chapter twenty-nine

Interference, diffraction, polarisation of light

INTERFERENCE OF LIGHT

THE beautiful colours seen in thin films of oil in the road, or in soap bubbles, are due to a phenomenon in light called *interference*. Newton discovered that circular coloured rings were obtained when white light illuminated a convex lens of large radius of curvature placed on a sheet of plane glass (p. 693), which is another example of interference. As we saw in Sound, interference can be used to measure the wavelength of sound waves (p. 617). By a similar method the phenomenon can be used to measure the wavelengths of different colours of light. Interference of light has also many applications in industry.

The essential conditions, and features, of interference phenomena have already been discussed in connection with sound waves. As there is an exact analogy between the interference of sound and light waves we can do no better than recapitulate here the results already obtained on pp. 616–617:

1. Permanent interference between two sources of light can only take place if they are *coherent* sources, i.e., they must have the same frequency and be always in phase with each other or have a constant phase difference. (This implies that the two sources of light must have the same colour.)

2. If the coherent monochromatic light sources are P, Q, a bright light is observed at B if the path-difference, QB—PB, is a whole number of wavelengths, Fig. 29.1. (This corresponds to the case of a loud sound

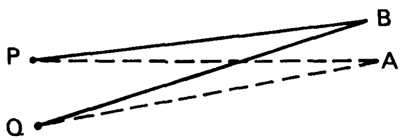

FIG. 29.1. Interference of light.

heard at B if P, Q were two coherent sources of sound.) A bright light is observed at A if PA = QA, in which case the path-difference is zero.

3. If the path-difference is an odd number of half wavelengths, darkness is observed at the point under consideration. (This corresponds to silence at the point in the case of two coherent sound sources.)

Young's Experiment

From the preceding, it can be understood that two conditions are essential to obtain an interference phenomenon. (i) Two coherent sources of light must be produced, (ii) the coherent sources must be very close to each other as the wavelength of light is very small, otherwise the bright and dark pattern in front of the sources tend to be too fine to see and no interference pattern is obtained.

One of the first demonstrations of the interference of light waves was given by YOUNG in 1801. He placed a source, S, of monochromatic light in front of a narrow slit C, and arranged two very narrow slits A, B, close to each other, in front of C. Much to his delight, Young observed bright and dark bands on either side of O on a screen T, where O is on the perpendicular bisector of AB, Fig. 29.2.

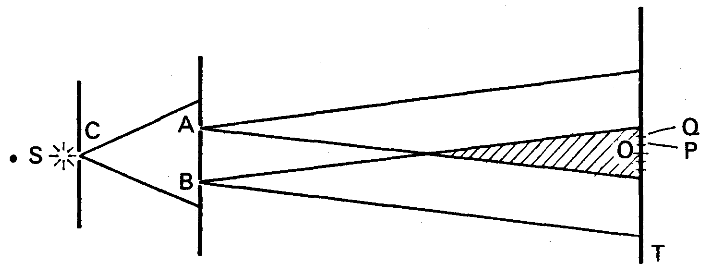

FIG. 29.2. Young's experiment.

Young's observations can be explained by considering the light from S illuminating the two slits A, B. Since the light diverging from A has exactly the same frequency as, and is always in phase with, the light diverging from B, A and B act as two close coherent sources. Interference thus takes place in the shaded region, where the light beams overlap, Fig. 29.2. As AO = OB, a bright band is obtained at O. At a point P close to O, such that BP − AP = $\lambda/2$, where λ is the wavelength of the light from S, a dark band is obtained. At a point Q such that BQ − AQ = λ, a bright band is obtained; and so on for either side of O. Young demonstrated that the bands were due to interference by covering A or B, when the bands disappeared.

Separation of Bands

Suppose P is the position of the mth bright band, so that BP − AP = $m\lambda$, Fig. 29.3. Let OP = x_m = distance from P to O, the centre of the band system, where MO is the perpendicular bisector of AB. If a length PN equal to PA is described on PB, then BN = BP − AP = $m\lambda$. Now in practice AB is very small, and PM is very much larger than AB. Thus AN meets PM practically at right angles. It then follows that

$$\text{angle PMO} = \text{angle BAN} = \theta \text{ say.}$$

From triangle BAN, $$\sin\theta = \frac{\text{BN}}{\text{AB}} = \frac{m\lambda}{a},$$

where a = AB = the distance between the slits. From triangle PMO,

$$\tan\theta = \frac{PO}{MO} = \frac{x_m}{D},$$

where D = MO = the distance from the screen to the slits. Since θ is very small, $\tan\theta = \sin\theta$.

$$\therefore \quad \frac{x_m}{D} = \frac{m\lambda}{a}$$

$$\therefore \quad x_m = \frac{mD\lambda}{a}$$

If Q is the neighbouring or $(m-1)$th bright band, it follows that

$$OQ = x_{m-1} = \frac{(m-1)D\lambda}{a}$$

$$\therefore \quad \text{separation } y \text{ between successive bands} = x_m - x_{m-1} = \frac{\lambda D}{a} \quad \text{(i)}$$

$$\therefore \quad \lambda = \frac{ay}{D} \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

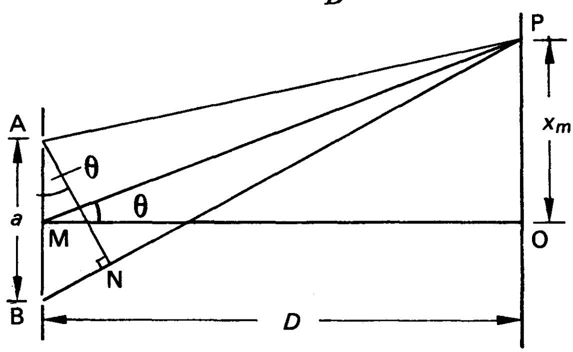

FIG. 29.3. Theory of Young's experiment (exaggerated).

Measurement of Wavelength by Young's Interference Bands

A laboratory experiment to measure wavelength by Young's interference bands is shown in Fig. 29.4. Light from a small filament lamp is focused by a lens on to a narrow slit S, such as that in the collimator of a spectrometer. Two narrow slits A, B, about a millimetre apart, are placed a short distance in front of S, and the light coming from A, B is viewed in a low-powered microscope or eyepiece M about two metres away. Some coloured interference bands are then observed by M. A red and then a blue filter, F, placed in front of the slits, produces red and then blue bands. Observation shows that the separation of the red bands is more than that of the blue bands. Now $\lambda = ay/D$, from (ii), where y is the separation of the bands. It follows that the wavelength of red light is *longer* than that of blue light.

An approximate value of the wavelength of red or blue light can be found by placing a Perspex rule R in front of the eyepiece and moving it until the graduations are clearly seen, Fig. 29.4. The average distance, y, between the bands is then measured on R. The distance a between

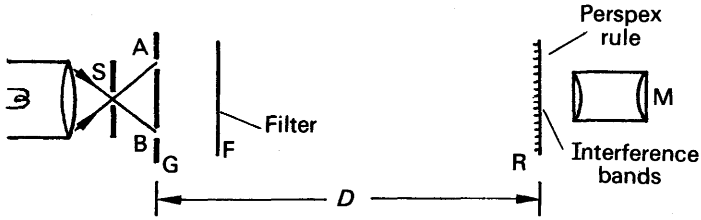

FIG. 29.4. Laboratory experiment on Young's interference bands.

the slits can be found by magnifying the distance by a convex lens, or by using a travelling microscope. The distance D from the slits to the Perspex rule, where the bands are formed, is measured with a metre rule. The wavelength λ can then be calculated from $\lambda = ay/D$, and is of the order 6×10^{-5} cm. Further details of the experiment can be obtained from *Advanced Level Practical Physics* by Nelkon and Ogborn (Heinemann).

The wavelengths of the extreme colours of the visible spectrum vary with the observer. This may be 4×10^{-5} cm for violet and 7×10^{-5} cm for red; an "average" value for visible light is $5{\cdot}5 \times 10^{-5}$ cm, which is a wavelength in the green.

Appearance of Young's Interference Bands

The experiment just outlined can also be used to demonstrate the following points:—

1. If the source slit S is moved nearer the double slits the separation of the bands is unaffected but their intensity increases. This can be seen from the formula y (separation) $= \lambda D/a$, since D and a are constant.

2. If the distance apart a of the slits is diminished, keeping S fixed, the separation of the bands increases. This follows from $y = \lambda D/a$.

3. If the source slit S is widened the bands gradually disappear. The slit S is then equivalent to a large number of narrow slits, each producing its own band system at different places. The bright and dark bands of different systems therefore overlap, giving rise to uniform illumination. It can be shown that, to produce interference bands which are recognisable, the slit width of S must be less than $\lambda D'/a$, where D' is the distance of S from the two slits A, B.

4. If one of the slits, A or B, is covered up, the bands disappear.

5. If white light is used the central band is white, and the bands either side are coloured. Blue is the colour nearer to the central band and red is farther away. The path difference to a point O on the perpendicular bisector of the two slits A, B is zero for all colours, and consequently each colour produces a bright band here. As they overlap, a white band is formed. Farther away from O, in a direction parallel to the slits, the shortest visible wavelengths, blue, produce a bright band first.

Fresnel's Biprism Experiment

Fresnel used a biprism R which had a very large angle of nearly 180°, and placed a narrow slit S, illuminated by monochromatic light, in

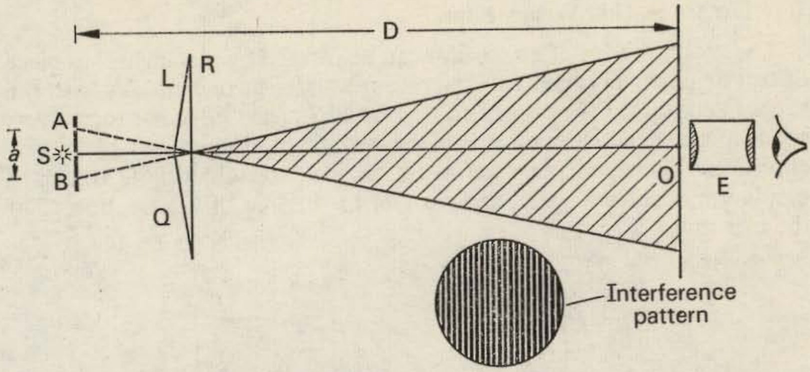

FIG. 29.5. Fresnel's biprism experiment (not to scale).

front of it so that the refracting edge was parallel to the slit, Fig. 29.5. The light emerging after refraction from the two halves, L, Q, of the prism can be considered to come from two sources, A, B, which are the virtual images of the slit S in L, Q respectively. Thus A, B are coherent sources; further, as R has a very large obtuse angle, A and B are close together. Thus an interference pattern is observed in the region of O where the emergent light from the two sources overlap, as shown by the shaded portion of Fig. 29.5, and bright and dark bands can be seen through an eyepiece E at O directed towards R, Fig. 29.6. By using cross-wires, and moving the eyepiece by a screw arrangement, the distance y between successive bright bands can be measured. Now it was shown on p. 689 that $\lambda = ay/D$, where a is the distance between A, B and D is the distance of the source slit from the eyepiece. The distance D is measured with a metre rule. The distance a can be found by moving a convex lens between the fixed biprism and eyepiece until a magnified image of the two slits A, B is seen clearly, and the magnified distance b between them is measured. The magnification m is (image distance $\div$ object distance) for the lens, and a can be calculated from $a = b/m$. Knowing a, y, D, the wavelength λ can be determined.

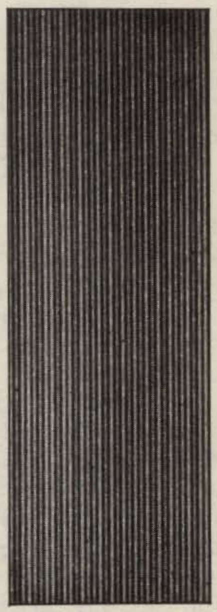

FIG. 29.6 Fresnel's biprism interference bands (magnified).

If A is the large angle, nearly 180°, of the biprism, each of the small base angles is $(180° - A)/2$, or $90° - A/2$. The small deviation d in radians of light from the slit S is $(n - 1)\,\theta$, where θ is the magnitude of the base angle in radians (p. 457), and hence the distance A, B between the virtual images of the slit $= 2td = 2t(n - 1)\,\theta$, where t is the distance from S to the biprism.

Interference in Thin Wedge Films

A very thin wedge of an air film can be formed by placing a thin piece of foil or paper between two microscope slides at one end Y, with the slides in contact at the other end X, Fig. 29.7. The wedge has then a very small angle θ, as shown. When the air-film is illuminated by monochromatic light from an extended source S, straight bright and dark bands are observed which are parallel to the line of intersection X of the two slides.

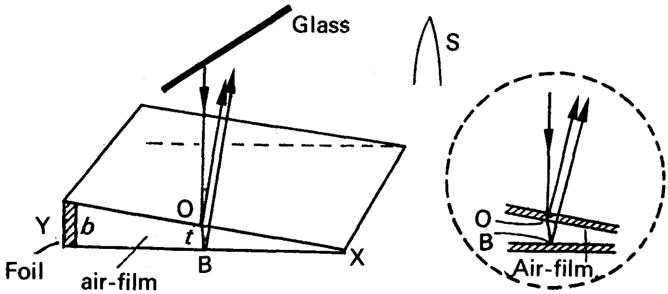

FIG. 29.7. Thin wedge film.

The light reflected down towards the wedge is partially reflected upwards from the lower surface O of the top slide. The remainder of the light passes through the slide and some is reflected upward from the top surface B of the lower slide. The two trains of waves are coherent, since both have originated from the same centre of disturbance at O, and they produce an interference phenomenon if brought together by the eye or in an eyepiece. Their path difference is $2t$, where t is the small thickness of the air-film at O. At X, where the path difference is apparently zero, we would expect a bright band. But a *dark* band is observed at X. This is due to a phase change of 180°, equivalent to an extra path difference of $\lambda/2$, which occurs when a wave is reflected at a denser medium. See pp. 641, 694. The optical path difference between the two coherent beams is thus actually $2t + \lambda/2$, and hence, if the beams are brought together to interfere, a bright band is obtained when $2t + \lambda/2 = m\lambda$, or $2t = (m - \frac{1}{2})\lambda$. A dark band is obtained at a thickness t given by $2t = m\lambda$.

The bands are located at the air-wedge film, and the eye or microscope must be focused here to see them. The appearance of a band is the contour of all points in the air-wedge film where the optical path difference is the same. If the wedge surfaces make perfect optical contact at one edge, the bands are straight lines parallel to the line of intersection of the surfaces. If the glass surfaces are uneven, and the contact at one edge is not regular, the bands are not perfectly straight. A particular band still shows the locus of all points in the air-wedge which have the same optical path difference in the air-film.

In *transmitted light*, the appearance of the bands are complementary to those seen by reflected light, from the law of conservation of energy. The bright bands thus correspond in position to the dark bands seen by reflected light, and the band where the surfaces touch is now bright instead of dark.

Thickness of Thin Foil. Expansion of Crystal

If there is a bright band at Y at the edge of the foil, Fig. 29.7, the thickness b of the foil is given by $2b = (m + \frac{1}{2})\lambda$, where m is the number of bright bands between X and Y. If there is a dark band at Y, then $2b = m\lambda$. Thus by counting m, the thickness b can be found. The small angle θ of the wedge is given by b/a, where a is the distance XY, and by measuring a with a travelling microscope focused on the air-film, θ can be found. If a liquid wedge is formed between the plates, the optical path difference becomes $2nt$, where the air thickness is t, n being the refractive index of the liquid. An optical path difference of λ now occurs for a change in t which is n times less than in the case of the air-wedge. The spacing of the bright and dark bands is thus n times closer than for air, and measurement of the relative spacing enables n to be found.

The coefficient of expansion of a crystal can be found by forming an air-wedge of small angle between a fixed horizontal glass plate and the upper surface of the crystal, and illuminating the wedge by monochromatic light. When the crystal is heated a number of bright bands, m say, cross the field of view in a microscope focused on the air-wedge. The increase in length of the crystal in an upward direction is $m\lambda/2$, since a change of λ represents a change in the thickness of the film is $\lambda/2$, and the coefficient of expansion can then be calculated.

Newton's Rings

Newton discovered an example of interference which is known as "Newton's rings". In this case a lens L is placed on a sheet of plane glass, L having a lower surface of very large radius of curvature, Fig. 29.8. By means of a sheet of glass G monochromatic light from a sodium

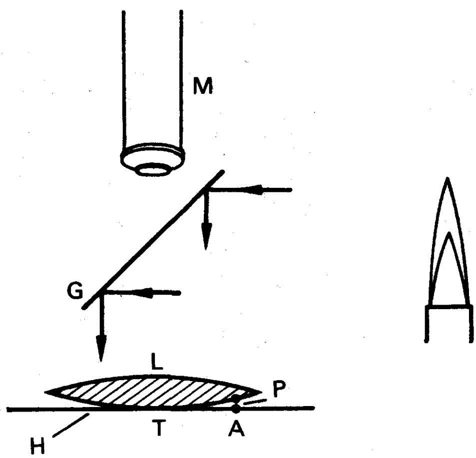

FIG. 29.8. Newton's rings.

flame, for example, is reflected downwards towards L; and when the light reflected upwards is observed through a microscope M focused on

H, a series of bright and dark rings is seen. The circles have increasing radius, and are concentric with the point of contact T of L with H.

Consider the air-film PA between A on the plate and P on the lower lens surface. Some of the incident light is reflected from P to the microscope, while the remainder of the light passes straight through to A, where it is also reflected to the microscope and brought to the same focus. The two rays of light have thus a net path difference of $2t$, where $t =$ PA. The same path difference is obtained at all points round T which are distant TA from T; and hence if $2t = m\lambda$, where m is an integer and λ is the wavelength, we might expect a bright *ring* with centre T. Similarly, if $2t = (m + \frac{1}{2})\lambda$, we might expect a dark ring.

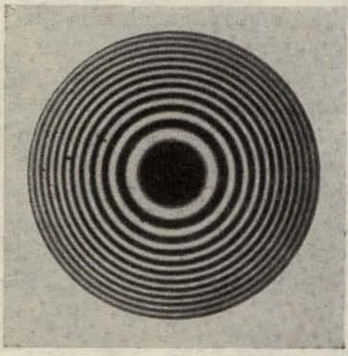

FIG. 29.9. Newton's rings, formed by interference of yellow light between a convex lens and a flat glass plate.

When a ray is reflected from an optically *denser* medium, however, a phase change of 180° occurs in the wave, which is equivalent to its acquiring an extra path difference of $\lambda/2$ (see also p. 692). The truth of this statement can be seen by the presence of the dark spot at the centre, T, of the rings. At this point there is no geometrical path difference between the rays reflected from the lower surface of the lens and H, so that they should be in phase when they are brought to a focus and should form a bright spot. The dark spot means, therefore, that one of the rays suffers a phase change of 180°. Taking the phase change into account, it follows that

$$2t = m\lambda \text{ for a } \textit{dark} \text{ ring} \quad . \quad . \quad . \quad (1)$$

and

$$2t = (m + \tfrac{1}{2})\lambda \text{ for a } \textit{bright} \text{ ring} \quad . \quad . \quad (2)$$

where m is an integer. Young verified the phase change by placing oil of sassafras between a crown and a flint glass lens. This liquid had a refractive index greater than that of crown glass and less than that of flint glass, so that light was reflected at an optically denser medium at each lens. A bright spot was then observed in the middle of the Newton's rings, showing that no net phase change had now occurred.

The grinding of a lens surface can be tested by observing the appearance of the Newton's rings formed between it and a flat glass plate when monochromatic light is used. If the rings are not perfectly circular, the grinding is imperfect. See Fig. 29.9.

Measurement of Wavelength by Newton's Rings

The radius r of a ring can be expressed in terms of the thickness, t, of the corresponding layer of air by simple geometry. Suppose TO is produced to D to meet the completed circular section of the lower surface PQ of the lens, PO being perpendicular to the diameter TD through T,

Fig. 29.10. Then, from the well-known theorem concerning the segments of chords in a circle, TO. OD = QO. OP. But AT = r = PO, QO = OP = r, AP = t = TO, and OD = $2a$ – OT = $2a - t$.

$$\therefore \quad t\,(2a - t) = r \times r = r^2$$

$$\therefore \quad 2at - t^2 = r^2$$

But t^2 is very small compared with $2at$, as a is large.

$$\therefore \quad 2at = r^2$$

$$\therefore \quad 2t = \frac{r^2}{a} \quad . \quad . \quad . \quad . \quad . \quad (i)$$

But $\quad 2t = (m + \frac{1}{2})\lambda$ for a bright ring.

$$\therefore \quad \frac{r^2}{a} = (m + \tfrac{1}{2})\lambda \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The first bright ring obviously corresponds to the case of $m = 0$ in equation (3); the second bright ring corresponds to the case of $m = 1$. Thus the radius of the 15th bright ring is given from (3) by $r^2/a = 14\frac{1}{2}\lambda$, from which $\lambda = 2r^2/29a$. Knowing r and a, therefore, the wavelength λ can be calculated. Experiment shows that the rings become narrower when blue or violet light is used in place of red light, which proves, from equation (3), that the wavelength of violet light is shorter than the wavelength of red light. Similarly it can be proved that the wavelength of yellow light is shorter than that of red light and longer than the wavelength of violet light.

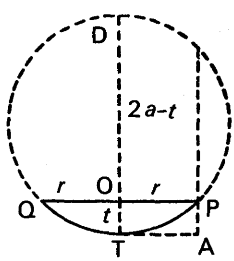

FIG. 29.10.
Theory of radius of Newton's rings.

The radius r of a particular ring can be found by using a travelling microscope to measure its diameter. The radius of curvature, a, of the lower surface of the lens can be measured accurately by using light of known wavelength λ', such as the green in a mercury-vapour lamp or the yellow of a sodium flame; since $a = r^2/(m + \frac{1}{2})\cdot\lambda'$ from (3), the radius of curvature a can be calculated from a knowledge of r, m, λ'.

Visibility of Newton's Rings

When white light is used in Newton's rings experiment the rings are coloured, generally with violet at the inner and red at the outer edge. This can be seen from the formula $r^2 = (m + \frac{1}{2})\,\lambda a$, (3), as $r^2 \propto \lambda$. Newton gave the following list of colours from the centre outwards:

First order: Black, blue, white, yellow, orange, red. *Second order:* Violet, blue, green, yellow, orange, red. *Third order:* Purple, blue, green, yellow, orange, red. *Fourth order:* Green, red. *Fifth order:* Greenish-blue, red. *Sixth order:* Green-blue, pale-red. *Seventh order:* Greenish-blue, reddish-white. Beyond the seventh order the colours overlap and hence white light is obtained. The list is known generally as "Newton's scale of colours". Newton left a detailed description of the colours obtained with different thicknesses of air.

When Newton's rings are formed by sodium light, close examination shows that the clarity, or visibility, of the rings gradually diminishes as one moves outwards from the central spot, after which the visibility improves again. The variation in clarity is due to the fact that sodium light is not monochromatic but consists of *two* wavelengths, λ_2, λ_1, close to one another. These are (i) $\lambda_2 = 5890 \times 10^{-8}$ cm (D_2), (ii) $\lambda_1 = 5896 \times 10^{-8}$ cm (D_1). Each wavelength produces its own pattern of rings, and the ring patterns gradually separate as m, the number of the ring, increases. When $m\lambda_1 = (m + \frac{1}{2})\lambda_2$, the bright rings of one wavelength fall in the dark spaces of the other and the visibility is a minimum. In this case

$$5896m = 5890\,(m + \tfrac{1}{2}).$$

$$\therefore \quad m = \frac{5890}{12} = 490 \text{ (approx.)}$$

At a further number of ring m_1, when $m_1\lambda_1 = (m_1 + 1)\lambda_2$, the bright (and dark) rings of the two ring patterns coincide again, and the clarity, or visibility, of the interference pattern is restored. In this case

$$5896m_1 = 5890\,(m_1 + 1),$$

from which $m_1 = 980$ (approx.). Thus at about the 500th ring there is a minimum visibility, and at about the 1000th ring the visibility is a maximum.

It may be noted here that the bands in films of varying thickness, such as Newton's rings and the air-wedge bands, p. 692, appear to be formed in the film itself, and the eye must be focused on the film to see them. We say that the bands are "localised" at the film. With a thin film of uniform thickness, however, bands are formed by parallel rays which enter the eye, and these bands are therefore localised at infinity.

"Blooming" of Lenses

Whenever lenses are used, a small percentage of the incident light is reflected from each surface. In compound lens systems, as in telescopes and microscopes, this produces a background of unfocused light, which results in a reduction in the clarity of the final image. There is also a reduction in the intensity of the image, since less light is transmitted through the lenses.

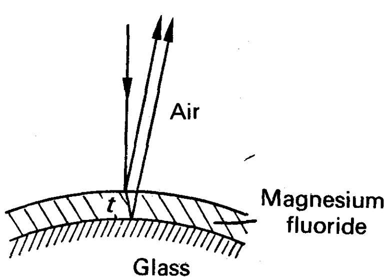

FIG. 29.11. Blooming of lens.

The amount of reflected light can be considerably reduced by evaporating a thin coating of a fluoride salt such as magnesium fluoride on to the surfaces, Fig. 29.11. Some of the light, of average wavelength λ, is then reflected from the air-fluoride surface and the remainder penetrates the coating and is partially reflected from the fluoride-glass surface. Destructive interference occurs between the two reflected beams when there is a phase difference of 180°, or a path difference of $\lambda/2$, as the refractive index of the fluoride is less than that of glass. Thus if t is the required thickness of the coating and n' its retractive index, $2n't = \lambda/2$. Hence $t = \lambda/4n' = 6 \times 10^{-5}/(4 \times 1{\cdot}38)$, assuming λ is 6×10^{-5} cm and n' is 1·38; thus $t = 1{\cdot}1 \times 10^{-5}$ cm.

For best results n' should have a value equal to about $\sqrt{n}$, where n is the refractive index of the glass lens. The intensities of the two reflected beams are then equal, and hence complete interference occurs between them. No light is then reflected back from the lens. In practice, complete interference is not possible simultaneously for every wavelength of white light, and an average wavelength for λ, such as green-yellow, is chosen. "Bloomed" lenses effect a marked improvement in the clarity of the final image in optical instruments.

Lloyd's Mirror

In 1834 LLOYD obtained interference bands on a screen by using a plane mirror M, and illuminating it with light nearly at grazing incidence,

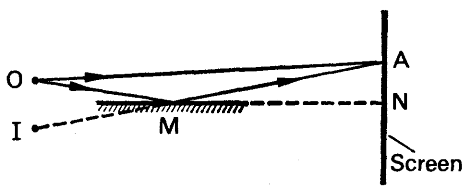

FIG. 29.12. Lloyd's mirror experiment.

coming from a slit O parallel to the mirror, Fig. 29.12. A point such as A on the screen is illuminated (i) by a ray OA and (ii) by a ray OM reflected along MA, which appears to come from the virtual image I of O in the mirror. Since O and I are close coherent sources interference bands are obtained on the screen.

Experiment showed that the band at N, which corresponds to the point of intersection of the mirror and the screen, was *dark;* since ON = IN, this band might have been expected, before the experiment was carried out, to be bright. Lloyd concluded that a phase change of 180°, equivalent to half a wavelength, occurred by reflection at the mirror surface, which is a denser surface than air (see p. 692).

Interference in Thin Films

The colours observed in a soap-bubble or a thin film of oil in the road are due to an interference phenomenon; they are also observed in thin transparent films of glass.

Consider a ray AO of monochromatic light incident on a thin parallel-sided film of thickness t and refractive index n. Fig. 29.13 is exaggerated for clarity. Some of the light is reflected at O along ON, while the remainder is refracted into the film, where reflection occurs at B. The ray BC then emerges into the air along CM, which is parallel to ON. The incident ray AO thus divides at O into two beams of different amplitude which are coherent, and if ON, CM are combined by a lens, or by the eye-lens, a bright or dark band is observed according to the path difference of the rays.

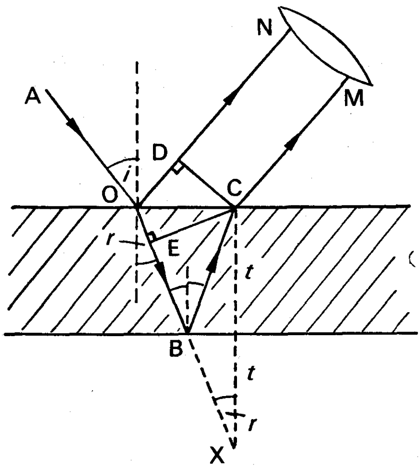

FIG. 29.13. Interference in thin films.

The time taken for light to travel a distance y in a medium of refractive index n is y/v, where v is the velocity of light in the medium. In this time, a distance $c \times y/v$ is travelled in air, where c is the velocity in air. But $n = c/v$. Hence the *optical* path of a length y in a medium of refractive index n is ny. The optical path difference between the two rays ON and OBCM is thus n (OB + BC) − OD, where CD is perpendicular to ON, Fig. 29.13. If CE is the perpendicular from C to OB, then OD/OE = $\sin i/\sin r = n$, so that nOE = OD.

$$\therefore \quad \text{optical path difference} = n\,(\text{EB} + \text{BC}) = n\,(\text{EB} + \text{BX}) = n.\,\text{EX}.$$
$$= 2\,nt \cos r,$$

where r is the angle of refraction in the film. With a phase change of 180° by reflection at a denser medium, a bright band is therefore obtained when $2\,nt \cos r + \lambda/2 = m\lambda$,

or $$2\,nt \cos r = (m - \tfrac{1}{2})\,\lambda \quad . \quad . \quad . \quad . \quad \text{(i)}$$

For a dark band, $$2\,nt \cos r = m\lambda \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

Colours in Thin Films

The colours in thin films of oil or glass are due to interference from an extended source such as the sky or a cloud. Fig. 29.14 illustrates interference between rays from points O_1, O_2 respectively on the extended source. Each ray is reflected and refracted at A_1, A_2 on the film, and enter the eye at E_1. Although O_1, O_2 are non-coherent, the eye will see the same colour of a particular wavelength λ if $2\,nt \cos r = (m - \tfrac{1}{2})\,\lambda$. The separation of the two rays from A_1 or from A_2 must be less than the diameter of the eye-pupil for interference to occur, and this is the case only for thin films. The angle of refraction r is determined by the angle of incidence, or reflection, at the film. The particular colour seen thus depends on the position of the eye. At E_2, for example, a different colour will be seen from another point O_3 on the extended source. The variation of θ and hence r is small when the eye observes a particular area of the film, and hence a band of a particular colour, such as A_1 A_2, is the contour of paths of *equal inclination* to the film. The bands are localised at infinity, since the rays reaching E_1 or E_2 are parallel.

If a thin wedge-shaped film is illuminated by an extended source, as shown on p. 692 or in Newton's rings, the bands seen are contours of *equal thickness* of the film.

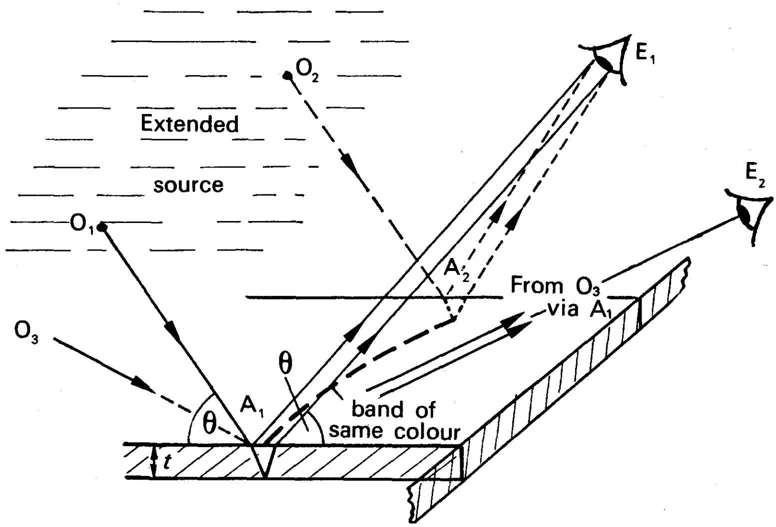

FIG. 29.14. Colours in thin films.

Vertical Soap Film Colours

An interesting experiment on thin films, due to C. V. Boys, can be performed by illuminating a vertical soap film with monochromatic light. At first the film appears uniformly coloured. As the soap drains to the bottom, however, a wedge-shaped film of liquid forms in the ring, the top of the film being thinner than the bottom. The thickness of the wedge is constant in a horizontal direction, and thus horizontal bright and dark bands are observed across the film. When the upper part of the film becomes extremely thin a black band is observed at the top (compare the dark central spot in Newton's rings experiment), and the film breaks shortly afterwards.

With white light, a succession of broad coloured bands is first observed in the soap film. Each band contains colours of the spectrum, red to violet. The bands widen as the film drains, and just before it breaks a black band is obtained at the top.

For normal incidence of white light, a particular wavelength λ is seen where the optical path difference due to the film $= (m - \frac{1}{2})\lambda$ and m is an integer. Thus a red colour of wavelength $7{\cdot}0 \times 10^{-5}$ cm is seen where the optical path difference is $3{\cdot}5 \times 10^{-5}$ cm, corresponding to $m = 1$. No other colour is seen at this part of the thin film. Suppose, however, that another part of the film is much thicker and the optical path difference here is $21 \times 3{\cdot}5 \times 10^{-5}$ cm. Then a red colour of wavelength $7{\cdot}0 \times 10^{-5}$ cm, $m = 11$, an orange colour of wavelength about

$6{\cdot}4 \times 10^{-5}$ cm, $m = 12$, a yellow wavelength about $5{\cdot}9 \times 10^{-5}$ cm, $m = 13$, and other colours of shorter wavelengths corresponding to higher integral values of m, are seen at the same part of the film. These colours all overlap and produce a white colour. If the film is thicker still, it can be seen that numerous wavelengths throughout the visible spectrum are obtained and the film then appears uniformly white.

Monochromatic light and Thin Parallel Films

If a thin parallel film is illuminated by a beam of monochromatic light, obtained by using an extended or broad source such as a bunsen burner sodium flame, a number of *circular* bright and dark curves can be seen. Fig. 29.15 illustrated how interference is obtained from the light originating from points a, b which is refracted at an angle α into the film. This is similar to Fig. 29.14 if B represents an eye-lens.

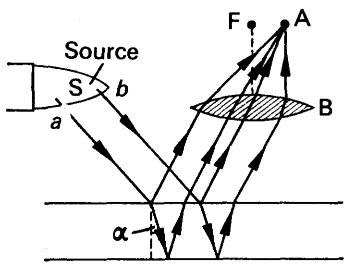

FIG. 29.15. Interference with extended source.

The emergent rays are combined by the eye-lens or a glass lens B, and a dark band is formed at A if $2nt \cos \alpha = m\lambda$, with the usual notation. If the light is incident on the film in every plane a circular band is obtained, whose centre is F, the focus of B. It is a band of 'equal inclination'.

When a *parallel* beam of monochromatic light is incident on the thin film, the angle of refraction r in the film and the thickness t are constant. The film thus appears uniformly bright at all points if the condition $2nt \cos r = (m + \frac{1}{2})\lambda$ is obeyed, and is uniformly dark if $2nt \cos r = m\lambda$. If the film is illuminated by a parallel beam of *white* light, the transmitted light appears to have dark bands across it when viewed through a spectroscope. The latter separates the colours, and a dark band is obtained where the condition $2nt \cos r = (m + \frac{1}{2})\lambda$ is satisfied for the particular wavelength, since we are now concerned with transmitted light.

EXAMPLE

What are Newton's rings and under what conditions can they be observed? Explain how they can be used to test the accuracy of grinding of the face of a lens. The face of a lens has a radius of curvature of 50 cm. It is placed in contact with a flat plate and Newton's rings are observed normally with reflected light of wavelength 5×10^{-5} cm. Calculate the radii of the fifth and tenth bright rings. (*C.*)

First parts. See text.

Second part. With the usual notation, for a bright ring we have

$$2t = (m + \tfrac{1}{2})\lambda, \qquad \text{(i)}$$

where t is the corresponding thickness of the layer of air.

But, from geometry, $$2t = \frac{r^2}{a} \qquad \text{(ii)}$$

where r is the radius of the ring and a is the radius of curvature of the lens face (p. 695).

$$\therefore \quad \frac{r^2}{a} = (m + \tfrac{1}{2})\lambda$$

$$\therefore \quad r^2 = (m + \tfrac{1}{2})\lambda a \qquad \text{(iii)}$$

The first ring corresponds to $m = 0$ from equation (iii). Hence the fifth ring corresponds to $m = 4$, and its radius r is thus given by

$$r^2 = (4 + \tfrac{1}{2}) \times 5 \times 10^{-5} \times 50$$

$$\therefore \quad r = \sqrt{\frac{9 \times 5 \times 10^{-5} \times 50}{2}} = 0{\cdot}106 \text{ cm.}$$

The tenth ring corresponds to $m = 9$ in equation (iii), and its radius is thus given by

$$r^2 = 9\tfrac{1}{2} \times 5 \times 10^{-5} \times 50$$

$$\therefore \quad r = 0{\cdot}154 \text{ cm.}$$

DIFFRACTION OF LIGHT

In 1665 GRIMALDI observed that the shadow of a very thin wire in a beam of light was much broader than he expected. The experiment was repeated by Newton, but the true significance was only recognised more than a century later, after Huygens' wave theory of light had been resurrected. The experiment was one of a number which showed that light could bend round corners in certain circumstances.

We have seen how interference patterns, for example, bright and dark bands, can be obtained with the aid of two sources of light close to each other. These sources must be coherent sources, i.e., they must have the same amplitude and frequency, and always be in phase with each other. Consider two points on the *same wavefront*, for example the two points A, B, on a plane wavefront arriving at a narrow slit in a screen, Fig. 29.16. A and B can be considered as secondary sources of light, an aspect introduced by Huygens in his wave theory of light (p. 676); and as they are on the same wavefront, A and B have identical amplitudes

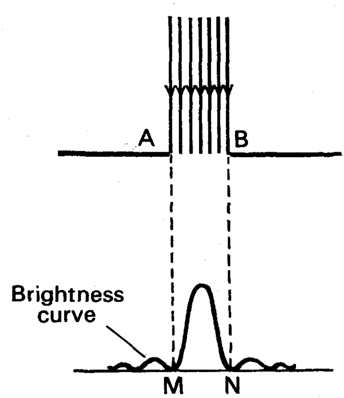

FIG. 29.16. Diffraction of light.

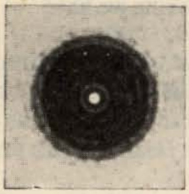

FIG. 29.17. Diffraction rings in the shadow of a small circular disc. The bright spot is at the centre of the geometrical shadow.

and frequencies and are in phase with each other. Consequently A, B, are coherent sources, and we can expect to find an interference pattern on a screen in front of the slit, provided the latter is small compared with the wavelength of light. For a short distance beyond the edges M, N, of the projection of AB, i.e., in the geometrical shadow, observation shows that there are some alternate bright and dark bands. See Fig. 29.20. Thus light can travel round corners. The phenomenon is called *diffraction*, and it has enabled scientists to measure accurately the wavelength of light.

If a source of white light is observed through the eyelashes, a series of coloured images can be seen. These images are due to interference between sources on the same wavefront, and the phenomenon is thus an example of diffraction. Another example of diffraction was unwittingly deduced by POISSON at a time when the wave theory was new. Poisson considered mathematically the combined effect of the wavefronts round a circular disc illuminated by a distant small source of light, and he came to the conclusion that the light should be visible beyond the disc in the middle of the geometrical shadow. Poisson thought this was impossible; but experiment confirmed his deduction, and he became a supporter of the wave theory of light. See Fig. 29.17.

Diffraction at Single Slit

We now consider diffraction at a single slit in more detail. Suppose parallel light is incident on a narrow rectangular slit AB, Fig. 29.18.

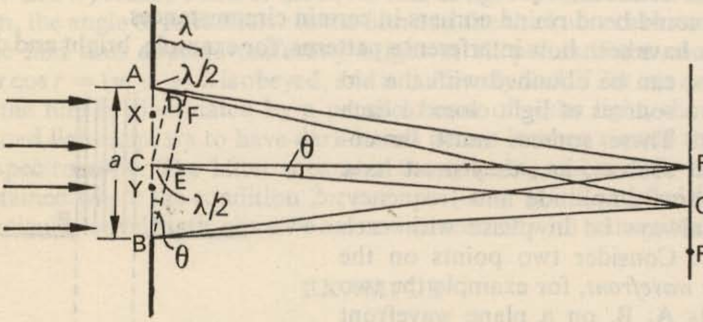

FIG. 29.18. Diffraction at single slit.

Each point on the same wavefront between A, B acts as a secondary centre of disturbance, and sends out wavelets beyond the slit. All the secondary centres are coherent, and their combined effect at any point such as P or Q can be found by summing the individual waves there, from the Principle of Superposition. The mathematical treatment is beyond the scope of this book. The general effect, however, can be derived by considering the two halves AC, CB of the wavefront AB. At a point P equidistant from A and B, corresponding secondary centres in AC, CB respectively, such as X and Y, are also equidistant from P. Consequently wavelets arrive in phase at P. When AB is of the order of a few wavelengths of light the resultant amplitude at P due to the whole wavefront AB is therefore large, and thus a bright band is obtained at P.

As we move from P parallel to AB, points are obtained where the secondary wavelets from the two halves of the wavefront become more and more out of phase on arrival and the brightness thus diminishes. Consider a point Q, where AQ is half a wavelength longer than CQ. A disturbance from A, and one from C, then arrive at Q 180° out of phase. This is also practically the case for all corresponding points such as X, Y on the two halves of the wavefront. In particular, CQ and BQ differ practically by $\lambda/2$, where C is the extreme point in the upper half of the wavefront and B is the extreme point on the lower half. Thus Q corresponds to the edge or minimum intensity of the central band round P, Fig. 29.19. As we move farther away from Q parallel to AB, the intensity rises again to a much smaller maximum at R, where AR − BR = $3\lambda/2$, Fig. 29.18. To explain this, one can imagine the wavefront AB in Fig. 29.18 divided into three equal parts. Two parts annul each other's displacements at R as just explained, leaving one-third of the wavefront, which produces a much less bright band at R than at P. Calculation shows that the maximum intensity of the band at R is less than 5 per cent of that of the central band at P. Other subsidiary maxima and minima diffraction bands are obtained if the slit is very narrow. See Fig. 29.20.

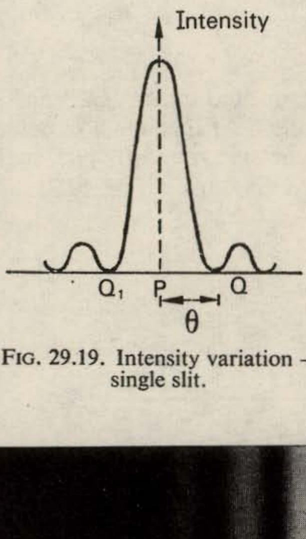

FIG. 29.19. Intensity variation – single slit.

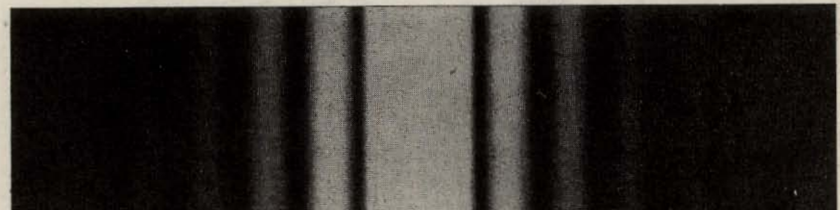
FIG. 29.20. Diffraction bands formed by a single small rectangular aperture.

Width of Central Band. Rectilinear Propagation

The angular width of the central bright band is 2θ, where θ is the angular width between the maximum intensity direction P and the minimum at Q, Fig. 29.19. From Fig. 29.18, it can be seen that the line CQ to the edge of the central band makes an angle θ with the direction CP of the incident light given by

$$\sin\theta = \frac{AF}{AB} = \frac{AD + CE}{AB} = \frac{\lambda/2 + \lambda/2}{a} = \frac{\lambda}{a},$$

where a = AB. When the slit is widened and a becomes large compared with λ, then $\sin\theta$ is very small and hence θ is very small. In this case the directions of the minimum and maximum intensities of the central band are very close to each other, and practically the whole of the light is confined to a direction immediately in front of the incident direction, that is, no spreading occurs. This explains the *rectilinear propagation of light*. When the slit width a is very small and equal to 2λ, for example, then $\sin\theta = \lambda/a = 1/2$, or $\theta = 30°$. The light waves now spread round through 30° on either side of the slit.

These results are true for any wave phenomenon. In the case of an electromagnetic wave of 3 cm wavelength, a slit of these dimensions produces sideways spreading. Sound waves of a particular frequency 256 Hz have a wavelength of about 1·3 m. Consequently, sound waves spread round corners or apertures such as a doorway, which have comparable dimensions to their wavelengths.

Diffraction in Telescope Objective

When a parallel beam of light from a distant object such as a star S_1 enters a telescope objective L, the lens collects light through a circular opening and forms a diffraction pattern of the star round its principal focus, F. This is illustrated in the exaggerated diagram of Fig. 29.21.

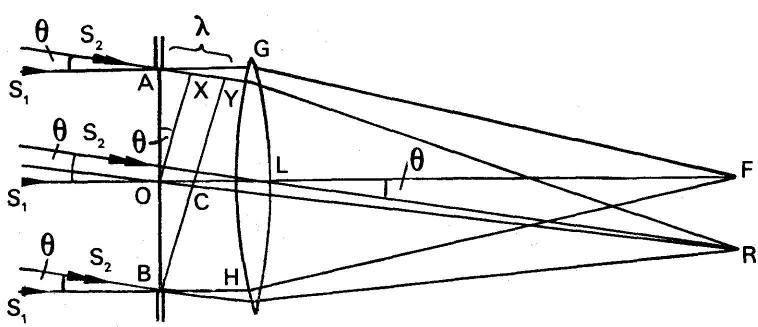

FIG. 29.21. Diffraction in telescope objective.

Consider an incident plane wavefront AB from the star S_1, and suppose for a moment that the aperture is rectangular. The diffracted rays such as AG, BH normal to the wavefront are incident on the lens in a direction parallel to the principal axis LF. The optical paths AGF, BHF are equal. This is true for all other diffracted rays from points between A, B which are parallel to LF, since the optical paths to an image produced by a lens are equal. The central part F of the star pattern is therefore bright.

Now consider those diffracted rays from all points between AB which enter the lens at an angle θ to the principal axis. This corresponds to a diffracted plane wavefront BY at an angle θ to AB. As described previously on p. 703, the wavefront AB can be divided into two halves, AO, OB. The rays from A and O in the two halves produce destructive interference if AX $= \lambda/2$, and likewise the extreme points O, B in the two halves produce destructive interference as OC $= \lambda/2$. Other corresponding points on the two halves also produce destructive interference. When the rays are collected and brought to a focus at R, darkness is thus obtained, that is, R is the edge of the central maximum of the star S_1. As explained on p. 704, other subsidiary maxima may be formed round F.

The angle θ corresponding to the edge R is given by

$$\sin\theta = \frac{\lambda/2 + \lambda/2}{D} = \frac{\lambda}{D},$$

where D is the diameter of the lens aperture. This is the case where the opening can be divided into a number of rectangular slits. For a circular opening such as a lens (or the concave mirror of the Palomar telescope), the formula becomes $\sin\theta = 1{\cdot}22\lambda/D$, and as θ is small, we may write $\theta = 1{\cdot}22\lambda/D$.

Resolving Power

Suppose now that another distant star S_2 is at an angular distance θ from S_1, Fig. 29.21. The maximum intensity of the central pattern of S_2 then falls on the minimum or edge of the central pattern of the star S_1, corresponding to R in Fig. 29.22 (i). Experience shows that the two stars

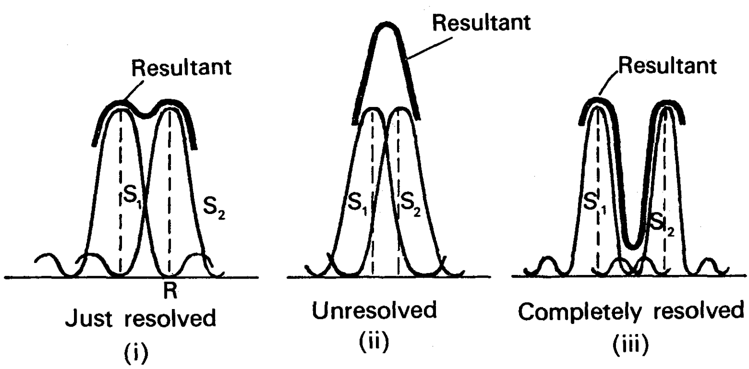

FIG. 29.22. Resolving power.

can then just be distinguished or *resolved.* Lord Rayleigh stated a criterion for the resolution of two objects, which is generally accepted: *Two objects are just resolved when the maximum intensity of the central pattern of one object falls on the first minimum or dark edge of the other.* Fig. 29.22 (i) shows the two stars just resolved. The resultant intensity in the middle dips to about 0·8 of the maximum, and the eye is apparently sensitive to the change here. Fig. 29.22 (ii) shows two stars S_1, S_2 unresolved, and Fig. 29.22 (iii) the same stars completely resolved.

The angular distance θ between two distant stars just resolved is thus given by $\sin\theta = \theta = 1{\cdot}22\ \lambda/D$, where D is the diameter of the objective. This is an expression for the *limit of resolution*, or *resolving power*, of a telescope. The limit of resolution or resolving power increases when θ is smaller, as two stars closer together can then be resolved. Consequently telescope objectives of large diameter D give high resolving power. The Yerkes Observatory has a large telescope objective of about 100 cm. The angular distance θ between two stars which can just be resolved is thus given by

$$\theta = \frac{1{\cdot}22\lambda}{D} = \frac{1{\cdot}22 \times 6 \times 10^{-5}}{100} = 7{\cdot}3 \times 10^{-7} \text{ radians},$$

assuming 6×10^{-5} cm for the wavelength of light. The Mount Palomar telescope has a parabolic mirror objective of aperture 5 metres, or 500 cm. The resolving power is thus five times as great as the Yerkes Observatory telescope. A large aperture D has also the advantage of collecting more light (p. 542). The Jodrell Bank radio telescope has a circular bowl of about 75 m, and for radio waves of 20 cm wavelength the resolving power, $\theta = 1{\cdot}22\lambda/D = 1{\cdot}22 \times 20/7500$ radians $= 3 \times 10^{-3}$ radians (approx.).

Magnifying Power of Telescope and Resolving Power

If the width of the emergent beam from a telescope is greater than the diameter of the eye-pupil, rays from the outer edge of the objective do not enter the eye and hence the full diameter D of the objective is not used. If the width of the emergent beam is less than the diameter of the eye-pupil, the eye itself, which has a constant aperture, may not be able to resolve the distant objects. Theoretically, the angular resolving power of the eye is $1{\cdot}22\ \lambda/a$, where a is the diameter of the eye-pupil, but in practice an angle of 1 minute is resolved by the eye, which is more than the theoretical value.

Now the angular magnification, or magnifying power, of a telescope is the ratio α'/α, where α' is the angle subtended at the eye by the final image and α is the angle subtended at the objective (p. 533). To make the fullest use of the diameter D of the objective, the magnifying power should therefore be increased to the angular ratio given by

$$\frac{\text{resolving power of eye}}{\text{resolving power of objective}} = \frac{\pi/(180 \times 60)}{1{\cdot}22 \times 6 \times 10^{-5}/D} = 4\,D \text{ (approx.)}$$

In this case the telescope is said to be in "normal adjustment". Any further increase in magnifying power will make the distant objects appear larger, but there is no increase in definition or resolving power.

Brightness of Images in Telescope

In a telescope, the eye is placed at the *exit-pupil* or *eye-ring*, the circle through which the emergent beam passes (p. 532). The *entrance pupil* of the telescope is the aperture or diameter of the objective. If the area of the latter is A, then the smaller area of the exit-pupil is A/M^2, where M is the angular magnification of the telescope in normal adjustment (see p. 535).

Consider a telescope used to observe (*a*) a small but finite area, or extended object, and (*b*) a point source, such as a star. Suppose that in each case the magnifying power is adjusted to make the exit pupil of the telescope equal to the eye pupil. In each case the luminous flux collected with the telescope is equal to the flux collected by an unaided eye multiplied by the ratio: area of objective/area of eye-pupil, which is M^2.

Geometrically, the telescope magnifies the finite object area by a factor M^2, but the image of the point object is still a point. Hence the area of the retinal image of the finite object is magnified by a factor M^2 when the telescope is used. On the other hand, since the eye-pupil is filled with light by the telescope, the retinal image of the point object with the telescope is the same as that without the telescope—it is the diffraction image for a point source. It will be seen that, for the finite object, the larger flux is spread over a larger image area, so that (apart from absorption losses in the telescope) the retinal illumination is unchanged. For the point object, however, the increased flux is spread over the same retinal area so that the brightness of the image is increased. On this account stars appear very much brighter when viewed by a telescope, whereas the brightness of the background, which acts as an extended object, remains about the same. Stars too faint to be seen with the naked eye become visible using a powerful telescope, and the number of stars seen thus increases considerably using a telescope.

Increasing Number of Slits. Diffraction Grating

On p. 703 we saw that the image of a single narrow rectangular slit is a bright central or principal maximum diffraction band, together with subsidiary maxima diffraction bands which are much less bright. Suppose that parallel light is incident on two more parallel close slits, and the light passing through the slits is received by a telescope focused at infinity. Since each slit produces a similar diffraction effect in the same direction, the observed diffraction pattern will have an intensity variation identical to that of a single slit. This time, however, the pattern is crossed by a number of interference bands, which are due to interference between slits (see *Young's experiment*, p. 688). The envelope of the intensity variation of the interference bands follow the diffraction pattern variation due to a single slit. In general, if I_s is the intensity at a point due to interference and I_d that due to diffraction, then the resultant intensity I is given by $I = I_d \times I_s$. Hence if $I_d = 0$ at any point, then $I = 0$ irrespective of the value of I_s.

As more parallel equidistant slits are introduced, the intensity and sharpness of the principal maxima increase and those of the subsidiary maxima decrease. The effect is illustrated roughly in Fig. 29.23. With several thousand lines per centimetre, only a few sharp principal maxima are seen in directions discussed shortly. Their angular separation depends only on the distance between successive slits. The slit width affects the intensity of the higher order principal maxima; the narrower the slit, the greater is the diffraction of light into the higher orders.

A *diffraction grating* is a large number of close parallel equidistant slits, ruled on glass or metal; it provides a very valuable means of studying spectra. If the width of a slit or clear space is a and the thickness of a ruled opaque line is b, the spacing d of the slits is $(a + b)$. Thus with a grating of 6000 lines per centimetre, the spacing $d = 1/6000$ centimetre $= 17 \times 10^{-5}$ cm, or a few wavelengths of visible light.

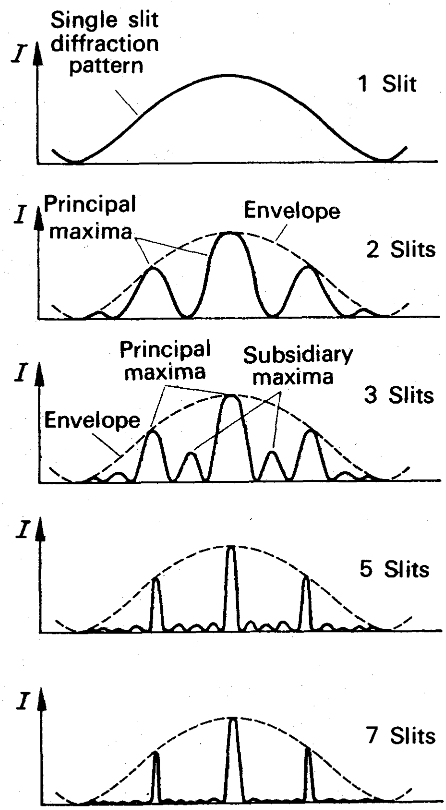

FIG. 29.23. Principal maxima with increasing slits.

Principal Maxima of Grating

The angular positions of the principal maxima produced by a diffraction grating can easily be found. Suppose X, Y are corresponding points in consecutive slits, where XY $= d$, and the grating is illuminated normally by monochromatic light of wavelength λ, Fig. 29.24. In a direction θ, the diffracted rays XL, YM have a path difference XA of $d \sin \theta$. The diffracted rays from all other corresponding points in the two slits have a path difference of $d \sin \theta$ in the same direction. Other pairs of slits throughout the grating can be treated in the same way. Hence bright or principal maxima are obtained when

$$d \sin \theta = m\lambda, \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

where m is an integer, if all the diffracted parallel rays are collected by a telescope focused at infinity. The images corresponding to

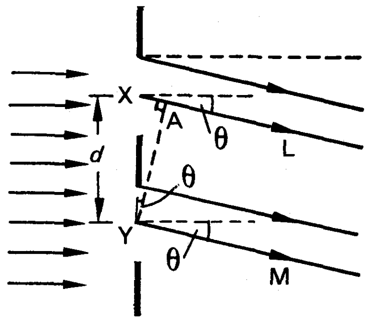

FIG. 29.24. Diffraction grating.

$m = 0, 1, 2, \ldots$ are said to be respectively of the zero, first, second . . . orders respectively. The zero order image is the image where the path difference of diffracted rays is zero, and corresponds to that seen directly opposite the incident beam on the grating. It should again be noted that all points in the slits are secondary centres on the same wavefront and therefore coherent sources.

Diffraction Images

The *first order* diffraction image is obtained when $m = 1$. Thus

$$d \sin \theta = \lambda,$$

or

$$\sin \theta = \frac{\lambda}{d}.$$

If the grating has 6000 lines per centimetre (6000 cm^{-1}), the spacing of the slits, d, is $\frac{1}{6000}$ cm. Suppose yellow light, of wavelength $\lambda = 5890 \times 10^{-8}$ cm, is used to illuminate the grating. Then

$$\sin \theta = \frac{\lambda}{d} = 5890 \times 10^{-8} \times 6000 = 0{\cdot}3534$$

$$\therefore \quad \theta = 20{\cdot}7°$$

The *second order* diffraction image is obtained when $m = 2$. In this case $d \sin \theta = 2\lambda$.

$$\therefore \quad \sin \theta = \frac{2\lambda}{d} = 2 \times 5890 \times 10^{-8} \times 6000 = 0{\cdot}7068$$

$$\therefore \quad \theta = 45{\cdot}0°$$

If $m = 3$, $\sin \theta = 3\lambda/d = 1{\cdot}060$. Since the sine of an angle cannot be greater than 1, it is impossible to obtain a third order image with this diffraction grating.

With a grating of 12000 lines per cm the diffraction images of sodium light would be given by $\sin \theta = m\lambda/d = m \times 5890 \times 10^{-8} \times 12000 = 0{\cdot}7068\, m$. Thus only $m = 1$ is possible here. As all the diffracted light is now concentrated in one image, instead of being distributed over several images, the first order image is very bright, which is an advantage.

Diffraction with Oblique Incidence

When a diffraction grating is illuminated by a monochromatic parallel beam PX, QY at an angle of incidence i, each point in the clear spaces acts as a secondary disturbance and diffracted beams emerge from the grating, Fig. 29.25. For a diffracted beam such as AB, making an angle of diffraction θ on the same side of the normal as PX or QY, the path difference between two typical rays PXA, QYB is $d(\sin i + \sin \theta)$. For a diffracted beam such as CD on the other side of the normal, the path difference between typical

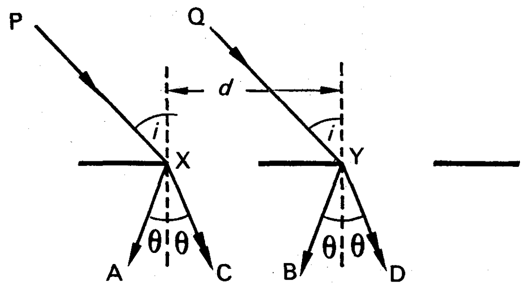

FIG. 29.25. Diffraction with oblique incidence.

rays PXC, QYD is $d(\sin i - \sin\theta)$. Thus, generally, a bright diffraction image is seen when $d(\sin i \pm \sin \theta) = m\lambda$, where m is an integer.

The zero order or central image is obtained in a direction opposite to the incident beam PX, QY. The first order diffraction image is obtained at angles θ on either side of this direction given respectively by $d(\sin i + \sin \lambda) = \lambda$ and $d(\sin i - \sin \theta) = \lambda$. Diffraction images of higher order are obtained from similar formulae.

Reflection gratings can be used when light of particular wavelengths are absorbed by materials used in making transmission gratings. In this case the light is diffracted back into the incident medium at the clear spaces, and the diffraction images of various orders are given by $d(\sin i \pm \sin \theta) = m\lambda$.

Measurement of Wavelength

The wavelength of monochromatic light can be measured by a diffraction grating in conjunction with a spectrometer. The collimator C and telescope T of the instrument are first adjusted for parallel light (p. 445), and the grating P is then placed on the table so that its plane is perpendicular to two screws, Q, R, Fig. 29.26 (i). To level the table so that the plane of P is parallel to the axis of rotation of the telescope, the latter is first placed in the position T_1 directly opposite the illuminated slit of the collimator, and then rotated exactly through 90° to a position T_2. The table is now turned until the slit is seen in T_2 by reflection at P, and one of the screws Q, R turned until the slit image is in the middle of the field of view. The plane of P is now parallel to the axis of rotation of the telescope. The table is then turned through 45° so that the plane of the grating is exactly perpendicular to the light from C, and the telescope is turned to a position T_3 to receive the first diffraction image, Fig. 29.26 (ii). If the lines of the grating are not parallel to the axis of rotation of the telescope, the image will not be in the middle of the field of view. The third screw is then adjusted until the image is central.

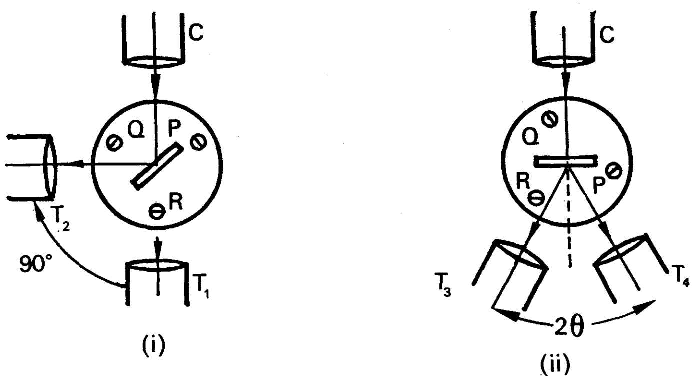

FIG. 29.26. Measurement of wavelength by diffraction grating.

The readings of the first diffraction image are observed on both sides of the normal. The angular difference is 2θ, and the wavelength is calculated from $\lambda = d \sin \theta$, where d is the spacing of the slits, obtained from the number of lines per centimetre of the grating. If a second order image is obtained for a diffraction angle θ, then $\lambda = d \sin \theta_1/2$.

Position of Image

If the grating lines are on the opposite side of the glass to the collimator C in Fig. 29.26 (ii), the light from C passes straight through the glass and the diffracted rays at the slits emerge into air. Suppose, however, that the grating is turned round so that the lines such as A, D face the collimator C, Fig. 29.27 (i). The rays are now diffracted into the glass and then refracted at B, F into the air at an angle θ to the normal. The optical path difference between the rays ABM, DFH from corresponding points A D, is then

$$n.\text{AB} + \text{BL} - n.\text{DF} = \text{BL}$$

since AB = DF. But BL = BF sin $\theta = d \sin \theta$. Consequently the angular

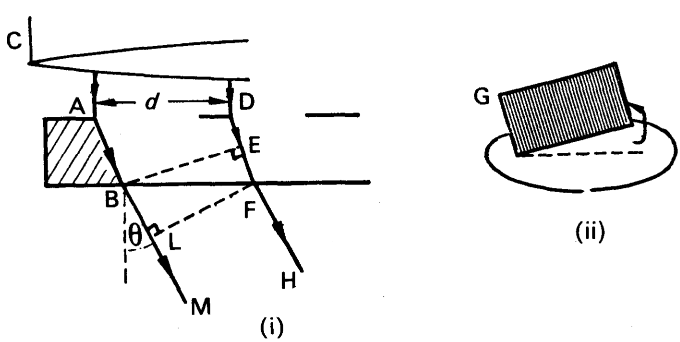

FIG. 29.27. Position of images with diffraction grating.

positions of the principal maxima diffraction images are given by $d \sin \theta = m\lambda$. Thus the images are observed at the same diffraction angles, no matter which side of the grating faces the collimator.

If the first order diffraction image is viewed in the telescope, and the grating G is turned round slightly in its own plane so that the lines are at a small angle to the vertical, the image of the slit moves round in the same direction, Fig. 29.27 (ii). The image then appears to move up or down in the field of view of the telescope, and disappears as the grating is turned round farther. The effect can be seen by viewing an electric lamp through a diffraction grating, and turning the grating in its own plane through 90°. The diffraction images of the lamp also rotate through 90°.

Spectra in Grating

If white light is incident normally on a diffraction grating, several coloured spectra are observed on either side of the normal, Fig. 29.28 (i). The first order diffraction images are given by $d \sin \theta = \lambda$, and as violet has a shorter wavelength than red, θ is less for violet than for red. Consequently the spectrum colours on either side of the incident white light are violet to red. In the case of a spectrum produced by dispersion in a glass prism, the colours range from red, the least deviated, to violet, Fig. 29.28 (ii). Second and higher order spectra are obtained with a diffraction grating on opposite sides of the normal, whereas only one spectrum is obtained with a glass prism. The angular spacing of the colours is also different in the grating and the prism.

If $d \sin \theta = m_1 \lambda_1 = m_2 \lambda_2$, where m_1, m_2 are integers, then a wavelength λ_1 in the m_1 order spectrum overlaps the wavelength λ_2 in the m_2 order. The

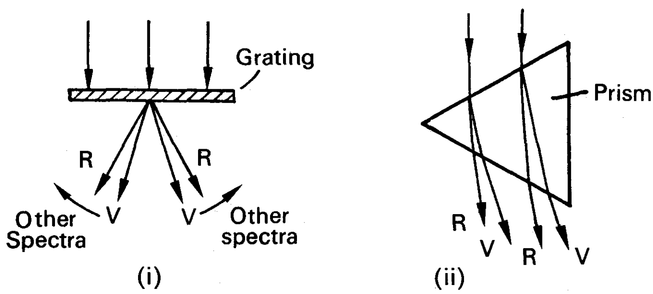

FIG. 29.28. Spectra in grating and prism.

extreme violet in the visible spectrum has a wavelength about $3{\cdot}8 \times 10^{-5}$ cm. The violet direction in the second order spectrum would thus correspond to $d \sin \theta = 2\lambda = 7{\cdot}6 \times 10^{-5}$ cm, and this would not overlap the extreme colour, red, in the first order spectrum, which has a wavelength about $7{\cdot}0 \times 10^{-5}$ cm. In the second order spectrum, a wavelength λ_2 would be overlapped by a wavelength λ_3 in the third order if $2\lambda_2 = 3\lambda_3$. If $\lambda_2 = 6{\cdot}9 \times 10^{-5}$ cm (red), then $\lambda_3 = 2\lambda_2/3 = 4{\cdot}6 \times 10^{-5}$ cm (blue). Thus overlapping of colours occurs in spectra of higher orders than the first.

Dispersion by Grating

The *dispersion* of a grating, $d\theta/d\lambda$, is a measure of the change in angular position per unit wavelength change. Now $d \sin \theta = m\lambda$,

$$\therefore \quad d \cos \theta \frac{d\theta}{d\lambda} = m$$

$$\therefore \quad \frac{d\theta}{d\lambda} = \frac{m}{d \cos \theta} \quad . \quad . \quad . \quad \text{(i)}$$

The dispersion thus increases with the order, m, of the image. It is also inversely proportional to the separation d of the slits, or, for a given grating width, directly proportional to the total number of slits on the grating. For a given order m, the dispersion increases when $\cos \theta$ is small, or when θ is large, which corresponds to the red wavelengths of the spectrum for normal incidence on the grating.

Resolving Power of Grating

For the mth order principal maximum of a grating, the path difference between diffracted rays from consecutive slits is $m\lambda$. The path difference AB between the extreme rays of the grating is thus $(N - 1) m\lambda$, where N is the total number of lines ruled on the grating, Fig. 29.29. The minimum intensity of the mth order principal maximum corresponds to a slightly different direction AC. Now the discussion about the disturbances from various points across a wide slit (p. 702) can be applied to disturbances from various slits across a grating. It therefore follows that, for the minimum intensity, the path difference between disturbances from the first to the last slit is one wavelength, λ, more than that for the maximum intensity position. The path difference to the minimum is thus $(N - 1) m\lambda + \lambda$. The mth order maximum of another wavelength λ', differing slightly from λ, is formed by extreme rays which have a path difference of $(N - 1) m\lambda'$ From Rayleigh's criterion, the two wavelengths λ' and λ are just resolved when the maximum of λ' falls on the first minimum of λ. In this case,

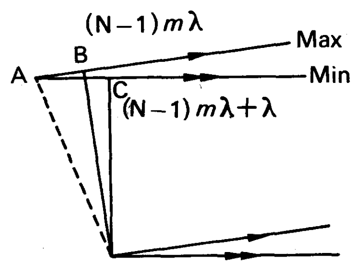

FIG. 29.29. Resolving power of diffraction grating.

$$(N - 1) m\lambda' = (N - 1) m\lambda + \lambda.$$

$$\therefore \quad (N - 1) m (\lambda' - \lambda) = \lambda,$$

$$\therefore \quad \frac{\lambda}{\lambda' - \lambda} = (N - 1) m = Nm,$$

since 1 is negligible compared with N.

$$\therefore \quad \textit{resolving power} = Nm$$

$$\therefore \quad \textit{resolving power} = \frac{Nd \sin \theta}{\lambda} \text{ or } \frac{Nd (\sin i \pm \sin \theta)}{\lambda},$$

the former being the expression for light incident normally on the grating and the latter if the angle of incidence is i. Either expression shows that for a given angle of incidence and diffraction, it is the *total width Nd of the grating which determines its resolving power.* The number of rulings in that width affects the dispersion in a given order but has no effect on the resolving power in that order. Thus a grating of 5 cm width and 6000 lines per cm has twice the resolving power of a grating 2·5 cm wide which also has 6000 lines per cm. If a grating is only 0·3 cm wide it has only about 2000 lines on it of the same spacing, whereas a grating 5 cm wide would have 12000 lines.

The two sodium lines or doublet have wavelengths $5{\cdot}890 \times 10^{-5}$ and $5{\cdot}896 \times 10^{-5}$ cm respectively. The resolving power, R.P., required to distinguish them is given by:

$$\text{R.P.} = \frac{\lambda}{\lambda' - \lambda} = \frac{5{\cdot}890 \times 10^{-5}}{0{\cdot}006 \times 10^{-5}} = 1000 \text{ (approx.)}$$

Thus if a grating has 800 lines per cm, and the width covered by a telescope objective is 2·5 cm the sodium lines are clearly resolved in the first order images. If three-quarters of the grating is covered there are only 500 lines left, and the lines are now no longer resolved in the first order. They are just resolved in the second order images.

Resolving Power of Microscope

ABBE proposed a theory of image formation in a microscope which stated basically that if the structure of the illuminated object is regular (periodic), it acts like an illuminated diffraction grating. In this case the structure appears uniformly bright and unrecognisable if only the zero order image is collected by the microscope objective. If, in addition, the first order diffraction image is collected, the image plane in the objective contains alternate bright and dark strips or fringes in positions corresponding to images of the grating elements. The observer then recognises the grating structure, that is, the grating is "resolved". The more orders collected by the objective, the closer does the intensity distribution across the image plane resemble that transmitted by the object itself. The effect is analogous to the recognition of a note from a violin in Sound. This consists of a fundamental of the same frequency, together with overtones of higher frequency which gives the sound its timbre or quality. If only the fundamental is received, the note will not be recognisable as the note from the violin. The more overtones received in addition to the fundamental, the more faithful is the reproduction of the note.

An expression for the resolving power of a microscope can now be obtained. We require, for resolution, that a first order diffraction image is collected in addition to the zero order. Suppose that an object of regular structure is illuminated at an angle of incidence i by an oblique beam (Fig. 29.30). Then, if the first order image is just collected by the microscope objective,

$$d(\sin i \pm \sin \alpha) = \lambda,$$

where α is the half-angle subtended by the objective at the object O and

d is the grating spacing of the object. The minimum value of d occurs when $i = \alpha$ and $d(\sin i + \sin \alpha) = \lambda$.

$$\therefore \quad 2d \sin \alpha = \lambda$$

$$\therefore \quad d = \frac{\lambda}{2 \sin \alpha}$$

This expression for d gives the grating spacing of the finest regular structure of the object which can just be resolved. If a medium such as oil of refractive index n is used in the object space beneath the objective,

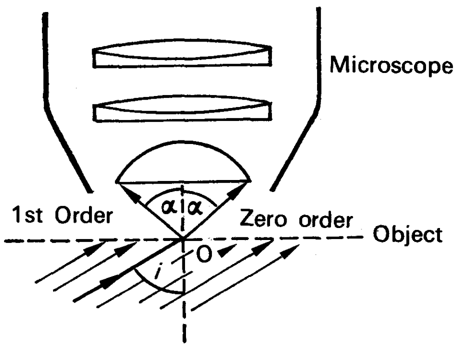

FIG. 29.30. Revolving power of microscope.

the least distance d or *limit of resolution* (also called the "*resolving power*") is:

$$\text{limit of resolution} = \frac{\lambda}{2n \sin \alpha}$$

The use of an oil-immersion objective was suggested by Abbe. The limit of resolution for the best optical microscopes is about 2×10^{-5} cm. The eye can resolve about 0·01 cm. The largest useful magnifying power of a microscope is one which magnifies the limit of resolution of the objective to that of the eye, and is about 1000 with glass lenses and visible light. Higher resolving powers may be obtained with ultra-violet light, from $\lambda/2n \sin \alpha$. An *electron microscope*, which contains electron lenses and utilises electrons in place of light, has a limit of resolution less than 10^{-7} cm owing to the much shorter wavelength of moving electrons compared with that of light (p. 1077). Much larger useful magnifying powers, such as 100 000, are thus obtained by using electron microscopes in place of optical microscopes.

Wavelengths of Electromagnetic Waves

In this book we have encountered rays which affect the sensation of vision (*visible rays*), rays which cause heat (*infra-red rays*, p. 456), and rays which cause chemical action (ultra-violet rays, p. 456). As these rays are all due to electric and magnetic vibrations they are examples of **electromagnetic** waves (see p. 719). Scientists have measured the wavelengths of these waves by a diffraction grating method, and results show a gradual transition in the magnitudes of the wavelength from one type of ray to another. Thus infra-red rays have a longer wavelength than visible rays, which in turn have a longer wavelength than ultra-violet rays. Radio waves are electromagnetic waves of longer wavelength

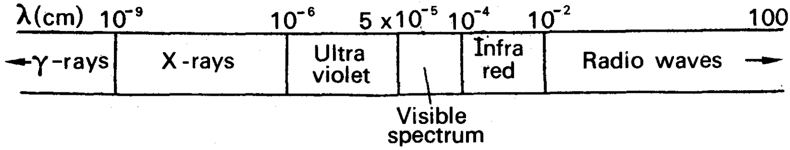

FIG. 29.31. Spectrum of electromagnetic waves (not to scale).

than infra-red rays, while X-rays and γ-rays are due to waves of shorter wavelength than ultra-violet waves. The whole spectrum of electromagnetic waves are shown in Fig. 29.31; this gives only an approximate value of the limits of the wavelength in the various parts of the spectrum, because these limits are themselves vague.

EXAMPLE

What is meant in optics by (*a*) interference, (*b*) diffraction? What part do each of these phenomena play in the production of spectra by a diffraction grating? A parallel beam of sodium light is incident normally on a diffraction grating. The angle between the two first order spectra on either side of the normal is 27° 42′. Assuming that the wavelength of the light is 5893×10^{-8} cm, find the number of rulings per cm on the grating. (*N.*)

First part. Briefly, interference is the name given to the phenomena obtained by the combined effect of light waves from two separate coherent sources; diffraction is the name given to the phenomena due to the combined effect of light waves from secondary sources on the same wavefront. In the diffraction grating, production of spectra is due to the interference between secondary sources on the same wavefront which are separated by a multiple of d, where d is the spacing of the grating rulings (p. 707).

Second part. The first order spectrum occurs at an angle $\theta = \frac{1}{2} \times 27° 42' = 13° 51'$.

But $$d \sin \theta = \lambda$$

$$\therefore \quad d = \frac{\lambda}{\sin \theta} = \frac{5893 \times 10^{-5}}{\sin 13° 51'} \text{ cm}$$

$$\therefore \text{ number of rulings per cm } = \frac{1}{d} = \frac{\sin 13° 51'}{5893 \times 10^{-8}}$$

$$= 4062$$

POLARISATION OF LIGHT

We have shown that light is a wave-motion of some kind, i.e., that it is a travelling vibration. For a long time after the wave-theory was revived it was thought that the vibrations of light occurred in the same direction as the light wave travelled, analogous to sound waves. Thus light waves were thought to be longitudinal waves (p. 584). Observations and experiments, however, to be described shortly, showed that the vibrations of light occur in planes *perpendicular* to the direction along which the light wave travels, and thus light waves are *transverse* waves.

Polarisation of Transverse Waves

Suppose that a rope ABCD passes through two parallel slits, B, C, and is attached to a fixed point at D, Fig. 29.32 (i). Transverse waves can be set up along AB by holding the end A in the hand and moving it up and down in all directions perpendicular to AB, as illustrated by the arrows in the plane X. A wave then emerges along BC, but unlike the

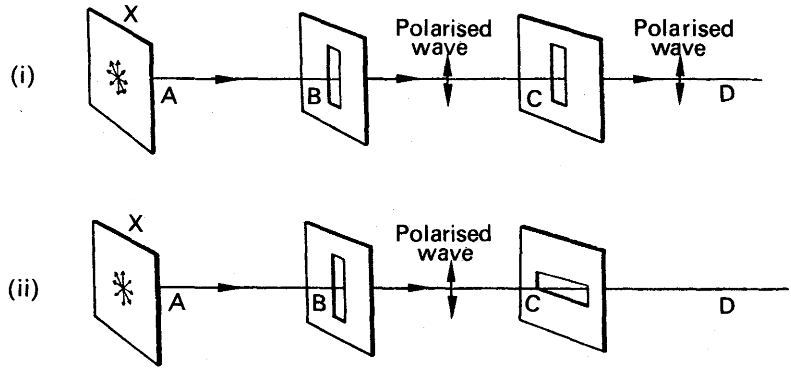

FIG. 29.32. Formation of plane-polarised waves.

waves along AB, which are due to transverse vibrations in every plane, it is due only to transverse vibrations parallel to the slit at B. This type of wave is called a *plane-polarised* wave. It shows a *lack of symmetry about the direction of propagation*, because a slit C allows the wave to pass through when it is parallel to B, but prevents it from passing when C is perpendicular to B, Fig. 29.32 (i), (ii). If B is turned so that it is perpendicular to the position shown in Fig. 29.32 (i), a polarised wave is again obtained along BC; but the vibrations which produce it are perpendicular to those shown between B and C in Fig. 29.32 (i).

Polarised Light

Years ago it was discovered accidentally that certain natural crystals affect light passing through them. *Tourmaline* is an example of such a crystal, *quartz* and *calcite* or *Iceland spar* are others (p. 720). Suppose two tourmaline crystals, P, Q, are placed with their axes, *a*, *b*, parallel, Fig. 29.33 (i). If a beam of light is incident on P, the light emerging from Q appears slightly darker. If Q is rotated slowly about the line of vision, with its plane parallel to P, the emergent light becomes darker and darker, and at one stage it disappears. In the latter case the axes *a*, *b* of the crystals are perpendicular, Fig. 29.33 (ii). When Q is rotated further the light reappears, and becomes brightest when the axes *a*, *b* are again parallel.

This simple experiment leads to the conclusion that light waves are *transverse* waves; otherwise the light emerging from Q could never be extinguished by simply rotating this crystal. The experiment, in fact, is analogous to that illustrated in Fig. 29.32, where transverse waves were set up along a rope and plane-polarised waves were obtained by means of a slit B. Tourmaline is a crystal which, because of its internal molecular structure, transmits only those vibrations of light parallel to its axis. Consequently plane-polarised light is obtained beyond the crystal P, and no light emerges beyond Q when its axis is perpendicular to P. Fig. 29.33 should be compared with Fig. 29.32.

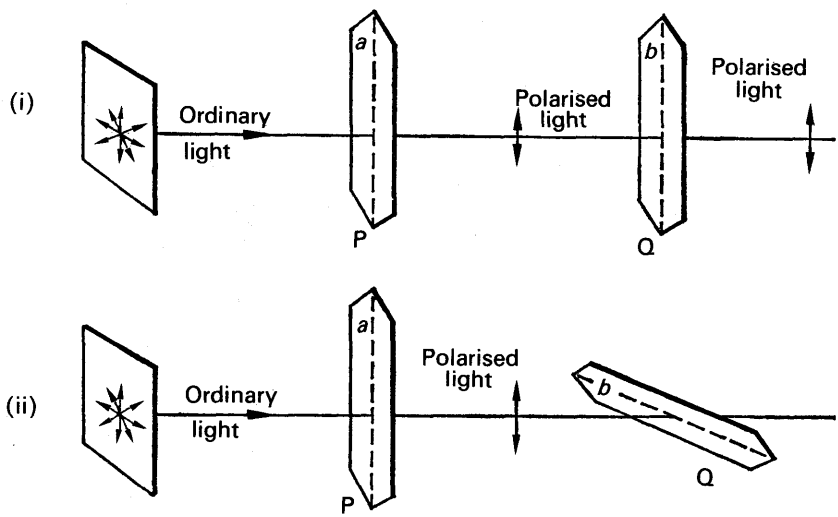

FIG. 29.33. Formation of plane-polarised light waves.

Vibrations in Unpolarised and Polarised Light

Fig. 29.34 (i) is an attempt to represent diagrammatically the vibrations of ordinary or unpolarised light at a point A when a ray travels in a direction AB. X is a plane perpendicular to AB, and ordinary (unpolarised) light may be imagined as due to vibrations which occur in

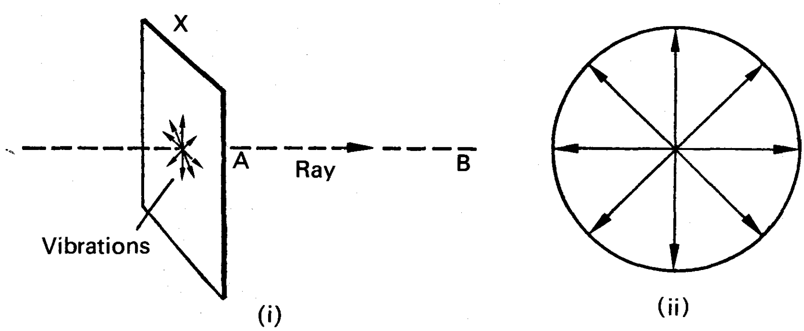

FIG. 29.34. (i). Vibrations occur in every plane perpendicular to AB.
(ii). Vibrations in ordinary light.

every one of the millions of planes which pass through AB and are perpendicular to X. As represented in Fig. 29.34 (ii), the amplitudes of the vibrations are all equal.

Consider the vibrations in ordinary light when it is incident on the tourmaline P in Fig. 29.33 (i). Each vibrations can be resolved into two components, one in a direction parallel to the axis *a* of the tourmaline P and the other in a direction *m* perpendicular to *a*, Fig. 29.35. Tourmaline absorbs the light due to the latter vibrations, known as the *ordinary* rays, allowing the light due to the former vibrations, known as the *extraordinary* rays, to pass through it. Thus plane-polarised light, due to the extraordinary rays, is produced by the tourmaline. Polaroid is a crystaline material, used in sun-glasses for example, which also has selective absorption.

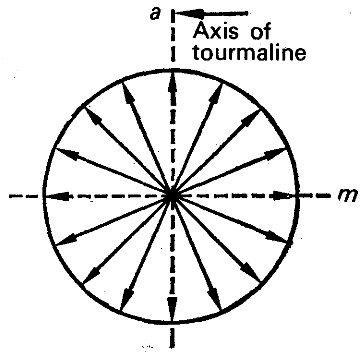

FIG. 29.35. Production of plane-polarised waves by tourmaline.

Light waves are electromagnetic waves. Theory and experiment show that the vibrations of light are *electromagnetic* in origin; a varying electric vector *E* is present, with a varying magnetic vector *B* which has the same frequency and phase. *E* and *B* are perpendicular to each other, and are in a plane at right angles to the ray of light, Fig. 29.36. Experiments have shown that the electric force in a light wave affects a photographic plate and causes fluorescence, while the magnetic force, though present, plays no part in this effect of a light wave. On this account the vibrations of the electric force, *E*, are now chosen as the "vibrations of light", and the planes containing the vibrations shown in Fig. 29.35 (i), (ii) are those in which only the electric forces are present.

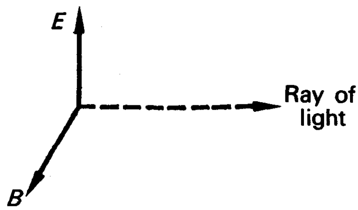

FIG. 29.36. Electromagnetic wave.

Polarised Light by Reflection

The production of polarised light by tourmaline is due to *selective absorption* of the "ordinary" rays. In 1808 MALUS discovered that polarised light is obtained when ordinary light is reflected by a plane sheet of glass (p. 719). The most suitable angle of incidence is about 56°, Fig. 29.37. If the reflected light is viewed through a tourmaline crystal which is slowly rotated about the line of vision, the light is practically extinguished at one position of the crystal. This proves that the light reflected by the glass is plane-polarised. Malus also showed that the light reflected by water is plane-polarised.

The production of the polarised light by the glass is explained as follows. Each of the vibrations of the incident (ordinary) light can be resolved into a component parallel to the glass surface and a component perpendicular to the surface. The light due to the components

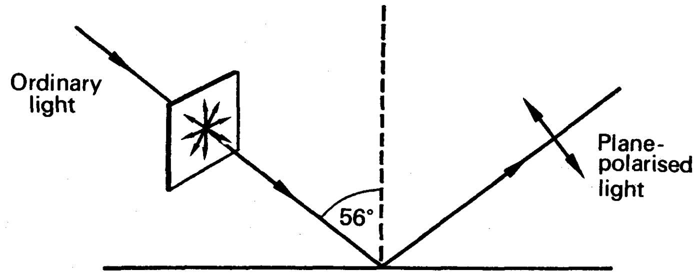

FIG. 29.37. Plane-polarised waves by reflection.

parallel to the glass is reflected, but the remainder of the light, due to the components perpendicular to the glass, is *refracted* into the glass. Thus the light reflected by the glass is plane-polarised.

Brewster's Law. Polarisation by Pile of Plates

The particular angle of incidence i on a transparent medium when the reflected light is almost completely plane-polarized is called the *polarising angle*. BREWSTER found that, in this case, *tan* $i = n$, where n is the refractive index of the medium (*Brewster's law*). Since sin i/sin r, where r is the angle of refraction, it then follows that cos i = sin r, or $i + r = 90°$. Thus the reflected and refracted beams are at 90° to each other.

The refracted beam contains light mainly due to vibrations perpendicular to that reflected and is therefore partially plane-polarised. Since refraction and reflection occur at both sides of a glass plate, the transmitted beam contains a fair percentage of plane-polarised light. A *pile of plates* increases the percentage, and thus provides a simple method of producing plane-polarised light. They are mounted inclined in a tube so that the ordinary (unpolarised) light is incident at the polarising angle, and the transmitted light it then fairly plane-polarised.

Polarisation by Double Refraction

We have already considered two methods of producing polarised light. The first observation of polarised light, however, was made by BARTHOLINUS in 1669, who placed a crystal of iceland spar on some words on a sheet of paper. To his surprise, two images were seen through the crystal. Bartholinus therefore gave the name of *double refraction* to the phenomenon, and experiments more than a century later showed that the crystal produced plane-polarised light when ordinary light was incident on it. See Fig. 29.38.

Iceland spar is a crystalline form of calcite (calcium carbonate) which cleaves in the form of a "rhomboid" when it is lightly tapped; this is a solid whose opposite faces are parallelograms. When a beam of unpolarised light is incident on one face of the crystal, its internal molecular structure produces two beams of polarised light, E, O, whose vibrations are perpendicular to each other, Fig. 29.39. If the incident direction AB is parallel to a plane known as the "principal section" of the crystal, one

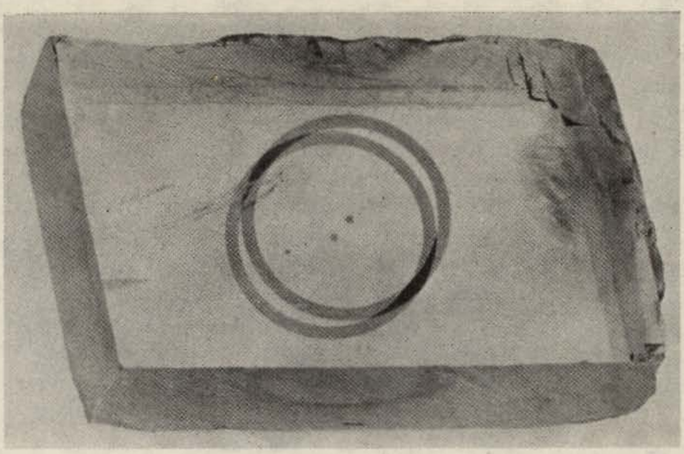

FIG. 29.38. DOUBLE REFRACTION. A ring with a spot in the centre, photographed through a crystal of Iceland spar. The light forms two rings and two spots.

beam O emerges parallel to AB, while the other beam E emerges displaced in a different direction. As the crystal is rotated about the line of vision the beam E revolves round O. On account of this abnormal behaviour the rays in E are called "extraordinary" rays; the rays in O are known as "ordinary" rays (p. 719). Thus two images of a word on a paper, for example, are seen when an Iceland spar crystal is placed on top of it; one image is due to the ordinary rays, while the other is due to the extra-ordinary rays.

With the aid of an Iceland spar crystal Malus discovered the polarisation of light by reflection (p. 719). While on a visit to Paris he gazed

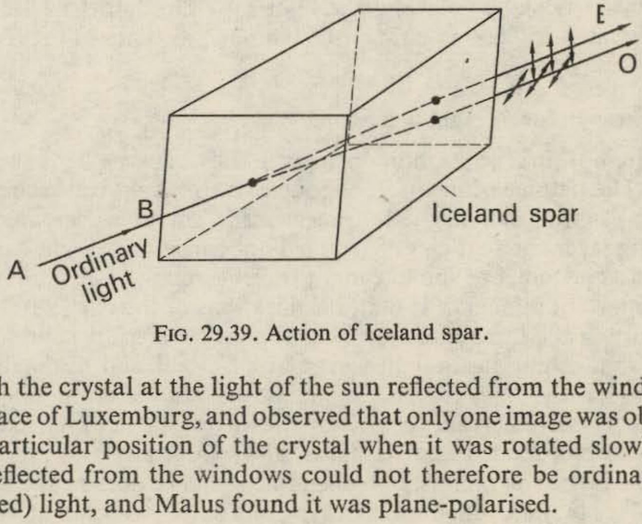

FIG. 29.39. Action of Iceland spar.

through the crystal at the light of the sun reflected from the windows of the Palace of Luxemburg, and observed that only one image was obtained for a particular position of the crystal when it was rotated slowly. The light reflected from the windows could not therefore be ordinary (un-polarised) light, and Malus found it was plane-polarised.

Nicol Prism

We have seen that a tourmaline crystal produces polarised light, and that the crystal can be used to detect such light (p. 717). NICOL designed a crystal of Iceland spar which is widely used for producing and detecting

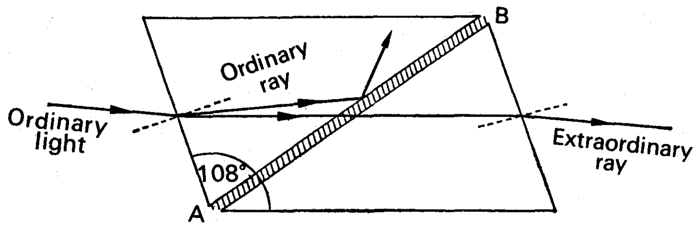

FIG. 29.40. Action of Nicol prism.

polarised light, and it is known as a *Nicol prism.* A crystal whose faces contain angles of 72° and 108° is broken into two halves along the diagonal AB, and the halves are cemented together by a layer of Canada balsam, Fig 29. 40. The refractive index of the crystal for the ordinary rays is 1·66, and is 1·49 for the extraordinary rays; the refractive index of the Canada balsam is about 1·55 for both rays, since Canada balsam does not produce polarised light. A critical angle thus exists between the crystal and Canada balsam for the ordinary rays, but not for the extraordinary rays. Hence total reflection of the former rays takes place at the canada balsam if the angle of incidence is large enough, as it is with the Nicol prism. The emergent light is then due to the extraordinary rays, and is polarised.

The prism is used like a tourmaline crystal to detect plane-polarised light, namely, the prism is held in front of the beam of light and is rotated. If the beam is plane-polarised the light seen through the Nicol prism varies in intensity, and is extinguished at one position of the prism.

Differences Between Light and Sound Waves

We are now in a position to distinguish fully between light and sound waves. The physical difference, of course, is that light waves are due to varying electric and magnetic forces, while sound waves are due to vibrating layers or particles of the medium concerned. Light can travel through a vacuum, but sound cannot travel through a vacuum. Another very important difference is that the vibrations of the particles in sound waves are in the same direction as that along which the sound travels, whereas the vibrations in light waves are perpendicular to the direction along which the light travels. Sound waves are therefore *longitudinal* waves, whereas light waves are *transverse* waves. As we have seen, sound waves can be reflected and refracted, and can give rise to interference phenomena; but no polarisation phenomena can be obtained with sound waves since they are longitudinal waves, unlike the case of light waves.

EXERCISES 29

Interference

1. Describe how to set up apparatus to observe and make measurements on the interference fringes produced by Young's slits. Explain how (i) the wavelengths of two monochromatic light sources could be compared, (ii) the separation of the slits could be deduced using a source of known wavelength. Establish any formula required.

State, giving reasons, what you would expect to observe (*a*) if a white light source were substituted for a monochromatic source, (*b*) if the source slit were then displaced slightly at right angles to its length in the plane parallel to the plane of the Young's slits. (*L*.)

2. Explain the formation of interference fringes by an air wedge and describe how the necessary apparatus may be arranged to demonstrate them.

Fringes are formed when light is reflected between the flat top of a crystal resting on a fixed base and a sloping glass plate. The lower end of the plate rests on the crystal and the upper end on a fixed knife-edge. When the temperature of the crystal is raised the fringe separation changes from 0·96 mm to 1·00 mm. If the length of the glass plate from knife-edge to crystal is 5·00 cm, and the light of wavelength $6{\cdot}00 \times 10^{-5}$ cm is incident normally on the wedge, calculate the expansion of the crystal. (*L*.)

3. Describe in detail how the radius of curvature of the spherical face of a planoconvex lens may be found by observations made on Newton's rings.

Two plane glass plates which are in contact at one edge are separated by a piece of metal foil 12·50 cm from that edge. Interference fringes parallel to the line of contact are observed in reflected light of wavelength 5460 Å and are found to be 1·50 mm apart. Find the thickness of the foil. (*L*.)

4. Describe, with the aid of a labelled diagram, how the wavelength of monochromatic light may be found using Young's slits. Give the theory of the experiment.

State, and give physical reasons for the features which are common to this method and to *either* the method based on Lloyd's mirror *or* that based on Fresnel's biprism.

In an experiment using Young's slits the distance between the centre of the interference pattern and the tenth bright fringe on either side is 3·44 cm and the distance between the slits and the screen is 2·00 m. If the wavelength of the light used is $5{\cdot}89 \times 10^{-7}$ m determine the slit separation. (*N*.)

5. Explain what is meant by the term *path-difference* with reference to the interference of two wave-motions.

Why is it not possible to see interference where the light beams from the headlamps of a car overlap?

Interference fringes were produced by the Young's slits method, the wavelength of the light being 6×10^{-5} cm. When a film of material $3{\cdot}6 \times 10^{-3}$ cm thick was placed over *one* of the slits, the fringe pattern was displaced by a distance equal to 30 times that between two adjacent fringes. Calculate the refractive index of the material. To which side are the fringes displaced?

(When a layer of transparent material whose refractive index is n and whose thickness is d is placed in the path of a beam of light, it introduces a path difference equal to $(n-1)d$.) (*O. & C.*)

6. Show how, with the aid of Huygens' idea of secondary wavelets, the wave theory of light will account for the laws of refraction and of reflexion at a plane surface.

Describe briefly Young's two-slit experiment and explain how it confirms the wave nature of light. (*L.*)

7. Describe, giving both theory and experimental detail, how you would find the radius of curvature of one surface of a convex lens by means of Newton's rings. You may assume that monochromatic light of a known wavelength is available.

Newton's rings are formed by reflexion between an equiconvex lens of focal length 100 cm made of glass of refractive index 1·50 and in contact with a plane glass plate of refractive index 1·60. Find the radius of the 5th bright ring using monochromatic light of wavelength 6000 Å.

Explain the changes which occur when oil of refractive index 1·55 fills the space between the lens and plate. (1 Å $= 10^{-8}$ cm.) (*N.*)

8. Define *velocity*, *frequency* and *wavelength* for any wave motion, and deduce a relation between them. What do you understand by 'interference between waves' and 'coherent wave trains'? Explain why interference is not observed between the beams of two electric torches.

Deduce the relation connecting the refractive index of a material with the velocities of light in vacuo and in the material. State clearly the assumptions you make about wave fronts in order to do this.

Light passes through a single crystal of ruby 10·0 cm long and emerges with a wavelength of $6{\cdot}94 \times 10^{-5}$ cm. If the critical angle of ruby for light of this wavelength if 34° 50′, calculate the number of wavelengths inside the crystal. (*C.*)

9. State the conditions necessary for the production of interference effects by two overlapping beams of light.

Describe fully *one* method for the production of interference fringes using light from a given monochromatic source. Show how with the aid of suitable measurements the wavelength of light emitted by the source may be determined with your apparatus.

Describe how the fringes produced by your apparatus would appear if a source of white light were employed instead of a monochromatic one. (*O. & C.*)

10. Explain how Newton's rings are formed, and describe how you would demonstrate them experimentally. How is it possible to predict the appearance of the centre of the ring pattern when (*a*) the surfaces are touching, and (*b*) the surfaces are not touching?

In a Newton's rings experiment one surface was fixed and the other movable along the axis of the system. As the latter surface was moved the rings appeared to contract and the centre of the pattern, initially at its darkest, became alternately bright and dark, passing through 26 bright phases and finishing at its darkest again. If the wavelength of the light was 5461 Å, how far was the surface moved and did it approach, or recede from, the fixed surface? Suggest one possible application of this experiment. (*O. & C.*)

11. Explain the formation of Newton's rings and describe how you would use them to measure the radius of curvature of the convex surface of a long-focus planoconvex lens.

The diameters of the mth and $(m + 10)$th bright rings formed by such a lens resting on a plane glass surface are respectively 0·14 cm and 0·86 cm. When the space between lens and glass is filled with water the diameters of the qth and $(q + 10)$th bright rings are respectively 0·23 cm and 0·77 cm. What is the refractive index of water? (*L.*)

12. What are the conditions essential for the production of optical interference fringes?

Explain how these conditions are satisfied in the case of (*a*) Young's fringes, and (*b*) thin film interference fringes. (*N.*)

13. Describe, in detail how you would arrange apparatus to observe, in monochromatic light, interference fringes formed by light reflected from two glass plates enclosing an air wedge. Show how the angle of the wedge could be obtained from measurements on the fringes.

Newton's rings are formed with light of wavelength $5{\cdot}89 \times 10^{-5}$ cm between the curved surface of a planoconvex lens and a flat glass plate, in perfect contact. Find the radius of the 20th dark ring from the centre if the radius of curvature of the lens surface is 100 cm. How will this ring move and what will its radius become if the lens and the plate are slowly separated to a distance apart of $5{\cdot}00 \times 10^{-4}$ cm? (*L.*)

14. What are the necessary conditions for interference of light to be observable? Describe with the aid of a labelled diagram how optical interference may be demonstrated using Young's slits. Indicate suitable values for all the distances shown.

How are the colours observed in thin films explained in terms of the wave nature of light? Why does a small oil patch on the road often show approximately circular coloured rings? (*L.*)

Diffraction

15. Describe and give the theory of an experiment to compare the wavelengths of yellow light from a sodium and red light from a cadmium discharge lamp, using a diffraction grating. Derive the required formula from first principles.

White light is reflected normally from a soap film of refractive index 1·33 and then directed upon the slit of a spectrometer employing a diffraction grating at normal incidence. In the first-order spectrum a dark band is observed with minimum intensity at an angle of 18° 0′ to the normal. If the grating has 5000 lines per cm, determine the thickness of the soap film assuming this to be the minimum value consistent with the observations. (*L.*)

16. Describe the phenomena which occur when plane waves pass (*a*) through a wide aperture, (*b*) through an aperture whose width is comparable with the wavelength of the waves.

How does the wave theory of light account for the apparent rectilinear propagation of light?

A diffraction grating has 6000 lines per cm. Calculate the angular separation between wavelengths $5{\cdot}896 \times 10^{-5}$ cm and $5{\cdot}461 \times 10^{-5}$ cm respectively after transmission through it at normal incidence, in the first-order spectrum. (*O. & C.*)

17. Describe two experiments to show the diffraction of light.

Describe how a diffraction grating may be used to measure the wavelength of sodium light, deriving any formulae employed. (*L.*)

18. What are the advantages and disadvantages of a diffraction grating as compared with a prism for the study of spectra?

A rectangular piece of glass 2 cm × 3 cm has 18000 evenly spaced lines ruled across its whole surface, parallel to the shorter side, to form a diffraction grating. Parallel rays of light of wavelength 5×10^{-5} cm fall normally on the grating. What is the highest order of spectrum in the transmitted light?

What is the minimum diameter of a camera lens which can accept all the light of this wavelength in this order which leaves the grating on one side of the normal? *(O. & C.)*

19. In an experiment using a spectrometer in normal adjustment fitted with a plane transmission grating and using monochromatic light of wavelength $5{\cdot}89 \times 10^{-5}$ cm, diffraction maxima are obtained with telescope settings of 153° 44′, 124° 5′, 76° 55′ and 47° 16′, the central maximum being at 100° 30′. Show that these observations are consistent with normal incidence and calculate the number of rulings per cm of the grating.

If this grating is replaced by an opaque plate having a single vertical slit $2{\cdot}00 \times 10^{-2}$ cm wide, describe and explain the diffraction pattern which may now be observed. Contrast the appearance of this pattern with that produced by the grating. (*N.*)

20. (*a*) What is meant by (i) *diffraction*, (ii) *superposition* of waves? Describe *one* phenomenon to illustrate each in the case of *sound waves*.

(*b*) The floats of two men fishing in a lake from boats are 22·5 metres apart. A disturbance at a point in line with the floats sends out a train of waves along the surface of the water, so that the floats bob up and down 20 times per minute. A man in a third boat observes that when the float of one of his colleagues is on the crest of a wave that of the other is in a trough, and that there is then one crest between them. What is the velocity of the waves? *(O. & C.)*

21. Give an account of the theory of the production of a spectrum by means of a plane diffraction grating. How does it differ from the spectrum produced by means of a prism?

Parallel light consisting of two monochromatic radiations of wavelengths 6×10^{-5} cm and 4×10^{-5} cm falls normally on a plane transmission grating ruled with 5000 lines per cm. What is the angular separation of the second-order spectra of the two wavelengths? (*C.*)

22. A pure spectrum is one in which there is no overlapping of light of different wavelengths. Describe how you would set up a diffraction grating to display on a screen as close an approximation as possible to a pure spectrum. Explain the purpose of each optical component which you would use.

A grating spectrometer is used at normal incidence to observe the light from a sodium flame. A strong yellow line is seen in the first order when the telescope axis is at an angle of 16° 26′ to the normal to the grating. What is the highest order in which the line can be seen?

The grating has 4800 lines per cm; calculate the wavelength of the yellow radiation.

What would you expect to observe in the spectrometer set to observe the first-order spectrum if a small but very bright source of white light is placed close to the sodium flame so that the flame is between it and the spectrometer? (*O. & C.*)

23. Describe how you would determine the wavelength of monochromatic light using a diffraction grating and a spectrometer. Give the theory of the method.

A filter which transmits only light between 6300 Å and 6000 Å is placed between a source of white light and the slit of a spectrometer; the grating has 5000 lines to the centimetre; and the telescope has an objective of focal length 15 cm with an eyepiece of focal length 3 cm. Find the width in millimetres of the first-order spectrum formed in the focal plane of the objective. Find also the angular width of this spectrum seen through the eyepiece. (*O.*)

Polarisation

24. What is meant by *plane of polarisation*? Explain why the phenomenon of polarisation is met with in dealing with light waves, but not with sound waves.

Describe and explain the action of (*a*) a nicol prism, (*b*) a sheet of Polaroid.

How can a pair of Polaroid sheets and a source of natural light be used to produce a beam of light the intensity of which may be varied in a calculable manner? (*L.*)

25. Explain what is meant by the statement that a beam of light is *plane polarised*. Describe *one* experiment in each instance to demonstrate (*a*) polarisation by reflexion, (*b*) polarisation by double refraction, (*c*) polarisation by scattering.

The refractive index of diamond for sodium light is 2·417. Find the angle of incidence for which the light reflected from diamond is completely plane polarised. (*L.*)

26. Give an account of the action of (*a*) a single glass plate. (*b*) a Nicol prism, in producing plane-polarised light. State *one* disadvantage of *each* method.

Mention *two* practical uses of polarising devices. (*N.*)

27. Describe how, using a long, heavy rope, you would demonstrate (*a*) a plane-polarised wave, and (*b*) a stationary wave.

Give a short account, with diagrams, of *three* ways in which plane-polarised light is obtained (other than by using 'polaroid'). State some uses of polarised light.

Two polaroid sheets are placed close together in front of a lamp so that no light passes through them. Describe and explain what happens when one sheet is slowly rotated, the other remaining in its original position. (*C.*)

28. Answer *two* of the following:

(i) How may it be shown that the radiation from (*a*) a sodium lamp, and (*b*) a radio transmitter (such as a broadcasting station or a microwave source) consists of waves?

(ii) Explain what is meant by the polarisation of light, and describe how you would demonstrate it. Why is light from most light sources unpolarised?

(iii) When a diffraction grating is illuminated normally by monochromatic light an appreciable amount of light leaves the grating in certain directions. Explain this phenomenon, and show how these directions may be predicted. (*O. & C.*)

29. What is meant by (*a*) polarised light, (*b*) polarising angle? Describe and explain two methods for producing plane-polarised light.

Calculate the polarising angle for light travelling from water, of refractive index 1·33, to glass, of refractive index 1·53. (*L.*)

30. A beam of plane-polarised light falls normally on a sheet of Polaroid, which is at first set so that the intensity of the transmitted light, as estimated by a photographer's light-meter, is a maximum. (The meter is suitably shielded from all other illumination.) Describe and explain the way in which you would expect the light-meter readings to vary as the Polaroid is rotated in stages through 180° about an axis at right angles to its plane.

How would you show experimentally (*a*) that calcite is doubly refracting, (*b*) that the two refracted beams are plane polarised, in planes at right angles to one another, and (*c*) that in general the two beams travel through the crystal with different velocities? (*O.*)

31. What is plane-polarised light?

Explain why two images of an object are seen through a crystal of Iceland Spar. What would be seen if the object were viewed through two crystals, one of which was slowly rotated about the line of vision?

How would you produce a plane-polarised beam of light by reflection from a glass surface? (*C.*)

32. What is meant by the polarisation of light? How is polarisation explained on the hypothesis that light has wave properties?

Describe how polarisation can be produced and detected by reflexion. Mention another way of obtaining polarised light and describe how you would determine which of the two methods is the more effective.

Describe briefly *two* uses of polarised light. (*N.*)

33. Give an account of the evidence for believing that light is a wave motion. What reason is there for believing light waves to be transverse waves?

Two dishes *A* and *B* each contain liquid to a depth of 3·000 cm. *A* contains alcohol, *B* a layer of water on which is a layer of transparent oil. The depths of the oil and water are adjusted so that for monochromatic light passing vertically through them, the number of wavelengths is the same in *A* and *B*. Find the depth of the water layer, if the refractive indices of alcohol, water and oil are respectively 1·363, 1·333 and 1·475. (*L.*)