chapter four

Static Bodies. Fluids

STATIC BODIES

Statics

1. STATICS is a subject which concerns the *equilibrium* of forces, such as the forces which act on a bridge. In Fig. 4.1 (i), for example, the joint O of a light bridge is in equilibrium under the action of the two forces P, Q acting in the girders meeting at O and the reaction S of the masonry at O.

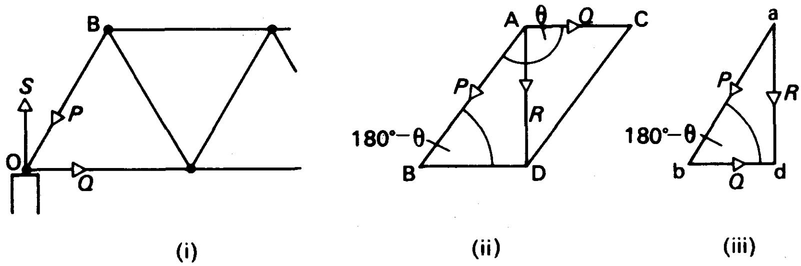

FIG. 4.1 Equilibrium of forces

Parallelogram of Forces

A force is a vector quantity, i.e., it can be represented in magnitude and direction by a straight line (p. 1). If AB, AC represent the forces P, Q respectively at the joint O, their *resultant*, R, is represented in magnitude and direction by the diagonal AD of the parallelogram ABDC which has AB, AC as two of its adjacent sides, Fig. 4.1 (ii). This is known as the *parallelogram of forces*, and is exactly analogous to the parallelogram of velocities discussed on p. 8. Alternatively, a line *ab* may be drawn to represent the vector P and *bd* to represent Q, in which case *ad* represents the resultant R.

By trigonometry for triangle ABD, we have

$$AD^2 = BA^2 + BD^2 - 2BA \,.\, BD \cos ABD.$$

$$\therefore\ R^2 = P^2 + Q^2 + 2PQ \cos \theta,$$

where θ = angle BAC; the angle between the forces P, Q, = $180° -$ angle ABD. This formula enables R to be calculated when P, Q

and the angle between them are known. The angle BAD, or α say, between the resultant R and the force P can then be found from the relation

$$\frac{R}{\sin\theta} = \frac{Q}{\sin\alpha},$$

applying the sine rule to triangle ABD and noting that angle ABD = $180° - \theta$.

Resolved component. On p. 8 we saw that the effective part, or resolved component, of a vector quantity X in a direction θ inclined to it is given by $X\cos\theta$. Thus the resolved component of a force P in a direction making an angle of 30° with it is $P\cos 30°$; in a perpendicular direction to the latter the resolved component is $P\cos 60°$, or $P\sin 30°$. In Fig. 4.1 (i), the downward component of the force P on the joint of O is given by $P\cos$ BOS.

Forces in Equilibrium. Triangle of Forces

Since the joint O is in equilibrium, Fig. 4.1 (i), the resultant of the forces P, Q in the rods meeting at this joint is equal and opposite to the reaction S at O. Now the diagonal AD of the parallelogram ABDC in Fig. 4.1 (ii) represents the resultant R of P, Q since ABDC is the parallelogram of forces for P, Q; and hence DA represents the force S. Consequently the sides of the triangle ABD represent the three forces at O in magnitude and direction: This result can be generalised as follows. *If three forces are in equilibrium, they can be represented by the three sides of a triangle taken in order.* This theorem in Statics is known as the *triangle of forces.* In Fig. 4.1 (ii), AB, BD, DA, in this order, represent, P, Q, S respectively in Fig. 4.1 (i)

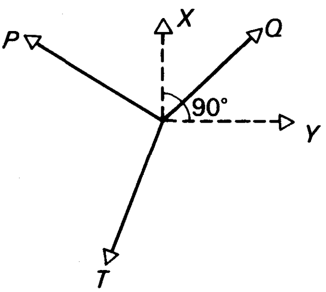

FIG. 4.2 Resolution of forces

We can derive another relation between forces in equilibrium. Suppose X, Y are the respective algebraic sums of the resolved components in two perpendicular directions of three forces P, Q, T in equilibrium, Fig. 4.2. Then, since X, Y can each be represented by the sides of a *rectangle* drawn to scale, their resultant R is given by

$$R^2 = X^2 + Y^2 \quad . \quad . \quad . \quad . \quad \text{(i)}$$

Now if the forces are in equilibrium, R is zero. It then follows from (i) that X must be zero and Y must be zero. Thus *if forces are in equilibrium the algebraic sum of their resolved components in any two perpendicular directions is respectively zero.* This result applies to any number of forces in equilibrium.

EXAMPLE

State what is meant by *scalar* and *vector* quantities, giving examples of each.

Explain how a flat kite can be flown in a wind that is blowing horizontally. The line makes an angle of 30° with the vertical and is under a tension of 0·1 newton; the mass of the kite is 5 g. What angle will the plane of the kite make with the vertical, and what force will the wind exert on it? (*O. & C.*)

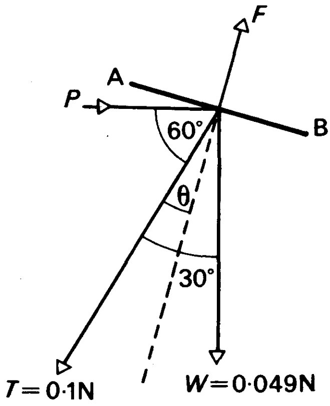

FIG. 4.3 Example

Second part. When the kite AB is inclined to the horizontal, the wind blowing horizontally exerts an upward force F normal to AB, Fig. 4.3. For equilibrium of the kite, F must be equal and opposite to the resultant of the tension T, 0·1 newton, and the weight W, $0{\cdot}005 \times 9{\cdot}8$ or 0·049 N. By drawing the parallelogram of forces for the resultant of T and W, F and the angle θ between F and T can be found. θ is nearly 10°, and F is about 0·148 N. The angle between AB and the vertical $= 60° + \theta = 70°$ (approx.). Also, since F is the component of the horizontal force P of the wind.

$$P\cos(60° + \theta) = F.$$

$$\therefore P = \frac{F}{\cos(60° + \theta)} = \frac{0{\cdot}148}{\cos 70°}$$

$$= 0{\cdot}43 \text{ N}.$$

Moments

When the steering-wheel of a car is turned, the applied force is said to exert a *moment*, or turning-effect, about the axle attached to the wheel. The magnitude of the moment of a force P about a point O is defined as *the product of the force P and the perpendicular distance OA* for all the forces in Fig. 4.4 (ii), we have

$$\text{moment} = P \times \text{AO}.$$

The magnitude of the moment is expressed in *newton metre* (N m) when

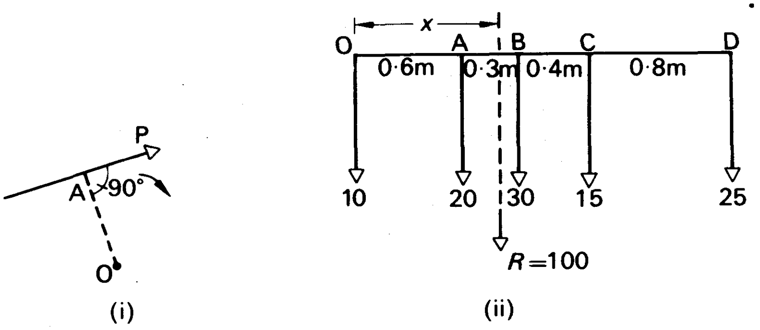

FIG. 4.4 Parallel forces

P is in newtons and AO is in metres. We shall take an anticlockwise moment as positive in sign and a clockwise moment as negative in sign.

Parallel Forces

If a rod carries loads of 10, 20, 30, 15, and 25 N at O, A, B, C, D respectively, the resultant R of the weights, which are parallel forces, for all the forces in Fig. 4.4 (ii), we have

$$\text{resultant, } R, = 10+20+30+15+25 = 100 \text{ N}.$$

Experiment and theory show that *the moment of the resultant of a number of forces about any point is equal to the algebraic sum of the moments of the individual forces about the same point.* This result enables us to find where the resultant R acts. Taking moments about O for all the forces in Fig. 4.4 (ii), we have

$$(20\times0{\cdot}6)+(30\times0{\cdot}9)+(15\times1{\cdot}3)+(25\times2{\cdot}1),$$

because the distances between the forces are 0·6 m, 0·3 m, 0·4 m, 0·8 m, as shown. If x m is the distance of the line of action of R from O, the moment of R about O $= R\times x = 100\times x$.

$$\therefore\ 100x = (20\times0{\cdot}6)+(30\times0{\cdot}9)+(15\times1{\cdot}3)+(25\times2{\cdot}1),$$

from which $x = 1{\cdot}1$ m.

Equilibrium of Parallel Forces

The resultant of a number of forces *in equilibrium* is zero; and the moment of the resultant about any point is hence zero. It therefore follows that the algebraic sum of the moments of all the forces about any point is zero when those forces are in equilibrium. This means that the total clockwise moment of the forces about any point = the total anticlockwise moment of the remaining forces about the same point.

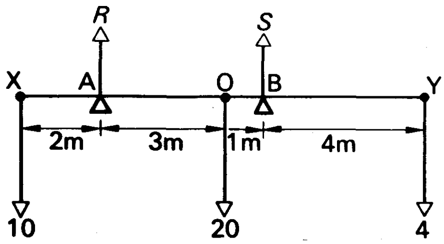

FIG. 4.5 Example

As a simple example of the equilibrium of parallel forces, suppose a light beam XY rests on supports, A, B, and has loads of 10, 20, and 4 N concentrated at X, O, Y respectively, Fig. 4.5. Let R, S be the reactions at A, B respectively. Then, for equilibrium in a vertical direction,

$$R+S = 10+20+4 = 34 \text{ N} \quad . \quad . \quad . \quad \text{(i)}$$

To find R, we take moments about a suitable point such as B, in which case the moment of S is zero. Then, for the remaining four forces,

$$+10.6+20.1-R.4-4.4 = 0,$$

from which $R = 16$ N. From (i), it follows that $S = 34-16 = 18$ N.

Equilibrium of Three Coplanar Forces

If any object is in equilibrium under the action of *three* forces, the resultant of two of the forces must be equal and opposite to the third force. Thus the line of action of the third force must pass through the point of intersection of the lines of action of the other two forces.

As an example of calculating unknown forces in this case, suppose that a 12 m ladder of 20·0 kgf is placed at an angle of 60° to the horizontal, with one end B leaning against a smooth wall and the other end

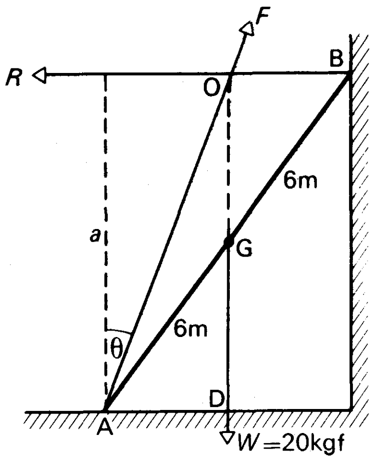

FIG. 4.6 Triangle of forces

A on the ground, Fig. 4.6. The force R at B on the ladder is called the *reaction* of the wall, and if the latter is smooth, R acts perpendicularly to the wall. Assuming the weight, W, of the ladder acts at its mid-point G, the forces W and R meet at O, as shown. Consequently the frictional force F at A passes through O.

The *triangle of forces* can be used to find the unknown forces R, F. Since DA is parallel to R, AO is parallel to F, and OD is parallel to W, the triangle of forces is represented by AOD. By means of a scale drawing R and F can be found, since

$$\frac{W(20)}{\text{OD}} = \frac{F}{\text{AO}} = \frac{R}{\text{DA}}.$$

A quicker method is to take moments about A for all the forces. The algebraic sum of the moments is zero about any point since the object is in equilibrium, and hence

$$R.a-W.\text{AD} = 0,$$

where a is the perpendicular from A to R. (F has zero moment about A.) But $a = 12 \sin 60°$, and AD $= 6 \cos 60°$.

$$\therefore R \times 12 \sin 60° - 20 \times 6 \cos 60° = 0.$$

$$\therefore R = 10 \frac{\cos 60°}{\sin 60°} = 5{\cdot}8 \text{ kgf}.$$

Suppose θ is the angle F makes with the vertical.

Resolving the forces vertically, $F \cos \theta = W = 20$ kgf.

Resolving horizontally, $F \sin \theta = R = 5{\cdot}8$ kgf.

$$\therefore F^2 \cos^2\theta + F^2 \sin^2\theta = F^2 = 20^2 + 5{\cdot}8^2.$$

$$\therefore F = \sqrt{20^2 + 5{\cdot}8^2} = 20{\cdot}8 \text{ kgf}.$$

Couples and Torque

There are many examples in practice where two forces, acting together, exert a moment or turning-effect on some object. As a very simple case, suppose two strings are tied to a wheel at X, Y, and *two equal and opposite forces*, F, are exerted tangentially to the wheel, Fig. 4.7 (i). If the wheel is pivoted at its centre, O, it begins to rotate about O in an anticlockwise direction.

Two equal and opposite forces whose lines of action do not coincide are said to constitute a *couple* in Mechanics. The two forces always have a turning-effect, or moment, called a *torque*, which is defined by

torque = one force × perpendicular distance between forces (1)

Since XY is perpendicular to each of the forces F in Fig. 46 (i), the

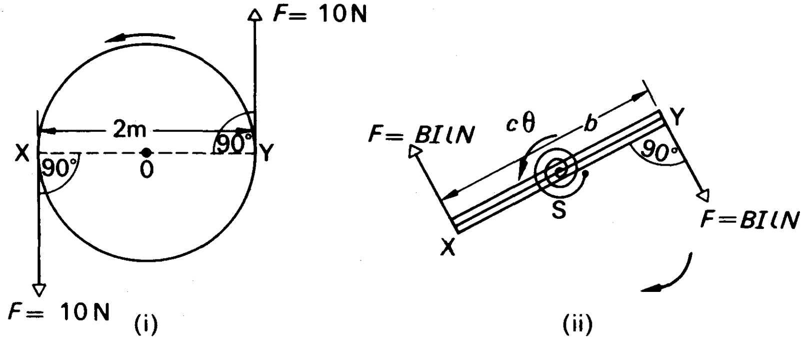

FIG. 4.7 Couple and torque

moment of the couple acting on the wheel $= F \times \text{XY} = F \times$ diameter of wheel. Thus if $F = 10$ newton and the diameter is 2 metre, the moment of the couple or torque = 20 newton metre (N m).

In the theory of the *moving-coil electrical instrument*, we meet a case where a coil rotates when a current I is passed into it and comes to rest after deflection through an angle θ. Fig. 4.7 (ii). The forces F on the two sides X and Y of the coil are both equal to $BIlN$, where B is the strength of the magnetic field, l is the length of the coil and N is the number of turns (see Electricity section, chapter 35). Thus the coil is deflected by a couple. The moment or torque of the deflecting couple $= F \times b$, where b = XY = breadth of coil. Hence

$$\text{torque} = BIlN \times b = BANI,$$

where $A = lb$ = area of coil. The opposing couple, due to the spring S, is $c\theta$, where c is its elastic constant (p.164). Thus, for equilibrium, $BANI = c\theta$.

Work Done by a Couple

Suppose two equal and opposite forces F act tangentially to a wheel W, and rotate it through an angle θ while the forces keep tangentially to the wheel, Fig. 4.8. The moment of the couple is then constant.

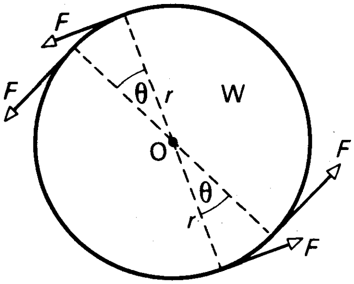

FIG. 4.8 Work done by couple

The work done by each force $= F \times \text{distance} = F \times r\theta$, since $r\theta$ is the distance moved by a point on the rim if θ is in radians.

$$\therefore \text{total work done by couple} = Fr\theta + Fr\theta = 2Fr\theta.$$

But $\quad$ moment of couple $= F \times 2r = 2Fr$

$\therefore$ *work done by couple* = *torque* or *moment of couple* $\times \theta$

Although we have chosen a simple case, the result for the work done by a couple is always given by *torque* × *angle of rotation*. In the formula, it should be carefully noted that θ is in radians. Thus suppose F = 100 gf = 0·1 kgf = 0·1 × 9·8 newton, r = 4 cm = 0·04 metre, and the wheel makes 5 revolutions while the moment of the couple is kept constant. Then

torque or moment of couple = 0·1 × 9·8 × 0·08 newton metre,

and $\quad$ angle of rotation = $2\pi \times 5$ radian.

$\therefore$ work done = 0·1 × 9·8 × 0·08 × 2π × 5 = 2·5 J

Centre of Gravity

Every particle is attracted towards the centre of the earth by the force of gravity, and the *centre of gravity* of a body is the point where the *resultant* force of attraction or *weight* of the body acts. In the simple case of a ruler, the centre of gravity is the point of support when the ruler is balanced. A similar method can be used to find roughly the centre of gravity of a flat plate. A more accurate method consists of suspending the object in turn from two points on it, so that it hangs freely in each case, and finding the point of intersection of a plumb-line, suspended in turn from each point of suspension. This experiment is described in elementary books.

An object can be considered to consist of many small particles. The forces on the particles due to the attraction of the earth are all parallel since they act vertically, and hence their resultant is the sum of all the forces. The resultant is the *weight* of the whole object, of course. In the case of a rod of uniform cross-sectional area, the weight of a particle A at one end, and that of a corresponding particle A′ at the other end, have a resultant which acts at the mid-point O of the rod, Fig. 4.9 (i).

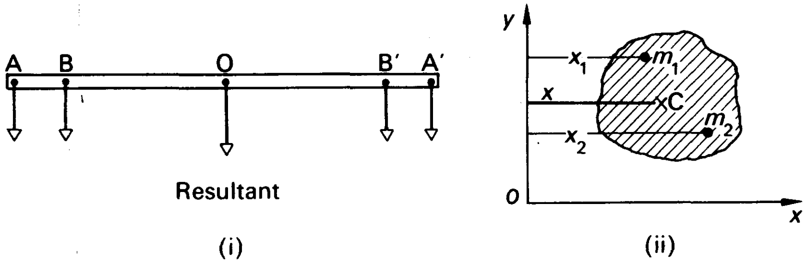

FIG. 4.9 Centre of gravity and mass

Similarly, the resultant of the weight of a particle B, and that of a corresponding particle at B′, have a resultant acting at O. In this way, i.e., by symmetry, it follows that the resultant of the weights of all the particles of the rod acts at O. Hence the centre of gravity of a uniform rod is at its mid-point.

The centre of gravity, C.G., of the curved surface of a hollow cylinder acts at the midpoint of the cylinder axis. This is also the position of the C.G. of a uniform solid cylinder. The C.G. of a triangular plate or lamina is two-thirds of the distance along a median from corresponding point of the triangle. The C.G. of a uniform right solid cone is three-quarters along the axis from the apex.

Centre of Mass

The 'centre of mass' of an object is the point where its total mass acts or appears to act. Fig. 4.9 (ii) illustrates how the position of the centre of mass of an object may be calculated, using axes Ox, Oy.

If m_1 is the mass of a small part of the object and x_1 is the perpendicular distance to Oy, then m_1x_1 represents a product similar to the moment of a weight at m_1 about Oy. Likewise, m_2x_2 is a 'moment' about Oy, where m_2 is another small part of the object. The sum of the total 'moments' about Oy of all the parts of the object can be written Σmx. The total mass $= \Sigma m = M$ say. The distance $\bar{x}$ of the centre of mass C from Oy is then given by

$$\bar{x} = \frac{\Sigma mx}{M}.$$

Similarly, the distance $\bar{y}$ of the centre of mass C from Ox is given by

$$\bar{y} = \frac{\Sigma my}{M}.$$

If the earth's field is uniform at all parts of the body, then the *weight* of a small mass m of it is typically mg. Thus, by moments, the distance of the centre of gravity from Oy is given by

$$\frac{\Sigma mg \times x}{\Sigma mg} = \frac{\Sigma mx}{\Sigma m} = \frac{\Sigma mx}{M}.$$

The acceleration due to gravity, g, cancels in numerator and denominator. It therefore follows that the centre of mass *coincides* with the centre of gravity. However, if the earth's field is *not* uniform at all parts of the object, the weight of a small mass m_1 is then m_1g_1 say and the weight of another small mass m_2 is m_2g_2. Clearly, the centre of gravity does not now coincide with the centre of mass. A very long or large object has different values of g at various parts of it.

EXAMPLE

What is meant by (*a*) the centre of mass of a body, (*b*) the centre of gravity of a body?

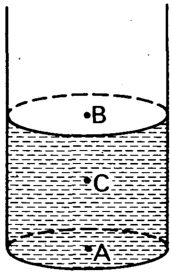

FIG. 4.10 Example

A cylindrical can is made of a material of mass 10 g cm^{-2} and has no lid. The diameter of the can is 25 cm and its height 50 cm. Find the position of the centre of mass when the can is half full of water. (*C.*)

The area of the base $= \pi r^2 = \pi \times (25/2)^2$ cm^2; hence the mass is $\pi \times (25/2)^2 \times 10$ g, and acts at A, the centre of the base, Fig. 4.10.

The mass of the curved surface of the centre $= 2\pi rh \times 10$ g $= 2\pi \times (25/2) \times 50 \times 10$ g, and acts at B, half-way along the axis.

The mass of water $= \pi r^2 h$ g $= \pi \times (25/2)^2 \times 25$ g, and acts at C, the mid-point of AB.

Thus the resultant mass in gramme

$$= \frac{\pi \times 625 \times 10}{4} + \frac{2\pi \times 25 \times 50 \times 10}{2} + \frac{\pi \times 625 \times 25}{4}$$

$$= \pi \times 625 \times 28\tfrac{3}{4}.$$

Taking moments about A,

$$\therefore \ \pi \times 625 \times 28\tfrac{3}{4} \times x = (\pi \times 12500) \times AB + \left(\pi \times \frac{625 \times 25}{4}\right) \times AC$$

where x is the distance of the centre of mass from A.

$$\therefore \ 28\tfrac{3}{4}x = 20 \times 25 + \frac{25}{4} \times 12\tfrac{1}{2}$$

$$\therefore \ x = 20 \text{ (approx).}$$

$\therefore$ centre of mass is 20 cm from the base.

Types of Equilibrium

If a marble A is placed on the curved surface of a bowl S, it rolls

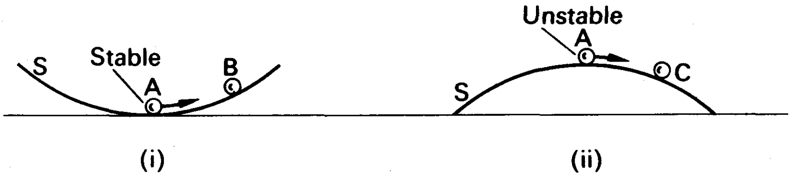

FIG. 4.11 Stable and unstable equilibrium

down and settles in equilibrium at the lowest point. Fig. 4.11 (i). *Its potential energy is then a minimum.* This is the case for objects in any field, gravitational, magnetic or electrical. The equilibrium position corresponds to minimum potential energy.

If the marble A is disturbed and displaced to B, its energy increases. When it is released, the marble rolls back to A. Thus the marble at A is said to be in *stable equilibrium.* Note that the centre of gravity of A is *raised* on displacement to B. On this account the forces in the field return the marble from B to A, where its potential energy is lower.

Suppose now that the bowl S is inverted and the marble is placed at its top point at A. Fig. 4.11 (ii). If A is displaced slightly to C, its potential energy and centre of gravity are then *lowered.* A now continues to move further away from B under the action of the forces in the field. Thus in Fig. 4.10 (ii), A is said to be in *unstable equilibrium.*

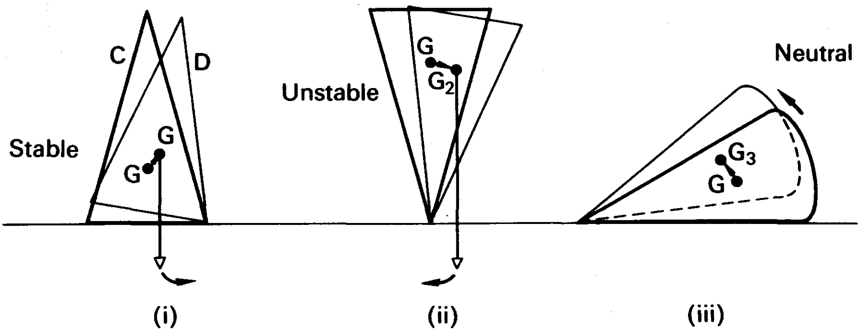

FIG. 4.12 Movement of C.G.

Fig. 4.12 (i) shows a cone C with its base on a horizontal surface. If it is slightly displaced to D, its centre of gravity G rises to G_1. As previously explained, D returns to C when the cone is released, so that the equilibrium is stable. In Fig. 4.12 (ii), the cone is balanced on its apex. When it is slightly displaced, the centre of gravity, G_1 is *lowered* to G_2. This is unstable equilibrium. Fig. 4.12 (iii) illustrates the case of the cone resting on its curved surface. If it is slightly displaced, the centre of gravity G remains at the same height G_3. The cone hence remains in its displaced position. This is called *neutral equilibrium.*

EXAMPLE

A rectangular beam of thickness a is balanced on the curved surface of a rough cylinder of radius r. Show that the beam is stable if r is greater than $a/2$.

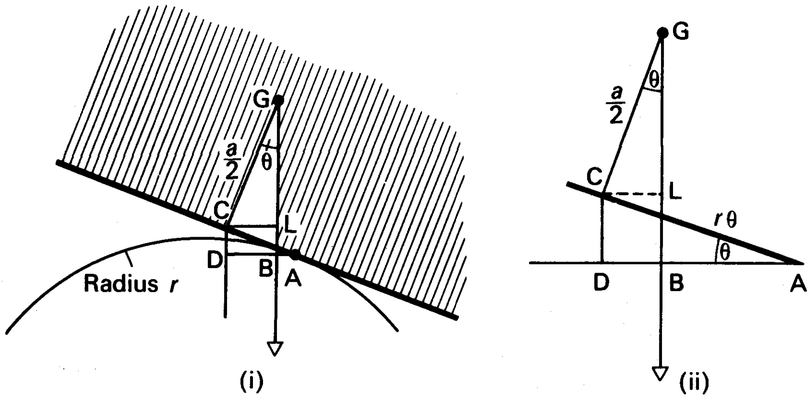

FIG. 4.13 Example

Suppose the beam is tilted through a small angle θ. The point of contact C then moves to A, the radius of the cylinder moves through an angle θ, and the vertical GB through the centre of gravity G of the beam makes an angle θ with CG. (Fig. 4.13 (i). As shown in the exaggerated sketch in Fig. 4.13 (ii), AC = $r\theta$.

The beam is in stable equilibrium if the vertical through G lies to the left of A, since a restoring moment is then exerted. Thus for stable equilibrium, AD must be greater than DB, where CD is the vertical through C.

Now $$\text{AD} = r\theta\cos\theta,\ \text{DB} = \text{CL} = \frac{a}{2}\sin\theta.$$

$$\therefore\ r\theta\cos\theta > \frac{a}{2}\sin\theta.$$

When θ is very small, $\cos\theta \rightarrow 1$, $\sin\theta \rightarrow \theta$.

$$\therefore\ r\theta > \frac{a}{2}\theta.$$

$$\therefore\ r > \frac{a}{2}.$$

Common Balance

The common balance is basically a lever whose two arms are equal, Fig. 4.14. The fulcrum, about which the beam and pointer tilt, is an agate wedge resting on an agate plate; agate wedges, B, at the ends of the beam, support the scale-pans. The centre of gravity of the beam and pointer is vertically below the fulcrum, to make the arrangement stable. The weights placed on the two scale-pans are equal when there is a 'balance'.

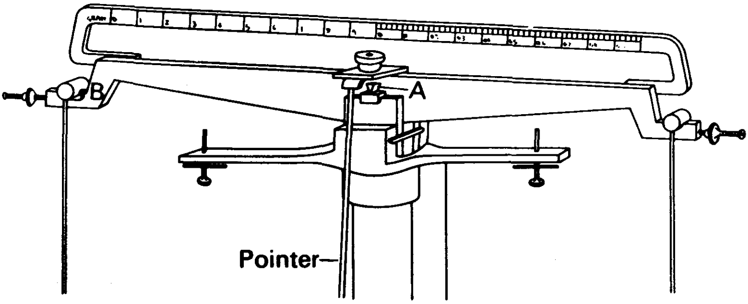

FIG. 4.14 Common balance

On rare occasions the arms of the balance are slightly unequal. The mass W of an object is then determined by finding the respective masses W_1, W_2 required to balance it on each scale-pan. Suppose a, b are the lengths of the respective arms. Then, taking moments,

$$\therefore W_1 . a = W . b, \text{ and } W . a = W_2 . b.$$

$$\therefore \frac{W}{W_1} = \frac{a}{b} = \frac{W_2}{W}$$

$$\therefore W^2 = W_1 W_2$$

$$\therefore W = \sqrt{W_1 W_2}.$$

Thus W can be found from the two masses W_1, W_2.

Sensitivity of a Balance

A balance is said to be very *sensitive* if a small difference in weights on the scale-pans causes a large deflection of the beam. To investigate the factors which affect the sensitivity of a balance, suppose a weight W_1 is placed on the left scale-pan and a slightly smaller weight W_2 is placed on the right scale-pan, Fig. 4.15. The beam AOB will then be inclined at some angle θ to the horizontal, where O is the fulcrum.

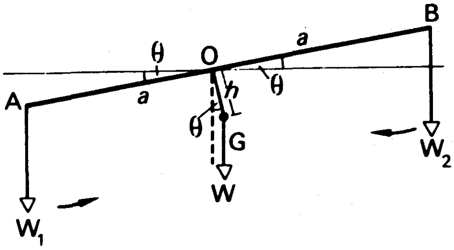

FIG. 4.15 Theory of balance

The weight W of the beam and pointer acts at G, at a distance h below O. Suppose AO = OB = a. Then, taking moments about O,

$$W_1 a \cos\theta = Wh \sin\theta + W_2 a \cos\theta$$

$$\therefore\ (W_1 - W_2)a \cos\theta = Wh \sin\theta$$

$$\therefore\ \tan\theta = \frac{(W_1 - W_2)a}{Wh}.$$

Thus for a given value of $(W_1 - W_2)$, the difference of the weights on the scale-pans, θ will increase when a increases and W, h both decrease. In theory, then, a sensitive balance must be light and have long arms, and the centre of gravity of its beam and pointer must be very close to the fulcrum. Now a light beam will not be rigid. Further, a beam with long arms will take a long time to settle down when it is deflected. A compromise must therefore be made between the requirements of sensitivity and those of design.

If the knife-edges of the scale-pan and beam are in the same plane, corresponding to A, B and O in Fig. 4.15, then the weights W_1, W_2 on them always have the same perpendicular distance from O, irrespective of the inclination of the beam. In this case the net moment about O is $(W_1 - W_2)a \cos\theta$. Thus the moment depends on the difference, $W_1 - W_2$, of the weights and not on their actual values. Hence the sensitivity is independent of the actual load value over a considerable range.

When the knife-edge of the beam is below the knife-edges of the two scale-pans, the sensitivity increases with the load; the reverse is the case if the knife-edge of the beam is above those of the scale-pans.

Buoyancy Correction in Weighing

In very accurate weighing, a correction must be made for the buoyancy of the air. Suppose the body weighed has a density ρ and a mass m. From Archimedes principle (p. 114), the upthrust due to the air of density σ is equal to the weight of air displaced by the body, and hence the net downward force $= \left(m - \frac{m}{\rho}\cdot\sigma\right)g$, since the volume of the body is m/ρ. Similarly, if the weights restoring a balance have a total mass m_1 and a density ρ_1, the net downward force $= \left(m_1 - \frac{m_1}{\rho_1}.\sigma\right)g$. Since there is a balance,

$$m - \frac{m\sigma}{\rho} = m_1 - \frac{m_1\sigma}{\rho_1}$$

$$\therefore\ m = m_1 \frac{\left(1 - \dfrac{\sigma}{\rho_1}\right)}{1 - \dfrac{\sigma}{\rho}}.$$

Thus knowing the density of air, σ, and the densities ρ, ρ_1, the true mass m can be found in terms of m_1. The pressure and temperature of air, which may vary from day to day, affects the magnitude of its density σ, from the gas laws; the humidity of the air is also taken into account in very accurate weighing, as the density of moist air differs from that of dry air.

FLUIDS

Pressure

Liquids and gases are called *fluids.* Unlike solid objects, fluids can flow.

If a piece of cork is pushed below the surface of a pool of water and then released, the cone rises to the surface again. The liquid thus exerts an upward force on the cork and this is due to the *pressure* exerted on the cork by the surrounding liquid. Gases also exert pressures. For example, when a thin closed metal can is evacuated, it

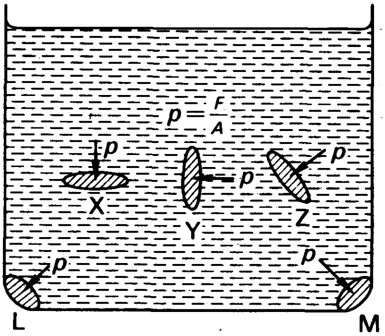

FIG. 4.16 Pressure in liquid

usually collapses with a loud explosion. The surrounding air now exerts a pressure on the outside which is no longer counter-balanced by the pressure inside, and hence there is a resultant force.

Pressure is defined as the *average force per unit area* at the particular region of liquid or gas. In Fig. 4.16, for example, X represents a small horizontal area, Y a small vertical area and Z a small inclined area, all inside a vessel containing a liquid. The pressure p acts normally to the planes of X, Y or Z. In each case

$$\text{average pressure, } p, = \frac{F}{A},$$

where F is the normal force due to the liquid on an area A of X, Y or Z. Similarly, the pressure p on the sides L or M of the curved vessel act normally to L and M have magnitude F/A. In the limit, when the area is very small, $p = dF/dA$.

At a given point in a liquid, the pressure can act in any direction. Thus *pressure is a scalar*, not a vector. The direction of the force on a particular surface is normal to the surface.

Formula for Pressure

Observation shows that the pressure increases with the depth, h, below the liquid surface and with its density ρ.

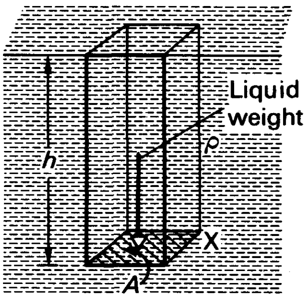

FIG. 4.17 Pressure and depth

To obtain a formula for the pressure, p, suppose that a horizontal plate X of area A is placed at a depth h below the liquid surface, Fig. 4.17. By drawing vertical lines from points on the perimeter of X, we can see that the force on X due to the liquid is equal to the weight of liquid of height h and uniform cross-section A. Since the volume of this liquid is Ah, the mass of the liquid $= Ah \times \rho$.

$$\therefore \text{ weight} = Ah\rho g \text{ newton,}$$

where g is 9·8, h is in m, A is in m^2, and ρ is in kg m^{-3}.

$$\therefore \text{ pressure, } p, \text{ on } X = \frac{\text{force}}{\text{area}} = \frac{Ah\rho g}{A}$$

$$\therefore \mathbf{p} = \mathbf{h\rho g} \quad . \quad . \quad . \quad . \quad (1)$$

When h, ρ, g have the units already mentioned, the pressure p is in *newton m*$^{-2}$ (N m^{-2}).

1 bar $= 10^6$ dyne cm^{-2}. To change 10^6 dyne cm^{-2} to N m^{-2}, we may proceed as follows:

$$10^6 \frac{\text{dyne}}{\text{cm}^2} = 10^6 \frac{\text{dyne}}{\text{N}} \cdot \frac{\text{N}}{\text{m}^2} \cdot \frac{\text{m}^2}{\text{cm}^2} = 10^6 \times \frac{1}{10^5} \cdot \frac{\text{N}}{\text{m}^2} \cdot 10^4 = 10^5 \frac{\text{N}}{\text{m}^2}$$

$$\therefore 1 \text{ bar} = 10^5 \text{ N m}^{-2} \quad . \quad . \quad . \quad . \quad (2)$$

Pressure is often expressed in terms of that due to a height of mercury (Hg). One unit is the *torr* (after Torricelli):

$$1 \text{ torr} = 1 \text{ mmHg} = 133{\cdot}3 \text{ N m}^{-2} \text{ (approx).}$$

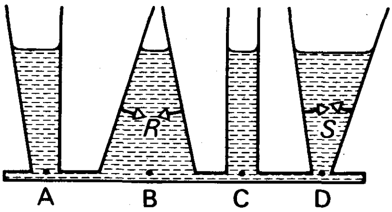

FIG. 4.18 Pressure and cross-section

From $p = h\rho g$ it follows that *the pressure in a liquid is the same at all points on the same horizontal level in it.* Experiment also gives the same result. Thus a liquid filling the vessel shown in Fig. 4.18 rises to the same height in each section if ABCD is horizontal. The cross-sectional area of B is greater than that of D; but the force on B is the sum of the weight of water above it together with the downward component of reaction R of the sides of the vessel, whereas the force on D is the weight of water above it *minus* the upward component of the reaction S of the sides of the vessel. It will thus be noted that the pressure in a vessel is independent of the cross-sectional area of the vessel.

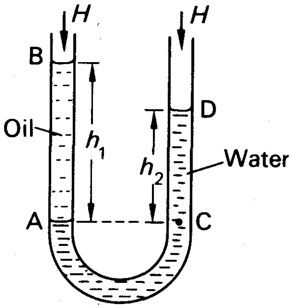

FIG. 4.19
Comparison of densities

Liquids in U-tube

Suppose a U-tube is partly filled with water, and oil is then poured into the left side of the tube. The oil will then reach some level B at a height h_1 above the surface of separation, A, of the water and oil, while the water on the right side of the tube will then reach some level D at a height h_2 above the level of A, Fig. 4.19.

Since the pressure in the water at A is equal to the pressure at C on the same horizontal level, it follows that

$$H + h_1\rho_1 g = H + h_2\rho_2 g,$$

where H is the atmospheric pressure, and ρ_1, ρ_2 are the respective densities of oil and water. Simplifying,

$$h_1\rho_1 = h_2\rho_2$$

$$\therefore \rho_1 = \rho_2 \times \frac{h_2}{h_1}.$$

Since ρ_2 (water) $= 1000\,\text{kg m}^{-3}$, and h_2, h_1 can be measured, the density ρ_1 of the oil can be found.

Atmospheric Pressure

The pressure of the atmosphere was first measured by Galileo, who observed the height of a water column in a tube placed in a deep well.

About 1640 TORRICELLI thought of the idea of using mercury instead of water, to obtain a much shorter column. He completely filled a glass tube about a metre long with mercury, and then inverted it in a vessel D containing the liquid, taking care that no air entered the tube. He observed that the mercury in the tube fell to a level A about 76 cm or 0·76 m above the level of the mercury in D, Fig. 4.20. Since there was

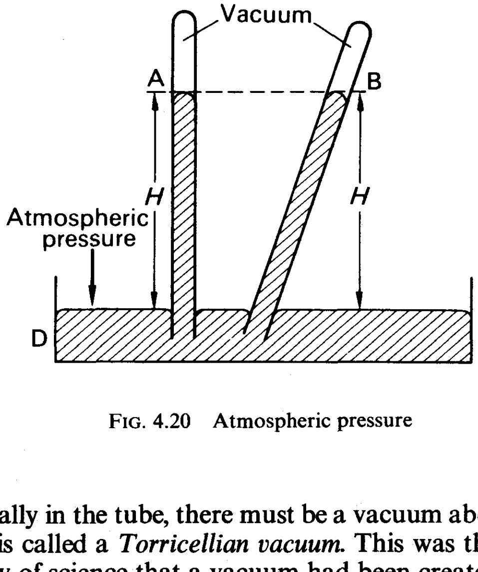

FIG. 4.20 Atmospheric pressure

no air originally in the tube, there must be a vacuum above the mercury at A, and it is called a *Torricellian vacuum.* This was the first occasion in the history of science that a vacuum had been created.

If the tube in Fig. 4.20 is inclined to the vertical, the mercury ascends the tube to a level B at the same vertical height H above the level of the mercury in D as A.

The pressure on the surface of the mercury in D is atmospheric pressure; and since the pressure is transmitted through the liquid, the atmospheric pressure supports the column of mercury in the tube. Suppose A is at a height H above the level of the mercury in D. Now the pressure, p, at the bottom of a column of liquid of height H and density ρ is given by $p = H\rho g$ (p. 110). Thus if $H = 760$ mm $= 0{\cdot}76$ m and $\rho = 13600$ kg m^{-3},

$$p = H\rho g = 0{\cdot}76 \times 13600 \times 9{\cdot}8 = 1{\cdot}013 \times 10^5 \text{ newton metre}^{-2}.$$

The pressure at the bottom of a column of mercury 76 cm high for a particular mercury density and value of g is known as *standard pressure* or *one atmosphere.* By definition, 1 atmosphere $= 1{\cdot}01325 \times 10^5$ N m^{-2}. *Standard temperature and pressure* (*S.T.P.*) is 0°C and 76 cm Hg pressure.

A *bar* is the name given to a pressure of one million (10^6) dyne cm^{-2}, and is thus very nearly equal to one atmosphere. 1 bar $= 10^5$ newton m^{-2} (p. 110).

Fortin's Barometer

A *barometer* is an instrument for measuring the pressure of the atmosphere, which is required in weather-forecasting, for example. The most accurate form of barometer is due to FORTIN, and like the simple arrangement already described, it consists basically of a barometer tube containing mercury, with a vacuum at the top, Fig. 4.21. One end of the tube dips into a pool of mercury contained in a washleather bag B. A brass scale C graduated in centimetres and millimetres is fixed at the top of the barometer. The zero of the scale correspondings to the tip of an ivory tooth P, and hence, before the level of the top of the mercury is read from the scales, the screw S is adjusted until the level of the mercury in B just reaches the tip of P. A vernier scale V can be moved by a screw D until the bottom of it just reaches the top of the mercury in the tube, and the reading of the height of the mercury is taken from C and V. Torricelli was the first person to observe the variation of the barometric height as the weather changed.

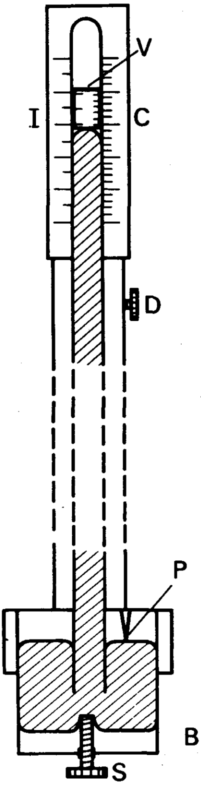

FIG. 4.21
Fortin barometer

'Correction' to the Barometric Height

For comparison purposes, the pressure read on a barometer is often 'reduced' or 'corrected' to the magnitude the pressure would have at 0°C and at sea-level, latitude 45°. Suppose the 'reduced' pressure is H_o cm of mercury, and the observed pressure is H_t cm of mercury, corresponding to a temperature of t°C. Then, since pressure = $h\rho g$ (p. 110),

$$H_o\rho_o g = H_t\rho_t g',$$

where g is the acceleration due to gravity at sea-level, latitude 45°, and g' is the acceleration at the latitude of the place where the barometer was read.

$$\therefore\ H_o = H_t\times\frac{\rho_t}{\rho_o}\times\frac{g'}{g}.$$

The magnitude of g'/g can be obtained from standard tables. The ratio ρ_t/ρ_o of the densities $= 1/(1+\gamma t)$. where γ is the absolute or true cubic expansivity of mercury. Further, the observed height H_t, on the brass scale requires correction for the expansion of brass from the temperature at which it was correctly calibrated. If the latter is 0°C, then the corrected height is $H_t(1+\alpha t)$, where α is the mean linear expansivity of brass. Thus, finally, the 'corrected' height H is given by

$$H_o = H_t \cdot \frac{1+\alpha t}{1+\gamma t} \cdot \frac{g'}{g}.$$

For further accuracy, a correction must be made for the surface tension of mercury (p. 132).

Variation of atmospheric pressure with height

The density of a liquid varies very slightly with pressure. The density of a gas, however, varies appreciably with pressure. Thus at sea-level the density of the atmosphere is about 1·2 kg m^{-3}; at 1000 m above sea-level the density is about 1·1 kg m^{-3}; and at 5000 m above sea-level it is about 0·7 kg m^{-3}. Normal atmospheric pressure is the pressure at the base of a column of mercury 760 mm high, a liquid which has a density of about 13600 kg m^{-3}. Suppose air has a constant density of about 1·2 kg m^{-3}. Then the height of an air column of this density which has a pressure equal to normal atmospheric pressure

$$= \frac{760}{1000} \times \frac{13600}{1 \cdot 2} \text{ m} = 8 \cdot 4 \text{ km}.$$

In fact, the air 'thins' the higher one goes, as explained above. The height of the air is thus much greater than 8·4 km.

Density, Relative Density

As we have seen, the pressure in a fluid depends on the density of the fluid.

The *density* of a substance is defined as its *mass per unit volume*. Thus

$$\text{density, } \rho, = \frac{\text{mass of substance}}{\text{volume of substance}} \quad . \quad . \quad (47)$$

The density of copper is about 9·0 g cm^{-3} or 9×10^3 kg m^{-3}; the density of aluminium is 2·7 g cm^{-3} or $2 \cdot 7 \times 10^3$ kg m^{-3}; the density of water at 4°C is 1 g cm^{-3} or 1000 kg m^{-3}.

Substances which float on water have a density less than 1000 kg m^{-3} (p. 117). For example, ice has a density of about 900 kg m^{-3}; cork has a density of about 250 kg m^{-3}. Steel, of density 8500 kg m^{-3}, will float on mercury, whose density is about 13600 kg m^{-3} at 0°C.

The density of a substance is often expressed relative to the density of water. This is called the *relative density* or *specific gravity* of the substance. It is a ratio or number, and has no units. The relative density of mercury is 13·6. Thus the density of mercury is 13·6 times the density of water, 1000 kg m^{-3}, and is hence 13600 kg m^{-3}. Copper has a relative density of 9·0 and hence a density of 9000 kg m^{-3}.

Archimedes' Principle

An object immersed in a fluid experiences a resultant upward force owing to the pressure of fluid on it. This upward force is called the *upthrust* of the fluid on the object. ARCHIMEDES stated that *the upthrust is equal to the weight of fluid displaced by the object*, and this is known as

Archimedes' Principle. Thus if an iron cube of volume 400 cm^3 is totally immersed in water of density 1 g cm^{-3}, the upthrust on the cube = 400×1 = 400 gf. If the same cube is totally immersed in oil of density 0·8 g cm^{-3}, the upthrust on it = $400 \times 0{\cdot}8$ = 320 gf.

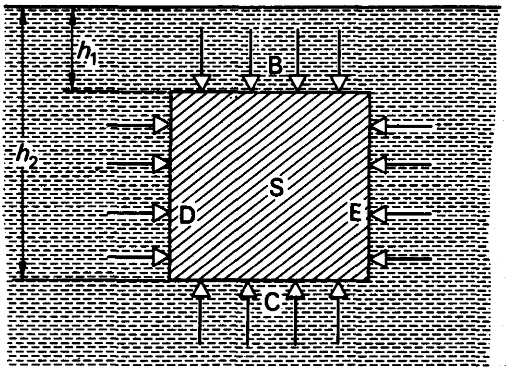

FIG. 4.22. Archimedes' Principle

Fig. 4.22 shows why Archimedes' Principle is true. If S is a solid immersed in a liquid, the pressure on the lower surface C is greater than on the upper surface B, since the pressure at the greater depth h_2 is more than that at h_1. The pressure on the remaining surfaces D and E act as shown. The *force* on each of the four surfaces is calculated by summing the values of *pressure × area* over every part, remembering that vector addition is needed to sum forces. With a simple *rectangular-shaped solid* and the sides, D, E vertical, it can be seen that (i) the resultant horizontal force is zero, (ii) the upward force on C = pressure × area $A = h_2 \rho g A$, where ρ is the liquid density and the downward force on B = pressure × area $A = h_1 \rho g A$. Thus

resultant force on solid = upward force (upthrust) = $(h_2 - h_1)\rho g A$.

But $(h_2 - h_1)A$ = volume of solid, V,

$\therefore$ *upthrust* = $V\rho g = mg$, where $m = V\rho$.

$\therefore$ *upthrust = weight of liquid displaced.*

With a solid of irregular shape, taking into account horizontal and vertical components of forces, the same result is obtained. The upthrust is the weight of *liquid* displaced whatever the nature of the object immersed, or whether it is hollow or not. This is due primarily to the fact that the pressure on the object depends on the liquid in which it is placed.

Density or Relative Density measurement by Archimedes' Principle

The upthrust on an object immersed in water, for example, is the difference between (i) its weight in air when attached to a spring-balance and (ii) the reduced reading on the spring-balance or 'weight' when it is totally immersed in the liquid. Suppose the upthrust is found to be 100 gf. Then, from Archimedes' Principle, the object displaces 100 gf of water. But the density of water is 1 g cm^{-3}. Hence the volume of the object = 100 cm^3, which is numerically equal to the difference in weighings in (i) and (ii).

The density or relative density of a *solid* such as brass or iron can thus be determined by (1) weighing it in air, m_0 gf say, (2) weighing it when it is totally immersed in water, m_1 gf say. Then

$$\text{upthrust} = m_0 - m_1 = \text{wt. of water displaced.}$$

$$\therefore \text{relative density of solid} = \frac{m_0}{m_0 - m_1},$$

$$\text{and density of solid, } \rho, = \frac{m_0}{m_0 - m_1} \times \text{density of water.}$$

The density or relative density of a *liquid* can be found by weighing a solid in air (m_0), then weighing it totally immersed in the liquid (m_1), and finally weighing it totally immersed in water (m_2).

Now $m_0 - m_2$ = upthrust in water = weight of water displaced,

and $m_0 - m_1$ = upthrust in liquid = weight of liquid displaced.

$$\therefore \frac{m_0 - m_1}{m_0 - m_2} = \text{relative density of liquid,}$$

or

$$\frac{m_0 - m_1}{m_0 - m_2} \times \text{density of water} = \text{density of liquid.}$$

Density of Copper Sulphate crystals

If a solid dissolves in water, such as a copper sulphate crystal for example, its density can be found by totally immersing it in a liquid in which it is insoluble. Copper sulphate can be weighed in paraffin oil, for example. Suppose the apparent weight is m_1, and the weight in air is m_0. Then

$$m_0 - m_1 = \text{upthrust in liquid} = V\rho,$$

where V is the volume of the solid and ρ is the density of the liquid.

$$\therefore V = \frac{m_0 - m_1}{\rho}.$$

$$\therefore \text{density of solid} = \frac{\text{mass}}{\text{volume}} = \frac{m_0}{V} = \frac{m_0}{m_0 - m_1} \cdot \rho.$$

The density, ρ, of the liquid can be found by means of a density bottle, for example. Thus knowing m_0 and m_1, the density of the solid can be calculated.

Density of Cork

If a solid floats in water, cork for example, its density can be found by attaching a brass weight or 'sinker' to it so that both solids become totally immersed in water. The apparent weight (m_1) of the sinker and cork together is then obtained. Suppose m_2 is the weight of the sinker in air, m_3 is the weight of the sinker alone in water, and m_0 is the weight of the cork in air.

Then $m_2 - m_3$ = upthrust on sinker in water.

$\therefore m_0 + m_2 - m_1 - (m_2 - m_3)$ = upthrust on cork in water $= m_0 - m_1 + m_3$

$$\therefore \text{relative density of cork} = \frac{m_0}{m_0 - m_1 + m_3}.$$

Flotation

When an object *floats* in a liquid, the upthrust on the object must be equal to its weight for equilibrium. Cork has a density of about 0·25 g cm^{-3}, so that 100 cm^3 of cork has a mass of 25 g. In water, then, cork sinks until the upthrust is 25 gf. Now from Archimedes' Principle, 25 gf is the weight of water displaced. Thus the cork sinks until 25 cm^3 of its 100 cm^3 volume is immersed. The fraction of the volume immersed is hence equal to the relative density.

Ice has a density of about 0·9 g cm^{-3}. A block of ice therefore floats in water with about $\frac{9}{10}$ths of it immersed.

Hydrometer

Hydrometers use the principle of flotation to measure density or relative density. Fig. 4.23. Since they have a constant weight, the upthrust when they float in a liquid is always the same. Thus in a liquid of density 1·0 g cm^{-3}, a hydrometer of 20 gf will sink until 20 cm^3 is immersed. In a liquid of 2·0 g cm^{-3}, it will sink until only 10 cm^3 is immersed. The density or relative density readings hence increase in a *downward* direction, as shown in Fig. 4.23.

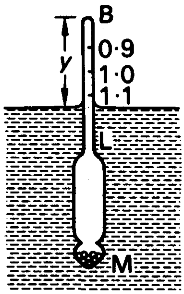

FIG. 4.23 Hydrometer

Practical hydrometers have a weighted end M for stability, a wide bulb to produce sufficient upthrust to counterbalance the weight, and a narrow stem BL for sensitivity. If V is the whole volume of the hydrometer in Fig. 4.23, a is the area of the stem and y is the length not immersed in a liquid of density ρ, then

$$\text{upthrust} = \text{wt. of liquid displaced} = (V - ay)\rho = w,$$

where w is the weight of the hydrometer.

EXAMPLE

An ice cube of mass 50·0 g floats on the surface of a strong brine solution of volume 200·0 cm^3 inside a measuring cylinder. Calculate the level of the liquid in the measuring cylinder (i) before and (ii) after all the ice is melted. (iii) What happens to the level if the brine is replaced by 200·0 cm^3 water and 50·0 g of ice is again added? (Assume density of ice, brine = 900, 1100 kg m^{-3} or 0·9, 1·1 g cm^{-3}.)

(i) Floating ice displaces 50 g of brine since upthrust is 50 gf.

$$\therefore \text{volume displaced} = \frac{\text{mass}}{\text{density}} = \frac{50}{1 \cdot 1} = 45 \cdot 5 \text{ cm}^3.$$

$\therefore$ level on measuring cylinder = 245·5 cm^3.

(ii) 50 g of ice forms 50 g of water when all of it is melted.

$\therefore$ level on measuring cylinder *falls* to 250·0 cm^3.

(iii) *Water*. Initially, volume of water displaced = 50 cm^3, since upthrust = 50 g.

$\therefore$ level on cylinder = 250·0 cm^3.

If 1 g of ice melts, volume displaced is 1 cm^3 less. But volume of water formed is 1 cm^3. Thus the net change in water level is zero. Hence the water level remains unchanged as the ice melts.

Fluids in Motion. Streamlines and velocity

A stream or river flows slowly when it runs through open country and faster through narrow openings or constrictions. As shown shortly, this is due to the fact that water is practically an incompressible fluid, that is, changes of pressure cause practically no change in fluid density at various parts.

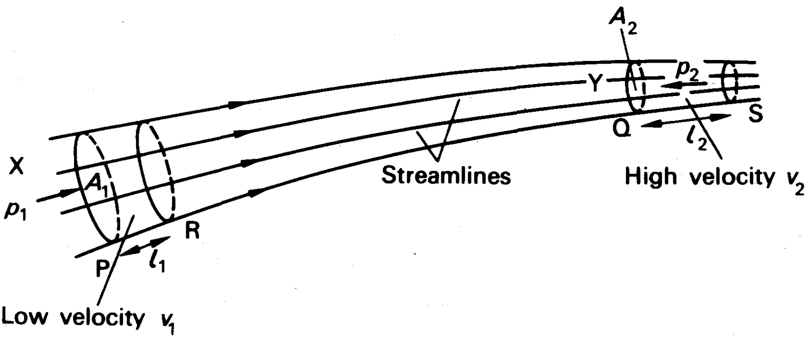

FIG. 4.24 Bernoulli's theorem

Fig. 4.24 shows a tube of water flowing steadily between X and Y, where X has a bigger cross-sectional area A_1 than the part Y, of cross-sectional area A_2. The *streamlines* of the flow represent the directions of the velocities of the particles of the fluid and the flow is uniform or laminar (p. 204). Assuming the liquid is incompressible, then, if it moves from PQ to RS, the volume of liquid between P and R is equal to the volume between Q and S. Thus $A_1l_1 = A_2l_2$, where l_1 is PR and l_2 is QS, or $l_2/l_1 = A_1/A_2$. Hence l_2 is greater than l_1. Consequently the *velocity* of the liquid at the narrow part of the tube, where, it should be noted, the streamlines are closer together, is greater than at the wider part Y, where the streamlines are further apart. For the same reason, slow-running water from a tap can be made into a fast jet by placing a finger over the tap to narrow the exit.

Pressure and velocity. Bernoulli's Principle

About 1740, Bernoulli obtained a relation between the pressure and velocity at different parts of a moving incompressible fluid. If the viscosity is negligibly small, there are no frictional forces to overcome (p. 174). In this case the work done by the pressure difference per unit volume of a fluid flowing along a pipe steadily is equal to the gain of kinetic energy per unit volume plus the gain in potential energy per unit volume.

Now the work done by a pressure in moving a fluid through a distance = force × distance moved = (pressure × area) × distance moved = pressure × volume moved, assuming the area is constant at a particular place for a short time of flow. At the beginning of the pipe where the pressure is p_1, the work done per unit volume on the fluid is thus p_1; at the other end, the work done per unit volume by the fluid is likewise p_2. Hence the net work done *on* the fluid per unit volume $= p_1 - p_2$. The kinetic energy per unit volume $= \frac{1}{2}$ mass per unit volume × velocity². $= \frac{1}{2}\rho \times$ velocity², where ρ is the density of the fluid. Thus if v_2 and v_1 are the final and initial velocities respectively at the end and the beginning of the pipe, the kinetic energy gained per unit volume = $\frac{1}{2}\rho(v_2 - v_1)^2$. Further, if h_2 and h_1 are the respective heights measured from a fixed level at the end and beginning of the pipe, the potential energy gained per unit volume = mass per unit volume $\times g \times (h_2 - h_1)$ $= \rho g(h_2 - h_1)$.

Thus, from the conservation of energy,

$$p_1 - p_2 = \tfrac{1}{2}\rho({v_2}^2 - {v_1}^2) + \rho g(h_2 - h_1)$$

$$\therefore\ p_1 + \tfrac{1}{2}\rho {v_1}^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho {v_2}^2 + \rho g h_2$$

$$\therefore\ p + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant},$$

where p is the pressure at any part and v is the velocity there. Hence it can be said that, for streamline motion of an incompressible non-viscous fluid,

the sum of the pressure at any part plus the kinetic energy per unit volume plus the potential energy per unit volume there is always constant.

This is known as *Bernoulli's principle.*

Bernoulli's principle shows that at points in a moving fluid where the potential energy change $\rho g h$ is very small, or zero as in flow through a horizontal pipe, the pressure is low where the velocity is high; conversely, the pressure is high where the velocity is low. The principle has wide applications.

EXAMPLE

As a numerical illustration of the previous analysis, suppose the area of cross-section A_1 of X in Fig. 4.25 is 4 cm², the area A_2 of Y is 1 cm², and water flows past each section in laminar flow at the rate of 400 cm³ s⁻¹. Then

$$\text{at X, speed } v_1 \text{ of water} = \frac{\text{vol. per second}}{\text{area}} = 100 \text{ cm s}^{-1} = 1 \text{ m s}^{-1};$$

$$\text{at Y, speed } v_2 \text{ of water} = 400 \text{ cm s}^{-1} = 4 \text{ m s}^{-1}.$$

The density of water, $\rho = 1000$ kg m⁻³.

$$\therefore\ p = \tfrac{1}{2}\rho({v_2}^2 - {v_1}^2) = \tfrac{1}{2} \times 1000 \times (4^2 - 1^2) = 7{\cdot}5 \times 10^3 \text{ newton m}^{-2}.$$

If h is in metres, $\rho = 1000$ kg m^{-3} for water, $g = 9{\cdot}8$ m s^{-2}, then, from $h\rho g$,

$$h = \frac{7{\cdot}5 \times 10^3}{1000 \times 9{\cdot}8} = 0{\cdot}77 \text{ m (approx.)}.$$

The pressure head h is thus equivalent to 0·77 m of water.

Applications of Bernoulli's Principle

1. A suction effect is experienced by a person standing close to the platform at a station when a fast train passes. The fast-moving air between the person and train produces a decrease in pressure and the excess air pressure on the other side pushes the person towards the train.

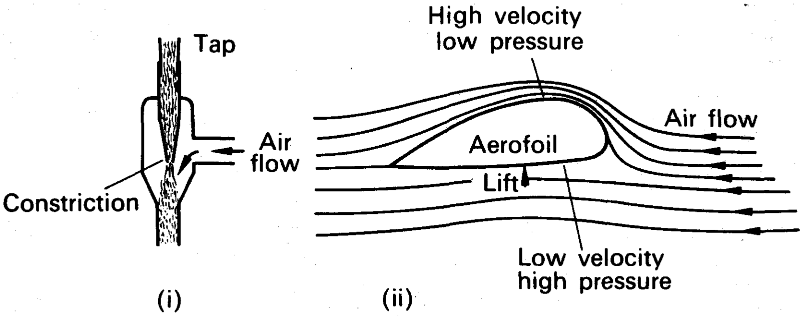

FIG. 4.25 Fluid velocity and pressure

2. *Filter pump.* A filter pump has a narrow section in the middle, so that a jet of water from the tap flows faster here. Fig. 4.25 (i). This causes a drop in pressure near it and air therefore flows in from the side tube to which a vessel is connected. The air and water together are expelled through the bottom of the filter pump.

3. *Aerofoil lift.* The curved shape of an aerofoil creates a fast flow of air over its top surface than the lower one. Fig. 4.25 (ii). This is shown by the closeness of the streamlines above the aerofoil compared with those below. From Bernoulli's principle, the pressure of the air below is greater than that above, and this produces the lift on the aerofoil.

4. *Flow of liquid from wide tank.* Suppose a liquid flows through a hole H at the bottom of a wide tank, as shown in Fig. 4.26. Assuming negligible viscosity and streamline flow at a small distance from the hole, which is an approximation, Bernoulli's theorem can be applied. At the top X of the liquid in the tank, the pressure is atmospheric, say B, the height measured from a fixed level such as the hole H is h, and the kinetic energy is negligible if the tank is wide so that the level falls very slowly. At the bottom, Y, near H, the pressure is again B, the height above H is now zero, and the kinetic energy is $\frac{1}{2}\rho v^2$, where ρ is the density and v is the

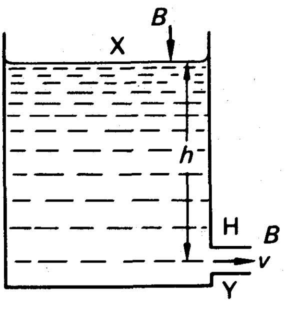

FIG. 4.26 Torricelli's theorem

velocity of emergence of the liquid. Thus, from Bernoulli's Principle,

$$B+\rho hg = B+\tfrac{1}{2}\rho v^2$$

$$\therefore v^2 = 2gh$$

Thus the velocity of the emerging liquid is the same as that which would be obtained if it fell freely through a height h, and this is known as *Torricelli's theorem*. In practice the velocity is less than that given by $\sqrt{2gh}$ owing to viscous forces, and the lack of streamline flow must also be taken into account.

EXERCISES 4

What are the missing words in the statements 1–6?

1. In SI units, the moment or torque of a couple is measured in . . .

2. In stable equilibrium, when an object is slightly displaced its centre of gravity . . .

3. When an object is in equilibrium under the action of three non-parallel forces, the three forces must . . . one point.

4. The component of a force F in a direction inclined to it at an angle θ is . . .

5. The sensitivity of a beam balance depends on the depth of the . . . below the fulcrum.

6. When an object floats, the weight of fluid displaced is equal to the . . .

7. In laminar flow of non-viscous fluid along a pipe, at regions of high pressure the . . . is low.

Which of the following answers, A, B, C, D or E, do you consider is the correct one in the statements 8–10?

8. If a cone is balanced on its apex on a horizontal table and then slightly displaced, the potential energy of the cone is then *A* increased, *B* decreased, *C* constant, *D* a minimum, *E* a maximum.

9. If a hydrometer of mass 20 g and volume 30 cm^3 has a graduated stem of 1 cm^2, and floats in water, the exposed length of stem is *A* 30 cm, *B* 25 cm, *C* 20 cm, *D* 10 cm, *E* 1 cm.

10. In laminar flow of a non-viscous fluid along a horizontal pipe, the work per second done by the pressure at any section is equal to *A* the pressure, *B* the volume per second there, *C* pressure × volume per second there, *D* pressure × volume, *E* pressure × area of cross-section.

11. A flat plate is cut in the shape of a square of side 20·0 cm, with an equilateral triangle of side 20·0 cm adjacent to the square. Calculate the distance of the centre of mass from the apex of the triangle.

12. The foot of a uniform ladder is on a rough horizontal ground, and the top rests against a smooth vertical wall. The weight of the ladder is 40 kgf, and a man weighing 80 kgf stands on the ladder one-quarter of its length from the bottom. If the inclination of the ladder to the horizontal is 30°, find the reaction at the wall and the total force at the ground.

13. A rectangular plate ABCD has two forces of 10 kgf acting along AB and DC in opposite directions. If AB = 3 m, BC = 5 m, what is the moment of the couple acting on the plate? What forces acting along BC and AD respectively are required to keep the plate in equilibrium?

14. A hollow metal cylinder 2 m tall has a base of diameter 35 cm and is filled with water to a height of (i) 1 m, (ii) 50 cm. Calculate the distance of the centre of gravity in metre from the base in each case if the cylinder has no top. (Metal weighs 20 kg m^{-2} of surface. Assume $\pi = 22/7$.)

15. A trap-door 120 cm by 120 cm is kept horizontal by a string attached to the mid-point of the side opposite to that containing the hinge. The other end of the string is tied to a point 90 cm vertically above the hinge. If the trap-door weight is 5 kgf, calculate the tension in the string and the reaction at the hinge.

16. Two smooth inclined planes are arranged with their lower edges in contact; the angles of inclination of the plane to the horizontal are 30°, 60° respectively, and the surfaces of the planes are perpendicular to each other. If a uniform rod rests in the principal section of the planes with one end on each plane, find the angle of inclination of the rod to the horizontal.

17. Describe and give the theory of an accurate beam balance. Point out the factors which influence the sensitivity of the balance. Why is it necessary, in very accurate weighing, to take into account the pressure, temperature, and humidity of the atmosphere? (*O.* & *C.*)

18. Summarise the various conditions which are being satisfied when a body remains in equilibrium under the action of three non-parallel forces.

A wireless aerial attached to the top of a mast 20 m high exerts a horizontal force upon it of 60 kgf. The mast is supported by a stay-wire running to the ground from a point 6 m below the top of the mast, and inclined at 60° to the horizontal. Assuming that the action of the ground on the mast can be regarded as a single force, draw a diagram of the forces acting on the mast, and determine by measurement or by calculation the force in the stay-wire. (*C.*)

19. The beam of a balance has mass 150 g and its moment of inertia is 5×10^{-4} kg m^2. Each arm of the balance is 10 cm long. When set swinging the beam makes one complete oscillation in 6 seconds. How far is the centre of gravity of the beam below its point of support, and through what angle would the beam be deflected by a weight of 1 milligram placed in one of the scale pans? (*C.*)

20. Under what conditions is a body said to be in equilibrium? What is meant by (*a*) *stable equilibrium* and (*b*) *unstable equilibrium*? Give one example of each.

A pair of railway carriage wheels, each of radius r, are joined by a thin axle; the mass of the whole is m. A light arm of length $l (< r)$ is attached perpendicularly to the axle and the free end of the arm carries a point mass M. The wheels rest, with the axle horizontal, on rails which are laid down a slope inclined at an angle ϕ to the horizontal. Show that, provided that ϕ is not too large and that the wheels do not slip on the rails, there are two values of the angle θ that the arm makes with the horizontal when the system is in equilibrium, and find these values of θ. Discuss whether, in each case, the equilibrium is stable or unstable. (*O.* & *C.*)

21. Give a labelled diagram to show the structure of a beam balance. Show if the knife-edges are collinear the sensitivity is independent of the load. Discuss other factors which then determine the sensitivity.

A body is weighed at a place on the equator, both with a beam balance and a very sensitive spring balance, with identical results. If the observations are repeated at a place near one of the poles, using the same two instruments, discuss whether identical results will again be obtained. (*L.*)

22. Three forces in one plane act on a rigid body. What are the conditions for equilibrium?

The plane of a kite of mass 6 kg is inclined to the horizon at 60°. The resultant thrust of the air on the kite acts at a point 25 cm above its centre of gravity, and the string is attached at a point 30 cm above the centre of gravity. Find the thrust of the air on the kite, and the tension in the string. (*C.*)

23. In what circumstances is a physical system in equilibrium? Distinguish between stable, unstable and neutral equilibria.

Discuss the stability of the equilibrium of a uniform rough plank of thickness t, balanced horizontally on a rough cylindrical-fixed log of radius r, it being assumed that the axes of plank and log lie in perpendicular directions. (*N.*)

24. State the conditions of equilibrium for a body subjected to a system of coplanar parallel forces and briefly describe an experiment which you could carry out to verify these conditions.

Show how the equilibrium of a beam balance is achieved and discuss the factors which determine its sensitivity. Explain how the sensitivity of a given balance may be altered and why, for a particular adjustment, the sensitivity may be practically independent of the mass in the balance pans. Why is it inconvenient in practice to attempt to increase the sensitivity of a given balance beyond a certain limit? (*O. & C.*)

Fluids

25. An alloy of mass 588 g and volume 100 cm^3 is made of iron of relative density 8·0 and aluminium of relative density 2·7. Calculate the proportion (i) by volume, (ii) by mass of the constituents of the alloy.

26. A string supports a solid iron object of mass 180 g totally immersed in a liquid of density 800 kg m^{-3}. Calculate the tension in the string if the density of iron is 8000 kg m^{-3}.

27. A hydrometer floats in water with 6·0 cm of its graduated stem unimmersed, and in oil of relative density 0·8 with 4·0 cm of the stem unimmersed. What is the length of stem unimmersed when the hydrometer is placed in a liquid of relative density 0·9?

28. An alloy of mass 170 g has an apparent weight of 95 gf in a liquid of density 1·5 g cm^{-3}. If the two constituents of the alloy have relative densities of 4·0 and 3·0 respectively, calculate the proportion by volume of the constituents in the alloy.

29. State the principle of Archimedes and use it to derive an expression for the resultant force experienced by a body of weight W and density σ when it is totally immersed in a fluid of density ρ.

A solid weighs 237·5 g in air and 12·5 g when totally immersed in a liquid of relative density 0·9. Calculate (*a*) the specific gravity of the solid, (*b*) the relative density of a liquid in which the solid would float with one-fifth of its volume exposed above the liquid surface. (*L.*)

30. Distinguish between *mass* and *weight*. Define *density*.

Describe and explain how you would proceed to find an accurate value for the density of gold, the specimen available being a wedding ring of pure gold.

What will be the reading of (*a*) a mercury barometer, (*b*) a water barometer, when the atmospheric pressure is 10^5 N m^{-2}? The density of mercury may be taken as 13600 kg m^{-3} and the pressure of saturated water vapour at room temperature as 13 mm of mercury. (*L.*)

31. Describe an experiment which demonstrates the difference between laminar and turbulent flow in a fluid.

A straight pipe of uniform radius R is joined, in the same straight line, to a narrower pipe of uniform radius r. Water (which may be assumed to be incompressible) flows from the wider into the narrower pipe. The velocity of flow in the wider pipe is V and in the narrower pipe is v. By equating work done against fluid pressures with change of kinetic energy of the water, show that the hydrostatic pressure is lower where the velocity of flow is higher.

Describe and explain one practical consequence or application of this difference in pressures. (*O. & C.*)

32. Describe some form of barometer used for the accurate measurement of atmospheric pressure, and point out the corrections to be applied to the observation.

Obtain an expression for the correction to be applied to the reading of a mercurial barometer when the reading is made at a temperature other than 0°C. (*L.*)

33. State the principle of Archimedes, and discuss its application to the determination of specific gravities by means of a common hydrometer. Why is this method essentially less accurate than the specific gravity bottle?

A common hydrometer is graduated to read specific gravities from 0·8 to 1·0. In order to extend its range a small weight is attached to the stem, above the liquid, so that the instrument reads 0·8 when floating in water. What will be the specific gravity of the liquid corresponding to the graduation 1·0? (*O. & C.*)

34. A hydrometer consists of a bulb of volume V and a uniform stem of volume v per cm of its length. It floats upright in water so that the bulb is just completely immersed. Explain for what density range this hydrometer may be used and how you would determine the density of such liquids. Describe the graph which would be obtained by plotting the reciprocal of the density against the length of the stem immersed.

A hydrometer such as that described sinks to the mark 3 on the stem, which is graduated in cm, when it is placed in a liquid of density 0·95 g cm^{-3}. If the volume per cm of the stem is 0·1 cm^3, find the volume of the bulb. (*L.*)

35. A straight rod of length l, small cross sectional area a and of material density ρ is supported by a thread attached to its upper end. Initially the rod hangs in a vertical position over a liquid of density σ and then is lowered until it is partially submerged. Derive and discuss the equilibrium conditions of the rod neglecting surface tension. (*N.*)