cm^2 in water at 10°C between two layers of water 0·1 cm apart which move with a relative velocity of 2 cm s^{-1} is found as follows:

Coefficient of viscosity $\eta = 1{\cdot}3 \times 10^{-3}$ newton m^{-2}, $A = 10 \times 10^{-4}$ m^2, velocity gradient $= 2 \times 10^{-2}$ m $s^{-1} \div 0{\cdot}1 \times 10^{-2}$ m $= 2/0{\cdot}1$ s^{-1}.

$$\therefore F = 1{\cdot}3 \times 10^{-3} \times 10 \times 10^{-4} \times 2/0{\cdot}1 = 2{\cdot}6 \times 10^{-5} \text{ newton.}$$

Dimensions. The dimensions of a force, F, (= mass × acceleration = mass × velocity change/time) are MLT^{-2}. See p. 31. The dimensions of an area, A, are L^2. The dimensions of velocity gradient

$$= \frac{\text{velocity change}}{\text{distance}} = \frac{L}{T} \div L = \frac{1}{T}.$$

Now

$$\eta = \frac{F}{A \times \text{velocity gradient}},$$

$$\therefore \text{dimensions of } \eta = \frac{MLT^{-2}}{L^2 \times 1/T},$$

$$= ML^{-1}T^{-1}.$$

Thus η may be expressed in units 'kg m^{-1} s^{-1}'.

Steady Flow of Liquid Through Pipe. Poiseuille's Formula

The steady flow of liquid through a pipe was first investigated thoroughly by POISEUILLE in 1844, who derived an expression for the volume of liquid issuing per second from the pipe. The proof of the formula is given on p. 208, but we can derive most of the formula by the *method of dimensions* (p. 31).

The volume of liquid issuing per second from the pipe depends on (i) the coefficient of viscosity, η, (ii) the radius, a, of the pipe, (iii), the *pressure gradient*, g, set up along the pipe. The pressure gradient $= p/l$, where p is the pressure difference between the ends of the pipe and l is its length. Thus x, y, z being indices which require to be found, suppose

$$\text{volume per second} = k\eta^x a^y g^z \quad . \quad . \quad . \quad (1)$$

Now the dimensions of volume per second are L^3T^{-1}; the dimensions of η are $ML^{-1}T^{-1}$, see p. 205; the dimension of a is L; and the dimensions of g are

$$\frac{[\text{pressure}]}{[\text{length}]}, \text{ or } \frac{[\text{force}]}{[\text{area}][\text{length}]}, \text{ or } \frac{MLT^{-2}}{L^2 \times L}, \text{ which is } ML^{-2}T^{-2}.$$

Thus from (i), equating dimensions on both sides,

$$L^3T^{-1} \equiv (ML^{-1}T^{-1})^x L^y (ML^{-2}T^{-2})^z.$$

Equating the respective indices of M, L, T on both sides, we have

$$x+z = 0,$$

$$-x+y-2z = 3,$$

$$x+2z = 1.$$

Solving, we obtain $x = -1$, $z = 1$, $y = 4$. Hence, from (1),

$$\text{volume per second} = k\frac{a^4 g}{\eta} = k\frac{pa^4}{l\eta}.$$

We cannot obtain the numerical factor k from the method of dimensions. As shown on p. 209, the factor of $\pi/8$ enters into the formula, which is:

$$\textbf{Volume per second} = \frac{\pi \mathbf{p} \mathbf{a}^4}{8\eta \mathbf{l}} \quad . \quad . \quad . \quad (2)$$

EXAMPLE

Explain as fully as you can the phenomenon of viscosity, using the viscosity of a gas as the basis of discussion. Show by the method of dimensions how the volume of liquid flowing in unit time along a uniform tube depends on the radius of the tube, the coefficient of viscosity of the liquid, and the pressure gradient along the tube.

The water supply to a certain house consists of a horizontal water main 20 cm in diameter and 5 km long to which is joined a horizontal pipe 15 mm in diameter and 10 m long leading into the house. When water is being drawn by this house only, what fraction of the total pressure drop along the pipe appears between the ends of the narrow pipe? Assume that the rate of flow of the water is very small. (*O. & C.*)

$$\text{Volume per second} = \frac{\pi p a^4}{8\eta l}, \text{ with usual notation.}$$

$$\text{Thus volume per second} = \frac{\pi p_1 \,.\, 0{\cdot}1^4}{8\eta \,.\, 5\times 10^3} = \frac{\pi p_2 \,.\, 0{\cdot}0075^4}{8\eta \,.\, 10}$$

where p_1, p_2 are the respective pressures in the two pipes, since the volume per second is the same.

$$\therefore \frac{p_1}{p_2} = \frac{0{\cdot}0075^4}{0{\cdot}1} \times \frac{5\times 10^3}{10} = \frac{1}{63} \text{ (approx.)}.$$

$$\therefore p_1 = \frac{1}{64} \times \text{total pressure} = 0{\cdot}016 \times \text{total pressure}.$$

Turbulent Motion

Poiseuille's formula holds as long as the velocity of each layer of the liquid is parallel to the axis of the pipe and the flow pattern has been developed. As the pressure difference between the ends of the pipe is increased, a critical velocity is reached at some stage, and the motion of the liquid changes from an orderly to a *turbulent* one. Poiseuille's formula does not apply to turbulent motion.

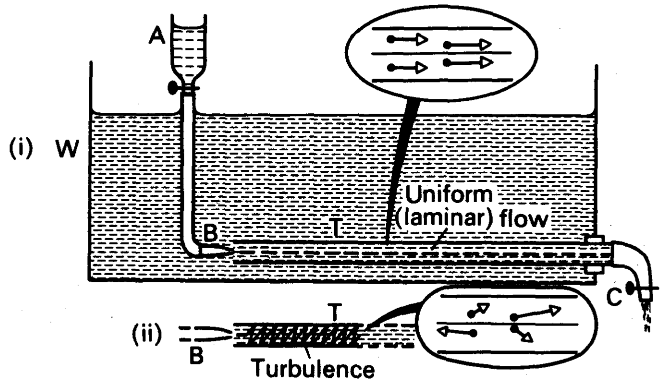

FIG. 7.4 Laminar and turbulent flow

The onset of turbulence was first demonstrated by O. REYNOLDS in 1883, and was shown by placing a horizontal tube T, about 0·5 cm in diameter, at the bottom of a tank W of water, Fig. 7.4 (i). The flow of water along T is controlled by a clip C on rubber tubing connected to T. A drawn-out glass jet B, attached to a reservoir A containing coloured water, is placed at one end of T, and at low velocities of flow a thin coloured stream of water is observed flowing along the middle of T. As the rate of flow of the water along T is increased, a stage is reached when the colouring in T begins to spread out and fill the whole of the tube, Fig. 7.4 (ii). The critical velocity has now been exceeded, and turbulence has begun.

Fig. 7.4 shows diagrammatically in inset: (i) laminar or uniform flow—here particles of liquid at the same distance from the axis always have equal velocities directed parallel to the axis, (ii) turbulence—here particles at the same distance from the axis have different velocities, and these vary in magnitude and direction with time.

Analogy with Ohm's Law

For orderly flow along a pipe, Poiseuille's formula in equation (2) states:

$$\text{Volume per second flowing} = \frac{\pi p a^4}{8\eta l},$$

$$= \frac{p \times \pi a^2}{8\pi\eta \times \dfrac{l}{\pi a^2}}.$$

Now $p \times \pi a^2$ = excess pressure × area of cross-section of liquid = excess force F on liquid, and $l/\pi a^2 = l/A$, where A is the area of cross-section.

$$\therefore \text{ volume per second flowing} = \frac{F}{8\pi\eta \times \dfrac{l}{A}} \quad . \quad . \quad \text{(i)}$$

The volume of liquid per second is analogous to electric current (I) if we compare the case of electricity flowing along a conductor, and the excess force F is analogous to the potential difference (V) along the conductor. Also, the resistance R of the conductor $= \rho l/A$, where ρ is its resistivity, l is its length, and A is the cross-sectional area. Since, from Ohm's law, $I = V/R$, it follows from (i) that

$8\pi\eta$ is analogous to ρ, the resistivity;

that is, the coefficient of viscosity η is a measure of the 'resistivity' of a liquid in orderly flow.

Proof of Poiseuille's Formula. Suppose a pipe of radius a has a liquid flowing steadily along it. Consider a cylinder of the liquid of radius r having the same axis as the pipe, where r is less than a. Then the force on this cylinder due to the excess pressure $p = p \times \pi r^2$. We can imagine the cylinder to be made up of cylindrical *shells*; the force on the cylinder due to viscosity is the algebraic sum of the viscous forces on these shells. The force on one shell is given by $\eta A dv/dr$, where dv/dr is the corresponding velocity gradient and A is the surface area of the shell. And although dv/dr changes as we proceed from the narrowest shell outwards, the forces on the neighbouring shells cancel each other out, by the law of action and reaction, leaving a net force of $\eta A dv/dr$, where dv/dr is the velocity gradient at the surface of the cylinder. The viscous force on the cylinder, and the force on it due to the excess pressure p, are together zero since there is no acceleration of the liquid, i.e., we have orderly or laminar flow.

$$\therefore\ \eta A\frac{dv}{dr}+\pi r^2 p = 0.$$

$$\therefore\ \eta . 2\pi r l\frac{dv}{dr}+\pi r^2 p = 0, \text{ since } A = 2\pi r l.$$

$$\therefore\ \frac{dv}{dr} = -\frac{pr}{2\eta l}.$$

$$\therefore\ v = -\frac{p}{4\eta l}r^2+c,$$

where c is a constant. Since $v = 0$ when $r = a$, at the surface of the tube, $c = pa^2/4\eta l$.

$$\therefore\ v = \frac{p}{4\eta l}(a^2-r^2) \qquad . \quad . \quad . \quad . \quad . \qquad \text{(i)}$$

Consider a cylindrical shell of the liquid between radii r and $(r+\delta r)$. The liquid in this shell has a velocity v given by the expression in (i), and the volume per second of liquid flowing along this shell $= v \times$ cross-sectional area of shell, since v is the distance travelled in one second, $= v \times 2\pi r . \delta r$.

$$\therefore \text{ total volume of liquid per second along tube} = \int_0^a v . 2\pi r . dr$$

$$= \int_0^a \frac{p}{4\eta l}(a^2-r^2) . 2\pi r . dr$$

$$= \frac{\pi p a^4}{8\eta l}.$$

Determination of Viscosity by Poiseuille's Formula

The viscosity of a liquid such as water can be measured by connecting one end of a capillary tube T to a constant pressure apparatus A, which provides a *steady* flow of liquid, Fig. 7.5. By means of a beaker B

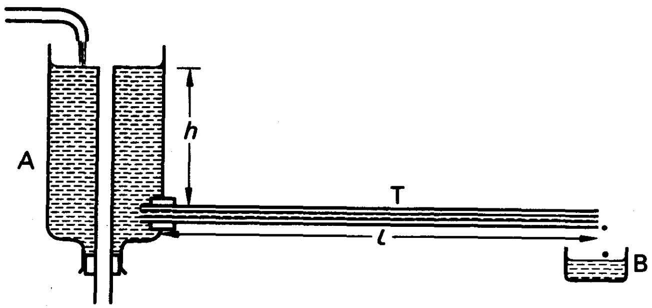

FIG. 7.5 Absolute measurement of viscosity

and a stop-clock, the volume of water per second flowing through the tube can be measured. The pressure difference between the ends of T is $h\rho g$, where h is the pressure head, ρ is the density of the liquid, and g is 9·8 m s^{-2}.

$$\therefore \text{ volume per second} = \frac{\pi p a^4}{8\eta l} = \frac{\pi h\rho g a^4}{8\eta l},$$

where l is the length of T and a is its radius. The radius of the tube can be measured by means of a mercury thread or by a microscope. The coefficient of viscosity η can then be calculated, since all the other quantities in the above equation are known.

Comparison of Viscosities. Ostwald Viscometer

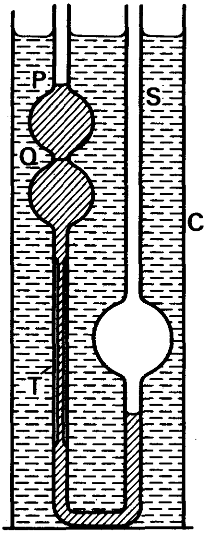

FIG. 7.6
Ostwald viscometer

An Ostwald viscometer, which contains a vertical capillary tube T, is widely used for comparing the viscosities of two liquids, Fig. 7.6. The liquid is introduced at S, drawn by suction above P, and the time t_1 taken for the liquid level to fall between the fixed marks P, Q is observed. The experiment is then repeated with the *same volume* of a second liquid, and the time t_2 for the liquid level to fall from P to Q is noted.

Suppose the liquids have respective densities ρ_1, ρ_2. Then, since the average head h of liquid forcing it through T is the same in each case, the pressure excess between the ends of T = $h\rho_1 g$, $h\rho_2 g$ respectively. If the volume between the marks P, Q is V, then, from Poiseuille's formula, we have

$$\frac{V}{t_1} = \frac{\pi(h\rho_1 g)a^4}{8\eta_1 l} \qquad \text{(i),}$$

where a is the radius of T, η_1 is the coefficient of viscosity of the liquid, and l is the length of T. Similarly, for the second liquid,

$$\frac{V}{t_2} = \frac{\pi(h\rho_2 g)a^4}{8\eta_2 l} \qquad \text{(ii)}$$

Dividing (ii) by (i),

$$\therefore \frac{t_1}{t_2} = \frac{\eta_1 \rho}{\eta_2 \rho_1}.$$

$$\therefore \frac{\eta_1}{\eta_2} = \frac{t_1}{t_2} \cdot \frac{\rho_1}{\rho_2} \qquad \text{(iii)}$$

Thus knowing t_1, t_2 and the densities ρ_1, ρ_2, the coefficients of viscosity can be compared. Further, if a pure liquid of a known viscosity is used, the viscometer can be used to measure the coefficient of viscosity of a liquid. Since the viscosity varies with temperature, the viscometer should be used in a cylinder C and surrounded by water at a constant temperature, Fig. 7.6. The arrangement can then also be used to investigate the variation of viscosity with temperature. In very accurate work a small correction is required in equation (iii). BARR, an authority on viscosity, estimates that nearly 90% of petroleum oil is tested by an Ostwald viscometer.

Experiment shows that the viscosity coefficient of a liquid diminishes as its temperature rises. Thus for water, η at 15°C is $1{\cdot}1 \times 10^{-3}$ N s m^{-2}, at 30°C it is $0{\cdot}8 \times 10^{-3}$ N s m^{-2} and at 50°C it is $0{\cdot}6 \times 10^{-3}$ N s m^{-2}. Lubricating oils for motor engines which have the same coefficient of viscosity in summer and winter are known as 'viscostatic' oils.

Stokes' Law. Terminal Velocity

When a small object, such as a steel ball-bearing, is dropped into a viscous liquid like glycerine it accelerates at first, but its velocity soon reaches a steady value known as the *terminal velocity*. In this case the viscous force acting upwards, and the upthrust due to the liquid on the object, are together equal to its weight acting downwards, so that the resultant force on the object is zero. An object dropped from an aeroplane at first increases its speed v, but soon reaches its terminal speed. Fig. 7.7 shows that variation of v with time as the terminal velocity v_0 is reached.

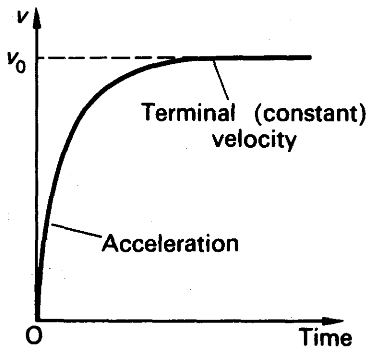

FIG. 7.7 Motion of falling sphere

Suppose a sphere of radius a is dropped into a viscous liquid of coefficient of viscosity η, and its velocity at an instant is v. The frictional force, F, can be partly found by the method of dimensions. Thus suppose $F = ka^x\eta^y v^z$, where k is a constant. The dimensions of F are MLT^{-2}; the dimension of a is L; the dimensions of η are $ML^{-1}T^{-1}$; and the dimensions of v are LT^{-1}.

$$\therefore\ MLT^{-2} \equiv L^x \times (ML^{-1}T^{-1})^y \times (LT^{-1})^z.$$

Equating indices of M, L, T on both sides,

$$\therefore\ y = 1,$$
$$x - y + z = 1,$$
$$-y - z = -2.$$

Hence $z = 1$, $x = 1$, $y = 1$. Consequently $F = k\eta av$. In 1850 STOKES showed mathematically that the constant k was 6π, and he arrived at the formula

$$F = 6\pi a\eta v \quad . \quad . \quad . \quad . \quad (1)$$

Comparison of Viscosities of Viscous Liquids

Stokes' formula can be used to compare the coefficients of viscosity of very viscous liquids such as glycerine or treacle. A tall glass vessel G is filled with the liquid, and a small ball-bearing P is dropped gently into the liquid so that it falls along the axis of G, Fig. 7.8. Towards the middle of the liquid P reaches its terminal velocity v_0, which is measured by timing its fall through a distance AB or BC.

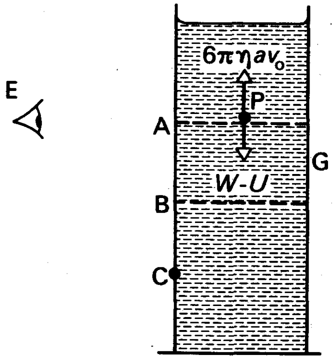

FIG. 7.8 Stokes' law

The upthrust, U, on P due to the liquid $= 4\pi a^3\sigma g/3$, where a is the radius of P and σ is the density of the liquid. The weight, W, of P is $4\pi a^3\rho g/3$, where ρ is density of the bearing's material. The net downward force is thus $4\pi a^3 g(\rho-\sigma)/3$. When the opposing frictional force grows to this magnitude, the resultant force on the bearing is zero. Thus for the terminal velocity v_0, we have

$$6\pi\eta a v_0 = \tfrac{4}{3}\pi a^3 g(\rho-\sigma),$$

$$\therefore \eta = \frac{2ga^2(\rho-\sigma)}{9v_0} \quad . \quad . \quad . \quad . \quad \text{(i)}$$

When the experiment is repeated with a liquid of coefficient of viscosity η_1 and density σ_1, using the same ball-bearing, then

$$\eta_1 = \frac{2ga^2(\rho-\sigma_1)}{9v_1} \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

where v_1 is the new terminal velocity. Dividing (i) by (ii),

$$\therefore \frac{\eta}{\eta_1} = \frac{v_1(\rho-\sigma)}{v_0(\rho-\sigma_1)} \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

Thus knowing v_1, v, ρ, σ_1, σ, the coefficients of viscosity can be compared. In very accurate work a correction to (iii) is required for the effect of the walls of the vessel containing the liquid.

Molecular theory of viscosity

Viscous forces are detected in gases as well as in liquids. Thus if a disc is spun round in a gas close to a suspended stationary disc, the latter rotates in the same direction. The gas hence transmits frictional forces. The flow of gas through pipes, particularly in long pipes as in transmission of natural gas from the North Sea area, is affected by the viscosity of the gas.

The viscosity of *gases* is explained by the transfer of momentum which takes place between neighbouring layers of the gas as it flows in a particular direction. Fast-moving molecules in a layer X cross with their own velocity to a layer Y say where molecules are moving with a slower velocity. Fig. 7.9. Molecules in Y likewise move to X. The net

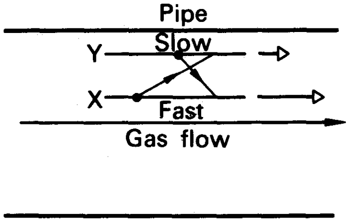

FIG. 7.9 Viscosity of gas–momentum effect

effect is an increase in momentum in Y and a corresponding decrease in X, although on the average the total number of molecules in the two layers is unchanged. Thus the layer Y speeds up and the layer X slows down, that is, a *force* acts on the layers of the gas while they move. This is the viscous force. We consider the movement of molecules in more detail shortly.

Although there is transfer of momentum as in the gas, the viscosity of a *liquid* is mainly due to the molecular attraction between molecules in neighbouring layers. Energy is needed to drag one layer over the other against the force of attraction. Thus a shear stress is required to make the liquid move in laminar flow.

EXERCISE 7

What are the missing words in the statements 1–6?

1. The coefficient of dynamic (kinetic) friction is the ratio . . .

2. The coefficient of friction between two given surfaces is . . . of the area in contact.

3. In orderly or laminar flow of liquids in a pipe, the volume per second flowing past any section is given by the formula . . .

4. The dimensions of coefficient of viscosity are . . .

5. When a small sphere of radius a falls through a liquid with a constant velocity v, the frictional force is given by the formula . . .

6. In comparing the viscosities of water and alcohol by an Ostwald viscometer, the same liquid . . . must used.

Which of the following answers, A, B, C, D or E, do you consider is the correct one in the statements 7–10?

7. In orderly or laminar flow of a liquid through a pipe, *A* tensile forces act on the layers and the volume per second V is proportional to the pressure at one end, *B* shear forces act on the layers and V is proportional to the pressure at one end, *C* shear forces act on the layers and V is proportional to the pressure difference between the ends, *D* bulk forces act throughout the liquid, *E* V is directly proportional to a^4 and to the coefficient of viscosity.

8. When a small steel sphere is dropped gently down the axis of a wide jar of glycerine, the sphere *A* travels with constant velocity throughout its motion, *B* accelerates at first and then reaches a constant velocity, *C* decelerates at first and then reaches a constant velocity, *D* accelerates throughout its motion, *E* slowly comes to rest.

9. When a gas flows steadily along a pipe, the viscous forces in it are due to *A* transfer of energy from one layer to another, *B* the uniform speed of the molecules, *C* the varying density along the pipe, *D* the transfer of momentum from one layer to another, *E* the varying pressure at a given section of the pipe.

10. A pipe P has twice the diameter of a pipe Q, and P has a liquid X flowing along it which has twice the viscosity of a liquid Y flowing through Q. If the flow is orderly or laminar in each, and the volume per second in P and Q is the same, the pressure difference at the ends of P compared to that of Q is *A* 1:8, *B* 1:4, *C* 8:1, *D* 4:1, *E* 1:1.

Solid Friction

11. State the laws of solid friction. Describe an experiment to determine the coefficient of dynamic (or sliding) friction between two surfaces.

A horizontal circular turntable rotates about its centre at the uniform rate of 120 revolutions per minute. Find the greatest distance from the centre at which a small body will remain stationary relative to the turntable, if the coefficient of static friction between the turntable and the body is 0·80. (*L.*)

12. State (*a*) the laws of solid friction, (*b*) the triangle law for forces in equilibrium. Describe an experiment to determine the coefficient of sliding (dynamic) friction between two wooden surfaces.

A block of wood of mass 150 g rests on an inclined plane. If the coefficient of static friction between the surfaces in contact is 0·30, find (*a*) the greatest angle to which the plane may be tilted without the block slipping, (*b*) the force parallel to the plane necessary to prevent slipping when the angle of the plane with the horizontal is 30°, showing that this direction of the force is the one for which the force required to prevent slipping is a minimum. (*L.*)

13. Distinguish between *static* and *sliding* (kinetic) friction and define the *coefficient of sliding friction.*

How would you investigate the laws of sliding friction between wood and iron?

An iron block, of mass 10 kg, rests on a wooden plane inclined at 30° to the horizontal. It is found that the least force parallel to the plane which causes the block to slide *up* the plane is 10 kgf. Calculate the coefficient of sliding friction between wood and iron. (*N.*)

14. Give an account of the factors which determine the force of friction (i) between solids, (ii) in liquids.

A block of mass 12 kg is drawn along a horizontal surface by a steadily applied force of 4 kg weight acting in the direction of motion. Find the kinetic energy acquired by the block at the end of 10 seconds and compare it with the total work done on the block in the same time. (Coefficient of friction = 0·28.) (*L.*)

15. State the laws of solid friction.

Describe experiments to verify these laws, and to determine the coefficient of static friction, for two wooden surfaces.

A small coin is placed on a gramophone turntable at a distance of 7·0 cm from the axis of rotation. When the rate of rotation is gradually increased from zero the coin begins to slide outwards when the rate reaches 60 revolutions per minute. Calculate the rate of rotation for which sliding would commence if (*a*) the coin were placed 12·0 cm from the axis, (*b*) the coin were placed in the original position with another similar coin stuck on top of it. (*L*

16. Define *coefficient of sliding friction, coefficient of viscosity*. Contrast the laws of solid friction with those which govern the flow of liquids through tubes.

Sketch the apparatus you would employ to determine the coefficient of sliding friction between a wood block and a board and show how you would deduce the coefficient from a suitable graph. (*L.*)

Viscosity

17. Define *coefficient of viscosity* of a fluid.

When the flow is orderly the volume V of liquid which flows in time t through a tube of radius r and length l when a pressure difference p is maintained between its ends is given by the equation $\frac{V}{t} = \frac{\pi p r^4}{8 l \eta}$ where η is the coefficient of viscosity of the liquid. Describe an experiment based on this equation *either* (*a*) to determine the value of η for a liquid, *or* (*b*) to compare the values of η for two liquids, pointing out the precautions which must be taken in the experiment chosen to obtain an accurate result.

Water flows steadily through a horizontal tube which consists of two parts joined end to end; one part is 21 cm long and has a diameter of 0·225 cm and the other is 7·0 cm long and has a diameter of 0·075 cm. If the pressure difference between the ends of the tube is 14 cm of water find the pressure difference between the ends of each part. (*L.*)

18. The dimensions of *energy*, and also those of *moment of a force* are found to be 1 in *mass*, 2 in *length* and -2 in *time*. Explain and justify this statement.

(*a*) A sphere of radius a moving through a fluid of density ρ with *high* velocity V experiences a retarding force F given by $F = k.a^x.\rho^y.V^z$, where k is a non-dimensional coefficient. Use the method of dimensions to find the values of x, y and z.

(*b*) A sphere of radius 2 cm and mass 100 g, falling vertically through air of density 1·2 kg m^{-3}, at a place where the acceleration due to gravity is 9·81 m s^{-2}, attains a steady velocity of 30 m s^{-1}. Explain why a constant velocity is reached and use the data to find the value of k in this case. (*O. & C.*)

19. Mass, length and time are *fundamental* units, whereas acceleration, force and energy are *derived* units. Explain the distinction between these two types of unit. Define each of the three derived units and apply your definition in each case to deduce its dimensions.

An incompressible fluid of viscosity η flows along a straight tube of length l and uniform circular cross-section of radius r. Provided the pressure difference p between the ends of the tube is not too great the velocity u of fluid flow along the axis of the tube is found to be directly proportional to p. Apply the method of dimensions to deduce this result assuming u depends only on r, l, η and p.

How may the viscosity of an ideal gas be accounted for by elementary molecular theory? (*O. & C.*)

20. Define *coefficient of viscosity*. Describe an experiment to compare the coefficients of viscosity of water and benzene at room temperature.

A small metal sphere is released from rest in a tall wide vessel of liquid. Discuss the forces acting on the sphere (*a*) at the moment of release, (*b*) soon after release, (*c*) after the terminal velocity has been attained.

Castor oil at 20°C has a coefficient of viscosity 2·42 N s m^{-2} and a density 940 kg m^{-3}. Calculate the terminal velocity of a steel ball of radius 2·0 mm falling under gravity in the oil, taking the density of steel as 7800 kg m^{-3}. (*L.*)

21. Define *coefficient of viscosity*. What are its dimensions?

By the method of dimensions, deduce how the rate of flow of a viscous liquid through a narrow tube depends upon the viscosity, the radius of the tube, and the pressure difference per unit length. Explain how you would use your results to compare the coefficients of viscosity of alcohol and water. (*C.*)

2 . Define *coefficient of viscosity*. For orderly flow of a given liquid through a capillary tube of length l, radius r, the volume of liquid issuing per second is proportional to pr^4/l where p is the pressure difference between the ends of the tube. How would you verify this relation experimentally for water at room temperature? How would you detect the onset of turbulence? (*N.*)

23. The viscous force acting on a small sphere of radius a moving slowly through a liquid of viscosity η with velocity v is given by the expression $6\pi\eta av$. Sketch the general shape of the velocity-time graph for a particle falling from rest through a viscous fluid, and explain the form of the graph. List the observations you would make to determine the coefficient of viscosity of the fluid from the motion of the particle.

Some particles of sand are sprinkled on to the surface of the water in a beaker filled to a depth of 10 cm. Estimate the least time for which grains of diameter 0·10 mm remain in suspension in the water, stating any assumptions made.

[Viscosity of water = $1{\cdot}1 \times 10^{-3}$ N s m^{-2}; density of sand = 2200 kg m^{-3}.] (*C.*)

24. Define *coefficient of friction* and *coefficient of viscosity*.

Describe how you would (*a*) measure the coefficient of sliding friction between iron and wood, and (*b*) compare the viscosities of water and paraffin oil. (*L.*)

PART TWO

Heat