OPTICS

chapter twenty-eight

Wave theory of light

Historical

IT has already been mentioned that light is a form of energy which stimulates our sense of vision. One of the early theories of light, about 400 B.C., suggested that particles were emitted from the eye when an object was seen. It was realised, however, that something is *entering* the eye when a sense of vision is caused, and about 1660 the great Newton proposed that particles, or corpuscles, were emitted from a luminous object. The *corpuscular theory of light* was adopted by many scientists of the day owing to the authority of Newton, but HUYGENS, an eminent Dutch scientist, proposed about 1680 that light energy travelled from one place to another by means of a wave-motion. If the *wave theory of light* was correct, light should bend round a corner, just as sound travels round a corner. The experimental evidence for the wave theory in Huygens' time was very small, and the theory was dropped for more than a century. In 1801, however, THOMAS YOUNG obtained evidence that light could produce wave effects (p. 688), and he was among the first to see clearly the close analogy between sound and light waves. As the principles of the subject became understood other experiments were carried out which showed that light could spread round corners, and Huygens' wave-theory of light was revived. Newton's corpuscular theory was rejected since it was incompatible with experimental observations (see p. 679), and the wave theory of light has played, and is still playing, an important part in the development of the subject.

In 1905 the great mathematical physicist EINSTEIN suggested that the energy in light could be carried from place to place by particles whose energy depended on the wavelength of the light. This was a return to a corpuscular theory, though it was completely different from that of Newton, as we see later. Experiments carried out at his suggestion showed that the theory was true, and the particles of light energy are known as "photons" (p. 1080). It is now considered that *either* the wave theory *or* the particle theory of light can be used in a problem on light, depending on the circumstances of the problem. In this book we shall first consider Huygens' wave theory, which was the foundation of many notable advances in the subject.

Wavefront. Rays

We have already considered the topic of waves in the Sound section

(p. 583). As we shall presently see, close analogies exist between light and sound waves.

Consider a point source of light, S, in air, and suppose that a disturbance, or wave, originates at S as a result of vibrations occurring inside the atoms of the source, and travels outwards. After a time t the wave has travelled a distance ct, where c is the velocity of light in air, and the light energy has thus reached the surface of a sphere of centre S and radius ct, Fig. 28.1. The surface of the sphere is called the *wavefront* of the light at this instant, and every point on it is vibrating "in step" or *in phase* with every other point. As time goes on the wave travels further and new wavefronts are obtained which are the surfaces of spheres of centre S.

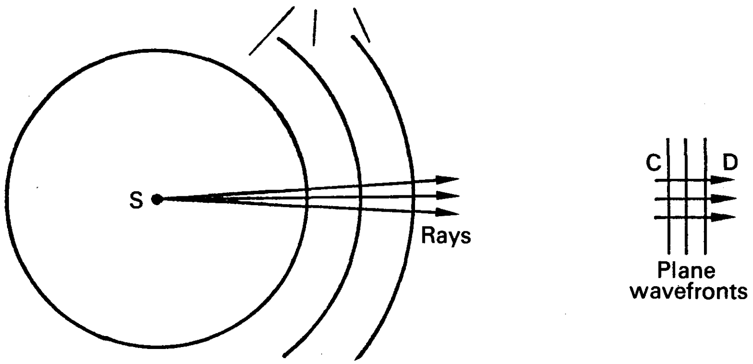

FIG. 28.1. Wavefronts and rays.

At points a long way from S, such as C or D, the wavefronts are portions of a sphere of very large radius, and the wavefronts are then substantially *plane*. Light from the sun reaches the earth in plane wavefronts because the sun is so far away; plane wavefronts also emerge from a convex lens when a point source of light is placed at its focus.

The significance of the wavefront, then, is that it shows how the light energy travels from one place in a medium to another. A *ray* is the name given to the direction along which the energy travels, and consequently a ray of light passing through a point is perpendicular to the wavefront at that point. The rays diverge near S, but they are approximately parallel a long way from S, as plane wavefronts are then obtained, Fig. 28.1.

Huygens' Construction for the New Wavefront

Suppose that the wavefront from a centre of disturbance S has reached the surface AB in a medium at some instant, Fig. 28.2. To obtain the position of the new wavefront after a further time t, Huygens postulated that *every point, A,..., C,..., E,..., B, on AB becomes a new or "secondary" centre of disturbance*. The wavelet from A then reaches the surface M of a sphere of radius vt and centre A, where v is the velocity

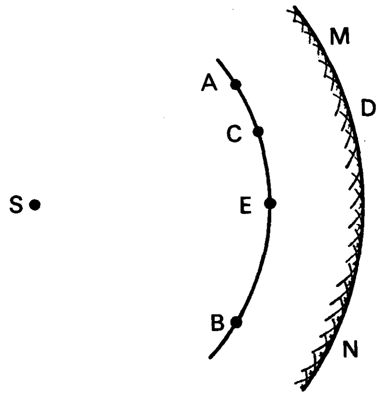

FIG. 28.2. Huygens' construction.

of light in the medium; the wavelet from C reaches the surface D of a sphere of radius *vt* and centre C; and so on for every point on AB. According to Huygens, *the new wavefront is the surface MN which touches all the wavelets from the secondary sources*; and in the case considered, it is the surface of a sphere of centre S.

In this simple example of obtaining the new wavefront, the light travels in the same medium. Huygens' construction, however, is especially valuable for deducing the new wavefront when the light travels from one medium to another, as we shall soon show.

Reflection at Plane Surface

Suppose that a beam of parallel rays between HA and LC is incident on a plane mirror, and imagine a plane wavefront AB which is normal to the rays, reaching the mirror surface, Fig. 28.3. At this instant the point A acts as a centre of disturbance. Suppose we require the new wavefront at a time corresponding to the instant when the disturbance at B reaches C. The wavelet from A reaches the surface of a sphere of radius AD at this instant; and as other points between AC on the mirror, such as P, are reached by the disturbances originating on AB, wavelets

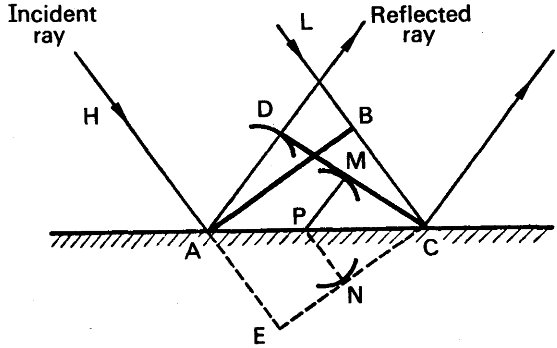

FIG. 28.3. Reflection at plane surface.

of smaller radius than AD are obtained at the instant we are considering. The new wavefront is the surface CMD which touches all the wavelets.

In the absence of the mirror, the plane wavefront AB would reach the position EC in the time considered. Thus AD = AE = BC, and PN = PM, where PN is perpendicular to EC. The triangles PMC, PNC are hence congruent, as PC is common, angles PMC, PNC are each 90°, and PN = PM. Thus angle PCM = angle PCN. But triangles ACD, AEC are also congruent. Consequently angle ACD = angle ACE = angle PCN = angle PCM, Since EC is a plane. Hence CMD is a *plane* surface.

Law of reflection. We can now deduce the law of reflection concerning the angles of incidence and reflection. From the above, it can be seen that the triangles ABC, AEC are congruent, and that triangles ADC, AEC are congruent. The triangles ABC, ADC are hence congruent, and therefore angle BAC = angle DCA. Now these are the angles made by the wavefront AB, CD respectively with the mirror surface AC. Since the incident and reflected rays, for example HA, AD, are normal to the wavefronts, these rays also make equal angles with AC. It now follows that the angles of incidence and reflection are equal.

Point Object

Consider now a point object O in front of a plane mirror M, Fig. 28.4. A spherical wave spreads out from O, and at some time the wavefront reaches ABC. In the absence of the mirror the wavefront would reach a position DEF in a time t thereafter, but every point between D and F on the mirror acts as a secondary centre of disturbance and wavelets are reflected back into the air. At the end of the time t, a surface DGC is drawn to touch all the wavelets, as shown. DGC is part of a spherical surface which advances into the air, and it appears to have come from a point I as a centre below the mirror, which is therefore a virtual image.

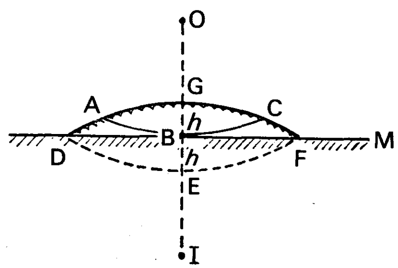

FIG. 28.4. Point object.

The sphere of which DGF is part has a chord DF. Suppose the distance from B, the midpoint of the chord, to G is h. The sphere of which DEF is part has the same chord DF, and the distance from B to E is also h. It follows, from the theorem of product of intersection of chords of a circle, that DB.BF $= h(2r - h) = h(2R - h)$, where r is the radius OE and R is the radius IG. Thus $R = r$, or IG = OE, and hence IB = OB. The image and object are thus equidistant from the mirror.

Refraction at Plane Surface

Consider a beam of parallel rays between LO and MD incident on the plane surface of a water medium from air in the direction shown, and suppose that a plane wavefront has reached the position OA at a certain

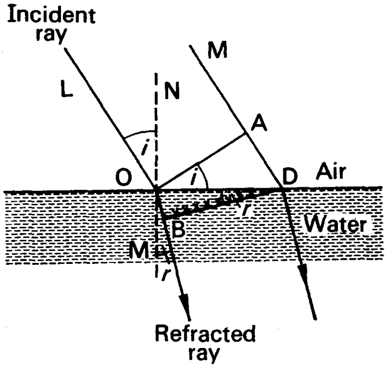

FIG. 28.5. Refraction at plane surface.

instant, Fig. 28.5. Each point between O, D becomes a new centre of disturbance as the wavefront advances to the surface of the water, and the wavefront changes in direction when the disturbance enters the liquid.

Suppose that t is the time taken by the light to travel from A to D. The disturbance from O travels a distance OB, or vt, in water in a time t, where v is the velocity of light in water. At the end of the time t, the wavefronts in the water from the other secondary centres between O, D reach the surfaces of spheres to each of which DB is a tangent. Thus DB is the new wavefront in the water, and the ray OB which is normal to the wavefront is consequently the refracted ray.

Since c is the velocity of light in air, AD $= ct$. Now

$$\frac{\sin i}{\sin r} = \frac{\sin \text{LON}}{\sin \text{BOM}} = \frac{\sin \text{AOB}}{\sin \text{ODB}}$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{\text{AD/OD}}{\text{OB/OD}} = \frac{\text{AD}}{\text{OB}} = \frac{ct}{vt} = \frac{c}{v} \quad . \quad . \quad . \quad \text{(i)}$$

But c, v are constants for the given media.

$$\therefore \quad \frac{\sin i}{\sin r} \text{ is a constant,}$$

which is Snell's law of refraction (p. 420).

It can now been seen from (i) that the refractive index, n, of a medium is given by $n = \frac{c}{v}$, where c is the velocity of light *in vacuo* and v is the velocity of light in the medium.

Newton's Corpuscular Theory of Light

Prior to the wave theory of light, Newton had proposed a corpuscular or particle theory of light. According to Newton, particles are emitted by a source of light, and they travel in a straight line until the boundary of a new medium is encountered.

In the case of *reflection at a plane surface*, Newton stated that at some very small distance from the surface M, represented by AB, the particles were acted upon by a repulsive force, which gradually diminished the component of the velocity v in the direction of the normal and then reversed it, Fig. 28.6. The *horizontal* component of the velocity remained unaltered, and hence the velocity of the particles of light as they moved away from M is again v. Since the horizontal components of the incident and reflected velocities are the same, it follows that

$$v \sin i = v \sin i' \quad . \quad . \quad . \quad . \quad \text{(i)}$$

where i' is the angle of reflection.

$$\therefore \quad \sin i = \sin i', \text{ or } i = i'$$

Thus the corpuscular theory explains the law of reflection at a plane surface.

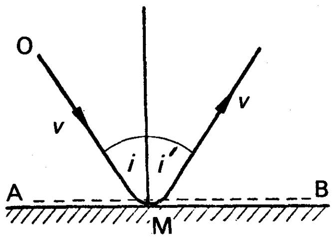

FIG. 28.6. Newton's corpuscular theory of reflection.

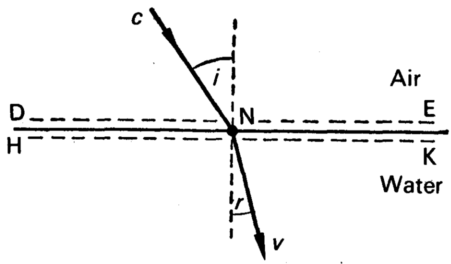

FIG. 28.7. Newton's corpuscular theory of refraction.

To explain *refraction at a plane surface* when light travels from air to a denser medium such as water, Newton stated that a force of attraction acted on the particles as they approached beyond a line DE very close to the boundary N, Fig. 28.7. The vertical component of the velocity of the particles was thus increased on entering the water, the horizontal component of the velocity remaining unaltered, and beyond a line HK close to the boundary the vertical component remained constant at its increased value. The resultant velocity, v, of the particles in the water is thus *greater* than its velocity, c, in air.

Suppose i, r are the angles of incidence and refraction respectively. Then, as the horizontal components of the velocity is unaltered.

$$c \sin i = v \sin r$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{v}{c}$$

$$\therefore \quad n = \frac{v}{c} = \text{the refractive index}$$

Since n is greater than 1, the velocity of light (v) in water is greater than the velocity (c) in air, as was stated above. This is according to Newton's corpuscular theory. On the wave theory, however, $n = \frac{c}{v}$ (see p. 679);

and hence the velocity of light (v) in water is *less* than the velocity (c) in air according to the wave theory. The corpuscular theory and wave theory are thus in conflict, and Foucault's experimental results showed that the corpuscular theory, as enunciated by Newton, could not be true (p. 559).

Dispersion

The dispersion of colours produced by a medium such as glass is due to the difference in speeds of the various colours in the medium. Thus suppose a plane wavefront AC of white light is incident in air on a plane glass surface, Fig. 28.8. In the time the light takes to travel in air from C to D, the red light from the centre of disturbance A reaches a position shown by the wavelet at R. The blue light from A reaches another position shown by the wavelet at B, since the speed of blue light in glass is less than that of red light, so that AB is less than AR. On drawing the new wavefronts DB, DR, it can be seen that the blue wavefront BD is refracted more in the glass than the red wavefront DR. The refracted blue ray is AB and the refracted red ray is AR, and hence dispersion occurs.

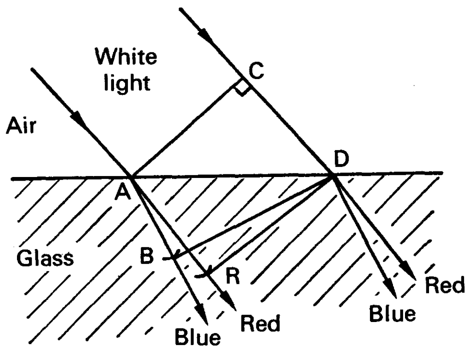

FIG. 28.8. Dispersion.

Refraction Through Prism at Minimum Deviation

Consider a wavefront HB incident on the face HB of a prism of angle A, Fig. 28.9. If the wavefront emerges along EC, then the light travels

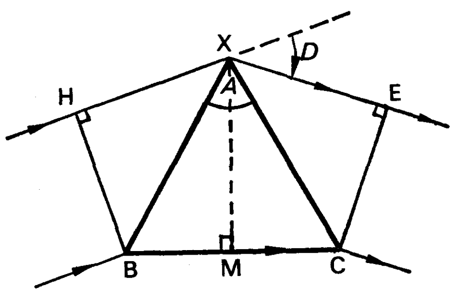

FIG. 28.9. Refraction at minimum deviation.

a distance HXE in air in the same time as the light travels a distance BC in glass.

$$\therefore \quad \frac{HX + XE}{c} = \frac{BC}{v},$$

where c is the velocity of light in air and v is the velocity in glass

$$\therefore \quad HX + XE = \frac{c}{v} BC = n\,BC \quad . \quad . \quad \text{(i)}$$

At minimum deviation, the wavefront passes symmetrically through the prism (p. 443).

$$\therefore \quad HX = XE$$

From (i),

$$\therefore \quad 2\,HX = n\,BC$$

$$\therefore \quad n = \frac{2\,HX}{BC} \quad . \quad . \quad . \quad \text{(ii)}$$

But $HX = XB \cos BXH = XB \cos\left[\frac{180° - (A + D)}{2}\right]$

$$= XB \sin\left(\frac{A + D}{2}\right),$$

and

$$BC = 2\,BM = 2\,XB \sin\frac{A}{2}$$

From (ii),

$$\therefore \quad n = \frac{\sin\left(\frac{A + D}{2}\right)}{\sin\frac{A}{2}}$$

Focal Length of Lens

The focal length of a lens (or curved spherical mirror) can also be found by wave theory. Suppose a plane wavefront AHB, parallel to

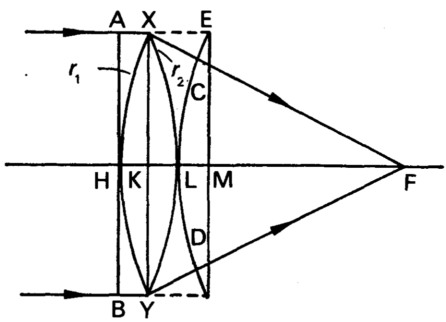

FIG. 28.10. Focal length of lens.

the principal axis, is incident on a converging lens, Fig. 28.10. After refraction the wavefront emerges in air as a converging spherical wavefront CLD of centre F, the principal focus, since the incident rays are parallel.

The time taken by the light to travel a distance AX + XE in air is equal to the time taken to travel a distance HKL in glass. Thus if c is the velocity in air and v is the velocity in glass,

$$\therefore \quad \frac{AX + XE}{c} = \frac{HKL}{v}$$

$$\therefore \quad AX + XE = \frac{c}{v} . HKL = n . HKL \qquad (i)$$

From the geometry of a circle, $AX = HK = \frac{h^2}{2r_1}$,

where XK is h and r_1 is the radius of curvature of the lens, assumed thin (see also p. 695).

Also, $$XE = KM = KL + LM = \frac{h^2}{2r_2} + \frac{h^2}{2f},$$

where r_2 is the radius of curvature of the surface XLY and FL $= f$. Substituting in (i),

$$\therefore \quad \frac{h^2}{2r_1} + \frac{h^2}{2r_2} + \frac{h^2}{2f} = n\left(\frac{h^2}{2r_1} + \frac{h^2}{2r_2}\right)$$

Simplifying, $$\therefore \quad \frac{1}{f} = (n - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right).$$

Power of a Lens

We have now to consider the effect of lenses on the *curvature* of wavefronts. A plane wavefront has obviously zero curvature and a spherical wavefront has a small curvature if the radius of the sphere is large. *The "curvature" of a spherical wavefront is defined as* $1/r$, where r is the radius of the surface which constitutes the wavefront, and hence the curvature is zero when r is infinitely large, as in the case of a plane wavefront.

When a plane wavefront is incident on a converging lens L, a spherical wavefront, S, of radius f emerges from L, where f is the focal length of the lens, Fig. 28.11 (i). Parallel rays, which are normal to the plane

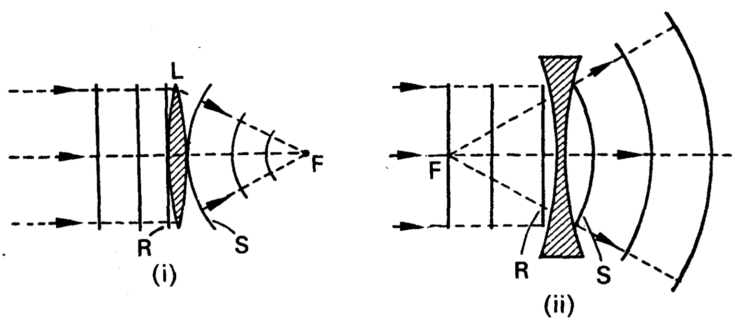

FIG. 28.11. (i). Converging lens. (ii). Diverging lens.

wavefront, are thus refracted towards F, the focus of the lens. Now the curvature of a plane wavefront is zero, and the curvature of the spherical wavefront S is $1/f$. Thus the convex lens impresses a curvature of $1/f$ on a wavefront incident on it, and $1/f$ is accordingly defined as the *converging power* of the lens.

$$\therefore \quad \text{Power } P, = \frac{1}{f} \quad . \quad . \quad . \quad . \quad (91)$$

Fig. 28.11 (ii) illustrates the effect of a *diverging* lens on a plane wavefront R. The front S emerging from the lens has a curvature opposite to S in Fig. 28.11 (i), and it appears to be diverging from a point F behind the concave lens, which is its focus. The curvature of the emerging wavefront is thus $1/f$, where f is the focal length of the lens, and the powers of the convex and concave lens are opposite in sign.

The power of a converging lens is positive, since its focal length is positive, while the power of a diverging lens is negative. Opticians use a unit of power called the *dioptre, D,* which is defined as the power of a lens of 100 cm focal length. A lens of focal length f cm has thus a power P given by

$$P = \frac{1/f}{1/100} \text{ dioptres}$$

$$\text{or} \quad P = \frac{100}{f} \text{ dioptres} \quad . \quad . \quad . \quad . \quad (92)$$

A lens of + 8 dioptres, or + $8D$, is therefore a converging lens of focal length 12·5 cm, and a lens of − $4D$ is a diverging lens of 25 cm focal length.

The Lens Equation

Suppose that an object O is placed a distance u from a converging lens, Fig. 28.12. The spherical wavefront. A from O which reaches the lens has a radius of curvature u, and hence a curvature $1/u$. Since the converging lens adds a curvature of $1/f$ to the wavefront (proved page 683), the spherical wavefront B emerging from the lens into the air has a curvature $\left(\frac{1}{u} + \frac{1}{f}\right)$. But the curvature is also given by $\frac{1}{v}$, where v is the image distance IB from the lens.

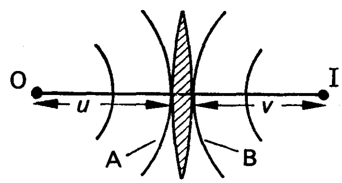

FIG. 28.12. Effect of lens on wavefront.

$$\therefore \quad \frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

It can be seen that the curvature of A is of an opposite sign to that of B; and taking this into account, the lens equation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ is obtained. A similar method can be used for a diverging lens, which is left as an exercise for the student.

EXERCISES 28

1. A parallel beam of monochromatic radiation travelling through glass is incident on the plane boundary between the glass and air. Using Huygens' principle draw diagrams (one in each case) showing successive positions of the wave fronts when the angle of incidence is (*a*) 0°, (*b*) 30°, (*c*) 60°. Indicate clearly and explain the constructions used. (The refractive index of glass for the radiation used is 1·5.) (*N.*)

2. State Snell's law of refraction. How is the law explained in terms of the wave theory of light?

An equiangular glass prism is placed in a broad beam of parallel monochromatic light as shown, the face AB of the prism being perpendicular to the direction of the incident light, Fig. 28A. By sketching typical rays, show that most of the light which is refracted by the prism emerges as two beams of parallel light deviated respectively through $\pm\theta$, and calculate the value of θ if the refractive index of the glass is 1·5. Why does not all the light falling on the prism emerge in this way?

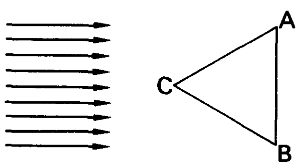

FIG. 28A.

The prism is turned round so that the light is incident normally on the face AB. Describe as fully as you can what happens to most of the light with the prism in this position. (*O. & C.*)

3. A plane wave-front of monochromatic light is incident normally on one face of a glass prism, of refracting angle 30°, and is transmitted. Using Huygens' construction trace the course of the wave-front. Explain your diagram and find the angle through which the wave-front is deviated. (Refractive index of glass = 1·5.) (*N.*)

4. State *Snell's law of refraction* and define *refractive index.*

Show how refraction of light at a plane interface can be explained on the basis of the wave theory of light.

Light travelling through a pool of water in a parallel beam is incident on the horizontal surface. Its speed in water is $2{\cdot}2 \times 10^8$ m s^{-1}. Calculate the maximum angle which the beam can make with the vertical if light is to escape into the air where its speed is $3{\cdot}0 \times 10^8$ m s^{-1}.

At this angle in water, how will the path of the beam be affected if a thick layer of oil, of refractive index 1·5, is floated on to the surface of the water? (*O. & C.*)

5. How did Huygens explain the reflection of light on the wave theory? Using Huygens' conceptions, show that a series of light waves diverging from a point source will, after reflection at a plane mirror, appear to be diverging from a second point, and calculate its position. (*C.*)

6. Explain how the *corpuscular theory* of Newton accounted for the laws of reflection and refraction. What experimental evidence showed that the theory was incorrect?

7. What is Huygens' principle?

Draw and explain diagrams which show the positions of a light wave-front at successive equal time intervals when (*a*) parallel light is reflected from a plane mirror, the angle of incidence being about 60°, (*b*) monochromatic light originating from a small source in water is transmitted through the surface of the water into the air.

Describe an experiment, and add the necessary theoretical explanation, to show that in air the wavelength of blue light is less than that of red light. (*N.*)

8. Using Huygens' concept of secondary wavelets show that a plane wave of monochromatic light incident obliquely on a plane surface separating air from glass may be refracted and proceed as a plane wave. Establish the physical significance of the refractive index of the glass.

In what circumstances does dispersion of light occur? How is it accounted for by the wave theory?

If the wavelength of yellow light in air is $6{\cdot}0 \times 10^{-7}$ m, what is its wavelength in glass of refractive index 1·5? (*N.*)

9. Describe fully a method for measuring the velocity of light in air. Explain, on the basis of the wave theory, the relation between the refractive index of a medium relative to air and the velocity of light. (*L.*)