chapter thirty-seven

Magnetic Fields of Current-Carrying Conductors

In the previous chapters, the induction or flux density B in a magnetic field was used to find the force on conductors and the e.m.f. induced in conductors. In this chapter we shall see how the magnitude of B is calculated. This depends on the geometry of the conductor, that is, whether it is a straight wire, or a coil, or a solenoid. The geometry also determines the pattern of the lines of force in the field.

Law of Biot and Savart

To calculate B for any shape of conductor, Biot and Savart gave a law which can now be stated as follows: The induction or flux density δB at a point P due to a small element δl of a conductor carrying a current is given by

$$\delta B \propto \frac{I\delta l \sin\alpha}{r^2}, \quad . \quad . \quad . \quad . \quad (1)$$

where r is the distance from the point P to the element and α is the angle between the element and the line joining it to P (Fig. 37.1).

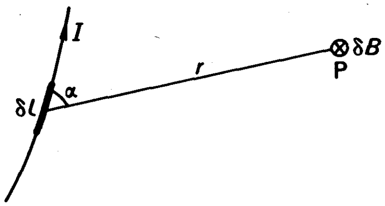


FIG. 37.1. Biot and Savart law.

The formula in (1) cannot be proved directly, as we cannot experiment with an infinitesimally small conductor. We believe in its truth because the deductions for large practical conductors turn out to be true.

The constant of proportionality in equation (1) depends on the medium in which the conductor is situated. In air (or, more exactly, in a vacuum), we write

$$\delta B = \frac{\mu_0}{4\pi}\frac{I\delta l \sin\alpha}{r^2} \quad . \quad . \quad . \quad . \quad (2)$$

The value of μ_0 is defined to be,

$$\mu_0 = 4\pi \times 10^{-7},$$

and its unit is '*henry per metre*' (H m^{-1}) as will be shown later.

Induction Formula for Narrow Coil

The formula for the induction B at the centre of a narrow circular coil can be immediately deduced from (2). Here the radius r is constant for all the elements δl, and the angle α is constant and equal to 90° (Fig. 37.2 (i)). If the coil has N turns, the length of wire in it is $2\pi rN$, and the field at its centre is therefore given, if the current is I, by

$$B = \int dB = \frac{\mu_0}{4\pi}\int_0^{2\pi rn} \frac{I\,dl \sin 90^\circ}{r^2}$$

$$= \frac{\mu_0 I}{4\pi r^2}\int_0^{2\pi rn} dl = \frac{\mu_0 I}{4\pi r^2} 2\pi rN$$

$$= \frac{\mu_0 NI}{2r} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From (1), $B \propto I$ when r and N are constant, $B \propto 1/r$ when I and N are constant, and $B \propto N$ when I and r are constant. Any one of these relations may be verified with the apparatus shown in Fig. 37.2 (ii). This

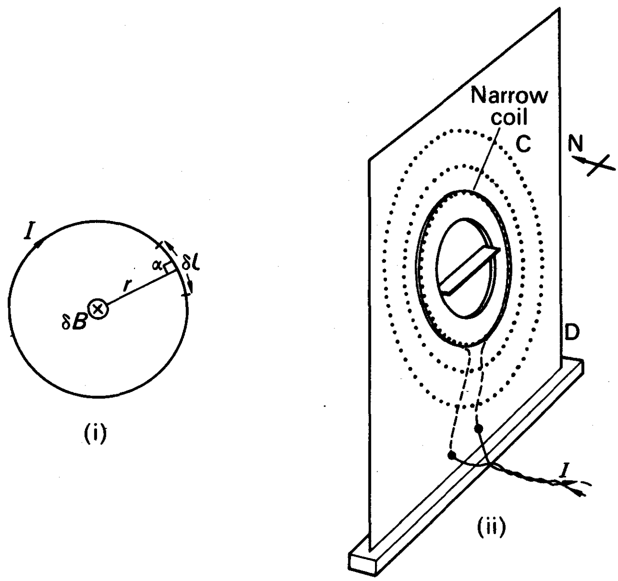


FIG. 37.2. Field of circular coil.

is a board D with sets of pegs, round which wire can be wound to form narrow coils of various radii. If a vibration magnetometer is used to measure B, the board is turned so that the axis of the coils C is in the direction of the magnetic meridian. The number of vibrations per minute is then found with the current in one direction and again when the current is reversed.

Suppose n_1 is the number when the field B of the current assists the field B_H due to the earth, so that $(B+B_H)$ is the resultant field. Suppose n_2 is the number when the field B opposes B_H and B is stronger than B_H, so that $(B-B_H)$ is the resultant field. Now on p. 887, it was shown that the flux density in a field was directly proportional to the square of the frequency of the vibration magnetometer. Hence

$$B+B_H = kn_1^2 \quad \text{and} \quad B-B_H = kn_2^2.$$

Adding,

$$\therefore 2B = k(n_1^2+n_2^2), \quad \text{or} \quad B \propto (n_1^2+n_2^2).$$

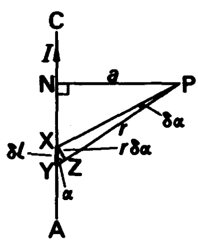
To verify $B \propto 1/r$ when I and N are constant, $(n_1^2+n_2^2)$ is evaluated when the wire is wound round other sets of pegs and I and N are kept constant each time. A graph of $(n_1^2+n_2^2)$ against $1/r$ is then plotted. This is found to be a straight line passing through the origin. Hence $B \propto 1/r$ when I and N are constant.

In a similar way, by varying the current I it can be shown that $B \propto I$ when N and r are constant. Similar experiments show that $B \propto N$ when I and r are constant. Thus $B \propto NI/r$ for a narrow circular coil.

The induction B in a magnetic field may also be measured by means of a *ballistic galvanometer* or by an *a.c. method*. See pp. 920, 934.

Field due to Long Straight Wire

We now deduce the induction at a point outside a long straight wire. In Fig. 37.3 (i), AC represents part of a long straight wire. P is taken as a point so near it that, from P, the wire looks infinitely long—it subtends very nearly 180°. An element XY of this wire, of length δl, makes an angle α with the radius vector, r, from P. It therefore contributes to the magnetic field at P an amount



(i)

FIG. 37.3 (i). Field of a long, straight wire.

$$\delta B = \frac{\mu_0 I \delta l \sin\alpha}{4\pi r^2} \quad . \quad \text{(i)}$$

when the wire carries a current I. If α is the perpendicular distance, PN, from P to the wire, then

$$\text{PN} = \text{PX} \sin\alpha$$

or

$$a = r\sin\alpha,$$

whence

$$r = \frac{a}{\sin\alpha}. \quad . \quad . \quad . \quad \text{(ii)}$$

Also, if we draw XZ perpendicular to PY, we have

$$\text{XZ} = \text{XY}\sin\alpha = \delta l \sin\alpha.$$

If δl subtends an angle $\delta\alpha$ at P, then

$$\text{XZ} = r\delta\alpha = \delta l \sin\alpha.$$

From (i),

$$\therefore \delta B = \frac{\mu_0 I \delta l \sin\alpha}{4\pi r^2} = \frac{\mu_0 I r \delta\alpha}{4\pi r^2} = \frac{\mu_0 I \delta\alpha}{4\pi r}$$

From (ii),

$$\therefore \delta B = \frac{\mu_0 I \sin\alpha\delta\alpha}{4\pi a}$$

When the point Y is at the bottom end A of the wire, $\alpha = 0$; and when Y is at the top C of the wire, $\alpha = \pi$. Therefore the total magnetic field at P is

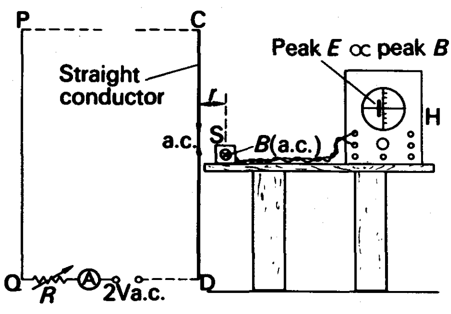
$$B = \frac{\mu_0}{4\pi}\int_0^{\pi} \frac{I \sin\alpha\delta\alpha}{a} = \frac{\mu_0 I}{4\pi a}\Big[-\cos\alpha\Big]_0^{\pi}$$

$$\therefore B = \frac{\mu_0 I}{2\pi a} \quad . \quad . \quad . \quad . \quad (1)$$

Equation (1) shows that the magnetic field of a long straight wire, at a point near it, is inversely proportional to the distance of the point from the wire. The result was discovered experimentally by Biot and Savart using a vibration magnetometer method, and led to their general formula in (i) which we used to derive (1).

Variation of *B* with Distance—A.C. Method

An apparatus suitable for finding the variation of *B* with distance from a long straight wire CD is shown in Fig. 37.3 (ii). Alternating current (a.c.) of the order of 10 A, from a low voltage mains transformer, is passed through CD by using another long wire PQ at least one metre away, a rheostat B and an a.c. ammeter A. A small search coil S, with thousands of turns of wire, such as the coil from an output transformer, is placed near CD. It is positioned with its axis at a small distance *r* from CD



(ii)

FIG. 37.3 (ii). Investigation of *B* due to long straight conductor.

and so that the flux from CD enters its face normally. S is joined by long twin flex to the Y-plates of an oscilloscope H and the greatest sensitivity, such as 5 mV/cm, is used.

When the a.c. supply is switched on, the varying flux through S produces an induced alternating e.m.f. E. The peak value of E can be determined by switching off the time-base and measuring the length of the line trace, Fig. 37.3 (ii). See p. 1014. Now the peak value of B, the magnetic induction, is proportional to the peak value of E, as shown in the case of the simple dynamo on p. 907. Thus the length of the trace gives a measure of the peak value of B.

The distance r of the coil from CD is then increased and the corresponding length of the trace is measured. The length of the trace plotted against $1/r$ gives a straight line graph passing through the origin. Hence $B \propto 1/r$. A similar method can be used for investigating the induction B for the case of a narrow circular coil or for a solenoid (p. 939).

EXAMPLE

Calculate the flux density at a distance of 1 cm or 0·01 m from a very long vertical straight wire carrying a current of 10 A. At what distance from the wire will the field induction neutralize that due to the earth's horizontal component flux-density, $0{\cdot}2 \times 10^{-4}$ T?

(i)
$$B = \frac{\mu_0 I}{2\pi a} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 10^{-2}}$$
$$= 2 \times 10^{-4} \text{ T}.$$

(ii) 1 cm from the wire, the induction due to the current is 2×10^{-4} T.

Now B is inversely-proportional to a, the distance from the wire. Thus B is $0{\cdot}2 \times 10^{-4}$ Wb m^{-2}, or ten times smaller than at 1 cm, at a distance 10 times as great. Thus the distance is 10 cm.

Note that the actual position of the point where the two fields neutralize must take account of the fact that B is a *vector*, that is, it has direction and magnitude. For a downward current of 10 A, the point concerned is due east of the wire. It is called a *neutral* point.

Field along Axis of a Narrow Circular Coil

We will now find the magnetic field at a point anywhere on the axis of a narrow circular coil (P in Fig. 37.4). We consider an element δl of the coil, at right angles to the plane of the paper. This sets up a field δB at P, in the plane of the paper, and at right angles to the radius vector r. If β is the angle between r and the axis of the coil, then the field δB has components $\delta B \sin\beta$ along the axis, and

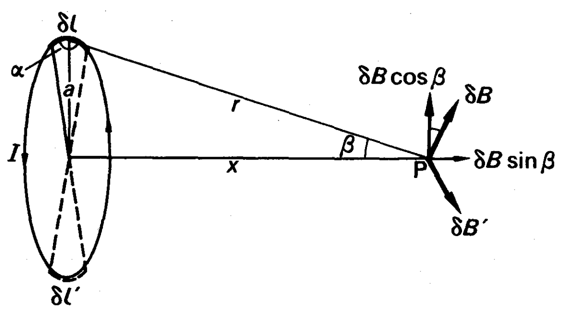


FIG. 37.4. Field on axis of flat coil.

$\delta B \cos\beta$ at right angles to the axis. If we now consider the element $\delta l'$ diametrically opposite to δl, we see that it sets up a field $\delta B'$ equal in magnitude to δB. This also has a component, $\delta B' \cos\beta$, at right angles to the axis; but this component acts in the opposite direction to $\delta B \cos\beta$ and therefore cancels it. By considering elements such as δl and $\delta l'$ all round the circumference of the coil, we see that the field at P can have no component at right angles to the axis. Its value along the axis is

$$B = \int dB \sin\beta.$$

From Fig. 37.4, we see that the length of the radius vector r is the same for all points on the circumference of the coil, and that the angle α is also constant, being 90°. This, if the coil has a single turn, and carries a current I,

$$\delta B = \frac{\mu_0 I \delta l \sin\alpha}{4\pi r^2} = \frac{\mu_0 I}{4\pi r^2}\delta l.$$

And, if the coil has a radius a, then

$$B = \int dB \sin\beta = \int_0^{2\pi a} \frac{\mu_0 I}{4\pi r^2} dl \sin\beta$$

$$= \frac{\mu_0 I a \sin\beta}{2r^2}. \qquad \text{(i)}$$

When the coil has more than one turn, the distance r varies slightly from one turn to the next. But if the width of the coil is small compared with all its other dimensions, we may neglect it, and write,

$$B = \frac{\mu_0 N I a \sin\beta}{2r^2}, \qquad \text{(ii)}$$

where N is the number of turns.

Equation (i) can be put into a variety of forms, by using the facts that

$$\sin\beta = \frac{a}{r},$$

and

$$r^2 = x^2 + a^2,$$

where x is the distance from P to the centre of the coil. Thus

$$B = \frac{\mu_0 N I a^2}{2r^3} = \frac{\mu_0 N A I}{2\pi(x^2+a^2)^{3/2}}, \qquad (1)$$

When the distance x is large compared with a, the expression (1) reduced to:

$$B = \frac{\mu_0 N A I}{2\pi x^3} = \frac{\mu_0 m}{2\pi x^3}, \qquad (2)$$

where $m = NAI =$ the *magnetic moment* of the circular coil, p. 885.

Helmholtz Coils

The field along the axis of a single coil varies with the distance x from the coil. In order to obtain a *uniform* field, Helmholtz used two coaxial parallel coils of equal radius R, separated by a distance R. In this case, when the same current flows round each coil in the same direction, the resultant field B is uniform for some distance on either side of the point on their axis midway between the coils. This may be seen roughly by adding the fields due to each coil alone. Helmholtz coils were used in Thomson's determination of e/m (p. 1003).

The magnitude of the resultant field B at the midpoint can be found from our previous formula for a single coil. We now have $a = R$ and $x = R/2$. Thus, for the two coils,

$$B = 2\times\frac{\mu_0 NIR^2}{2(R^2/4+R^2)^{3/2}} = \left(\frac{4}{5}\right)^{3/2}\times\frac{\mu_0 NI}{R}$$
$$= 0{\cdot}72\frac{\mu_0 NI}{R} \text{ (approx.).}$$

Field on Axis of a Long Solenoid

We may regard a solenoid as a long succession of narrow coils; if it has n turns per metre, then in an element δx of it there are $n\delta x$ coils (Fig. 37.5). At a point P on the axis of the solenoid, the field due to these is, by equation (ii),

$$\delta B = \frac{\mu_0 Ia \sin\beta}{2r^2}n\delta x,$$

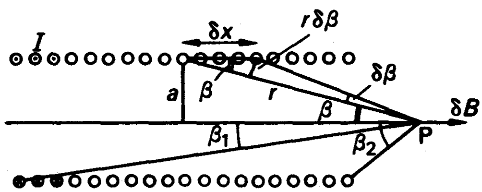


FIG. 37.5. Field on axis of solenoid.

in the notation which we have used for the flat coil. If the element δx subtends an angle $\delta\beta$ at P, then, from the figure,

$$r\delta\beta = \delta x \sin\beta;$$

whence
$$\delta x = \frac{r\delta\beta}{\sin\beta}.$$

Also,
$$a = r\sin\beta.$$

Thus
$$\delta B = \frac{\mu_0 Ir\sin^2\beta}{2r^2}n\frac{r\delta\beta}{\sin\beta}$$
$$= \frac{\mu_0 nI}{2}\sin\beta\delta\beta.$$

If the radii of the coil, at its ends, subtend the angles β_1 and β_2 at P, then the field at P is

$$H = \int_{\beta^1}^{\beta_2}\frac{\mu_0 nI}{2}\sin\beta d\beta$$
$$= \frac{\mu_0 nI}{2}\Big[-\cos\beta\Big]_{\beta_2}^{\beta_2},$$
$$= \frac{\mu_0 nI}{2}(\cos\beta_1 - \cos\beta_2). \quad . \quad . \quad . \quad . \quad (1)$$

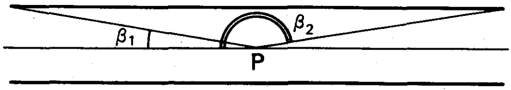


FIG. 37.6. A very long solenoid.

If the point P is inside a very long solenoid—so long that we may regard it as infinite—then $\beta_1 = 0$ and $\beta_2 = \pi$, as shown in Fig. 37.6. Then, by equation (1):

$$B = \frac{\mu_0 nI}{2}\Big[-\cos\beta\Big]_0^{\pi}.$$

whence

$$B = \mu_0 nI \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The quantity nI is often called the 'ampere-turns per metre'.

Very Long Solenoid or Toroid

Equation (1) shows that the field along the axis of an infinite solenoid is constant: it depends only on the number of turns per centimetre, and the current. By methods beyond the scope of this book, it can also be shown that the field is the same at points not on the axis. An infinite solenoid therefore gives us a means of producing a uniform magnetic field.

In practice, solenoids cannot be made infinitely long. But if the length of a solenoid is about ten times its diameter, the field near its middle is fairly uniform, and has the value given by equation (1).

A form of coil which gives a very nearly uniform field is shown in Fig. 37.7. It is a solenoid of N turns and length L metre wound on a

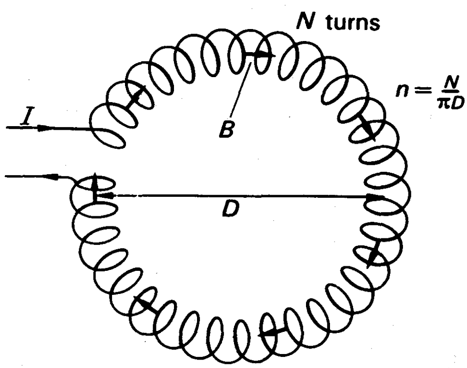


FIG. 37.7. A toroid.

circular support instead of a straight one, and is called a toroid. If its average diameter D is several times its core diameter d, then the turns of wire are almost equally spaced around its inside and outside circumferences; their number per metre is therefore

$$n = \frac{N}{L} = \frac{N}{\pi D} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The magnetic field within a toroid is very nearly uniform, because the coil has no ends. The coil is equivalent to an infinitely long solenoid, and the field-strength at all points within it is given by

$$B = \mu_0 nI \quad . \quad . \quad . \quad . \quad (2)$$

A 'Slinky' is a coil which can be stretched to provide simply a solenoid with a varying number of turns per metre, n. A small search coil with many thousands of turns, placed coaxially inside the solenoid, can be connected to an oscilloscope to provide a measure of B when alternating current is passed into the solenoid. See p. 934. Since $n \propto 1/L$, where L is the length of the coil, a graph of B against $1/L$ can be plotted for various values of L, the current being the same each time. A straight line through the origin is obtained, showing that $B \propto n$. A search coil connected to a ballistic galvanometer, and a direct current which is reversed in the coil, may also be used to provide a measure of B (p. 919).

Forces between Currents

Ampère carried out many experiments on the forces of attraction and repulsion between two current-carrying conductors. Each was acted on by the field of the other, as shown in Fig. 37.8. Currents flowing in the same direction ('like' currents) attracted each other,

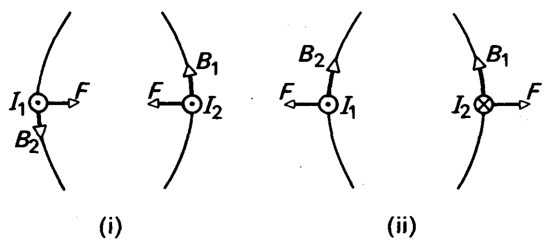


FIG. 37.8. Forces between currents.

Fig. 37.8 (i), while 'unlike' currents (in opposite directions) repelled each other, Fig. 37.8 (ii). This difference from the laws governing poles and charges greatly impressed Ampère.

If two long straight conductors lie parallel and close together at a distance r apart, and carry currents I, I' respectively, then the current I is in a magnetic field of flux density B equal to $\mu_0 I'/2\pi r$ due to the current I (p. 934). The force per metre length F is hence given by

$$F = BIl = BI \times 1 = \frac{\mu_0 I'}{2\pi r} \times I \times 1$$

$$\therefore F = \frac{\mu_0 II'}{2\pi r} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Nowadays the ampere is *defined* in terms of the force between conductors. It is *that current, which flowing in each of two infinitely-long* *parallel straight wires of negligible cross-sectional area separated by a distance of 1 metre in vacuo, produces a force between the wires of 2×10^{-7} newton metre^{-1}.*

Taking $I = I' = 1$ A, $r = 1$ metre, $F = 2 \times 10^{-7}$ newton metre^{-1}, then, from (1),

$$2 \times 10^{-7} = \frac{\mu_0 \times 1 \times 1}{2\pi \times 1}$$

$$\therefore \mu_0 = 4\pi \times 10^{-7} \text{ henry metre}^{-1},$$

which is the value quoted above.

Unit of μ_0

The permeability of free space μ_0 was defined previously from the relation

$$\delta B = \frac{\mu_0 I \delta s \sin\theta}{4\pi r^2}$$

From this relation, the unit of μ_0 is

$$\frac{\text{weber metre}^{-2} \times \text{metre}^2}{\text{ampere} \times \text{metre}} \text{ or Wb A}^{-1}\text{ m}^{-1} \quad . \quad (2)$$

Now the unit of inductance L is the henry (H), which can be defined from the relation $\Phi = LI$ (p. 925). Thus, since $L = \Phi/I$

$$1 \text{ H} = 1 \text{ Wb A}^{-1}.$$

From (2), it follows that the unit of μ_0 can be written as

$$\text{H m}^{-1} \text{ (henry per metre)},$$

and this is the SI unit of μ_0 and of permeability μ generally.

Absolute Determination of Current

A laboratory form of an *ampere balance*, which measures current by measuring the force between current-carrying conductors, is shown in Fig. 37.9.

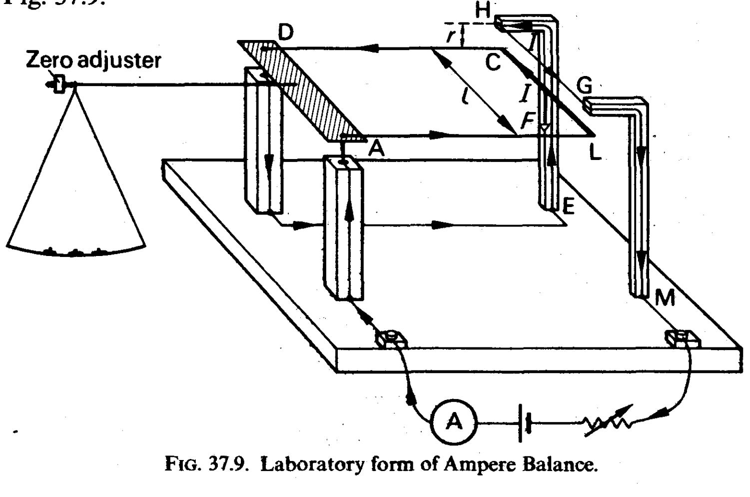


FIG. 37.9. Laboratory form of Ampere Balance.

With no current flowing, the zero screw is adjusted until the plane of ALCD is horizontal. The current I to be measured is then switched on so that it flows through ALCD and EHGM in series and HG repels CL. The mass m necessary to restore balance is then measured, and mg is the force between the conductors since the respective distances of CL and the scale pan from the pivot are equal. The equal lengths l of the straight wires CL and HG, and their separation r, are all measured.

From equation (1) on p. 939,

$$\text{force per metre} = \frac{4\pi \times 10^{-7} I^2}{2\pi r}$$

$$\therefore mg = \frac{4\pi \times 10^{-7} I^2 l}{2\pi r}$$

$$\therefore I = \sqrt{\frac{mgr}{2 \times 10^{-7} l}}$$

In this expression, I will be in amperes if m is in kilogrammes, $g = 9{\cdot}8\ \text{m s}^{-2}$ and l and r are measured in metres.

Magnetizing Force, or Intensity, *H*

Biot and Savart's law has been stated as

$$\delta B = \frac{\mu_0 I \delta l \sin \alpha}{4\pi r^2}$$

This is only true in air or a vacuum. In other materials the flux density may be altered, even though the currents remain the same. To take account of this we write

$$\delta B = \frac{\mu I \delta l \sin \alpha}{4\pi r^2}$$

where μ is the *permeability* of the medium. μ_0 is called the permeability of 'free space' or vacuum.

So far we have only used the flux density, B, in a field. Another field quantity, symbol H, is also used. It is called the *magnetizing force or intensity*, and is defined by the relation

$$H = \frac{B}{\mu}.$$

We may thus write δH arising from a current I in an element of length δl as

$$\delta H = \frac{I \delta l \sin \alpha}{4\pi r^2} \quad . \quad . \quad . \quad . \quad (1)$$

From this it can be seen that the unit of H is

$$\frac{\text{ampere metre}}{\text{metre}^2} \quad \text{or} \quad \text{ampere metre}^{-1}\ (\text{A m}^{-1});$$

whereas the unit of B in the tesla (T) or weber metre^{-2} (Wb m^{-2}).

In any medium δB has a value depending on the permeability of the medium. From (1), it can be seen that H does not depend on μ but only the currents and their geometry. H is *independent*, therefore, of the medium in which the conductors are situated. It is for this reason that H is regarded as being due directly to the currents. H is then a 'cause' which gives rise to a flux density B given by μH, and so B is dependent on the medium used. Because of this interpretation, H is often called the 'magnetizing force', or 'magnetizing intensity'.

From equation (1) on p. 934, the magnetizing force due to a straight wire is given by

$$H = \frac{I}{2\pi a}.$$

Ampère's Theorem

In the calculation of magnetic fields, we have used so far only the Biot and Savart law. Another law useful for calculating magnetic field strengths is *Ampère's theorem.*

Consider Fig. 37.10, in which a continuous closed line or loop L is drawn round the wires P, Q, R which carry currents of I_1, I_2, I_3 respectively. The total current enclosed by L is $(I_1+I_2+I_3)$. Now if H is the magnetizing force or *magnetic field intensity* at any element dl

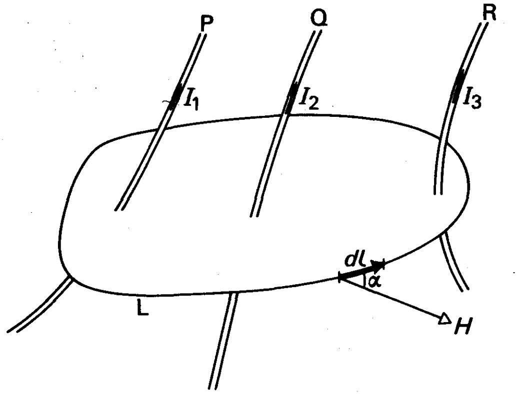


FIG. 37.10. Ampere's theorem.

in the loop L, obtained from the Biot and Savart law, it is possible to calculate the quantity $H.dl.\cos\alpha$, where α is the angle between the element and the field H. When the quantity $H.dl.\cos\alpha$ is summed for every element in the closed loop L, calculation beyond the scope of this book shows that the result is $(I_1+I_2+I_3)$. This is expressed by

$$\oint H.dl.\cos\alpha = I_1+I_2+I_3,$$

where the symbol $\oint$ represents the integral taken completely round the closed loop. Ampère's theorem is the general statement

$$\oint H.dl.\cos\alpha = I,$$

where I is the total current enclosed by the loop.

We now apply the theorem to two special cases of current-carrying conductors.

1. *Straight wire*

Fig. 37.11 shows a circular loop L of radius r, drawn concentrically round a straight wire carrying a current I. The lines of force are circles and hence, at *every* part of a closed line, H is directed along the line itself. Thus $\alpha = 0°$ all round the line. Further, by symmetry, H has the same value everywhere on the line.

$$\therefore \oint H\,.\,dl\,.\cos\alpha = H\oint dl = H\,.\,2\pi r,$$

since $\alpha = 0°$ and H is constant. Hence, from Ampère's theorem,

$$H\,.\,2\pi r = I$$

$$\therefore H = \frac{I}{2\pi r}, \quad \text{and} \quad B = \mu_0 H = \frac{\mu_0 I}{2\pi r}.$$

This agrees with the result for B on p. 934.

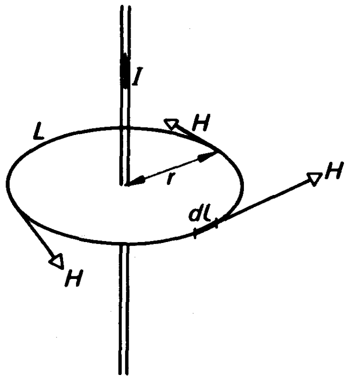


FIG. 37.11. Field intensity of straight wire.

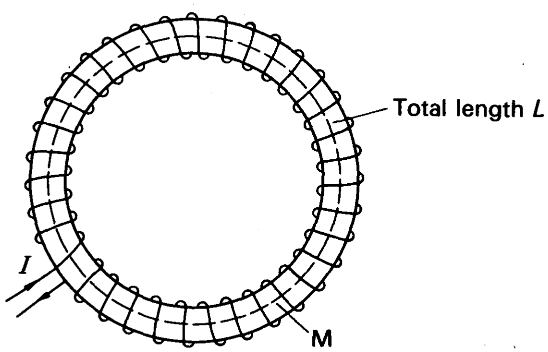


FIG. 37.12. Field intensity of toroid.

2. *Toroid*

Consider the closed loop M indicated by the broken line in Fig. 37.12. Again H is everywhere the same on M and is directed along the loop.

$$\therefore \oint H\,.\,dl\cos\alpha = H\oint dl$$
$$= HL,$$

where L is the total length of the loop M. Hence, from Ampere's theorem,

$$HL = NI$$

$$\therefore H = \frac{NI}{L} = nI,$$

where N is the total number of turns, and n is the number of turns per metre. This agrees with p. 938.

Earth's Magnetism

It was Dr. Gilbert who first showed that a magnetized needle, when freely suspended about its centre of gravity, dipped downwards towards the north at about 70° to the horizontal in England. He also found that this *angle of dip* increased with latitude, as shown in Fig. 37.13, and concluded that the earth itself was, or contained, a magnet. The points where the angle of dip is 90° are called the earth's magnetic poles; they

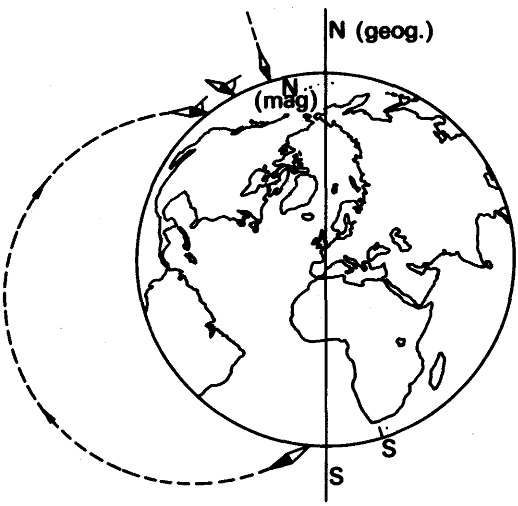


FIG. 37.13. Illustrating the angle of dip.

are fairly near to the geographic poles, but their positions are continuously, though slowly, changing. Gilbert's simple idea of the earth as a magnet has had to be rejected. The earth's crust does not contain enough magnetic material to make a magnet of the required strength; the earth's core is, we believe, molten—and molten iron is non-magnetic. The origin of the earth's magnetism is, in fact, one of the great theoretical problems of the present day.

Horizontal and Vertical Components. Variation and Dip

Since a freely suspended magnetic needle dips downward at some angle δ to the horizontal, the earth's *resultant magnetic field*, B_R acts at an angle δ to the horizontal. The 'angle of dip', or *inclination*, can thus be defined as the angle between the resultant earth's field and the horizontal. The earth's field has a *vertical component*, B_V, given by

$$B_V = B_R \sin \delta, \quad \quad (1)$$

and a *horizontal component*, B_H, given by

$$B_H = B_R \cos \delta. \quad \quad (2)$$

Also,

$$\frac{B_V}{B_H} = \tan \delta. \quad \quad (3)$$

To specify the earth's magnetic field at any point, we must state its

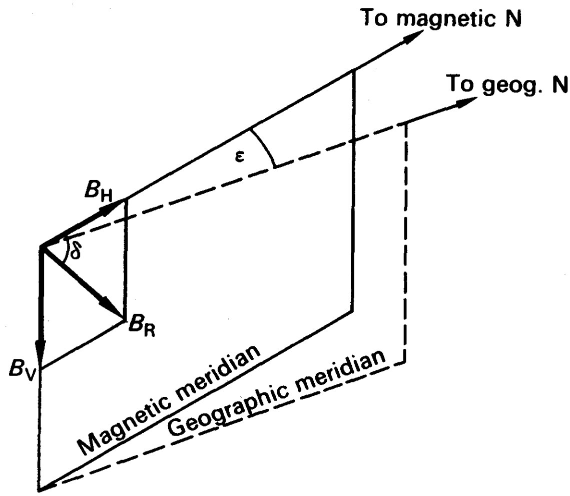


FIG. 37.14. Magnetic and geographic meridians. Dip.

strength and direction. To specify its direction we must give the direction of the magnetic meridian, and the angle of dip δ (Fig. 37.14). In most parts of the world the magnetic meridian does not lie along the geographic meridian (the vertical plane running geographically north–south). The angle between the magnetic and geographic meridians, ε, is called the magnetic variation, or sometimes the declination, at the place concerned; it is shown on the margins of maps. The horizontal and vertical components of the earth's field, and the angle of dip, can be measured by a large coil or 'earth inductor' (see p. 921).

EXERCISES 37

1. Define the *ampere*. Write down expressions for (i) the magnetic field strength (magnetizing force) at a distance of d from a very long straight conductor carrying a current I, and (ii) the mechanical force acting on a straight conductor of length l carrying a current I at right angles to a uniform magnetic field of flux density B.

Show how these two expressions may be used to deduce a formula for the force per unit length between two long straight parallel conductors in vacuo carrying currents I_1 and I_2 separated by a distance d.

A horizontal straight wire 5 cm long weighing 1·2 g m^{-1} is placed perpendicular to a uniform horizontal magnetic field of flux density 0·6 Wb m^{-2}. If the resistance of the wire is 3·8 ohm m^{-1}, calculate the p.d. that has to be applied between the ends of the wire to make it just self-supporting. Draw a diagram showing the direction of the field and the direction in which the current would have to flow in the wire. (*C.*)

2. State the law of force acting on a conductor carrying an electric current in a magnetic field. Indicate the direction of the force and show how its magnitude depends on the angle between the conductor and the direction of the field.

Sketch the magnetic field due solely to two long parallel conductors carrying respectively currents of 12 and 8 A in the same direction. If the wires are 10 cm apart, find where a third parallel wire also carrying a current must be placed so that the force experienced by it shall be zero. (*L.*)

3. Define the *ampere*.

Two long vertical wires, set in a plane at right angles to the magnetic meridian, carry equal currents flowing in opposite directions. Draw a diagram showing the pattern, in a horizontal plane, of the magnetic flux due to the currents alone—that is, for the moment ignoring the earth's magnetic field.

Next, taking into account the earth's magnetic field, discuss the various situations that can give rise to neutral points in the plane of the diagram.

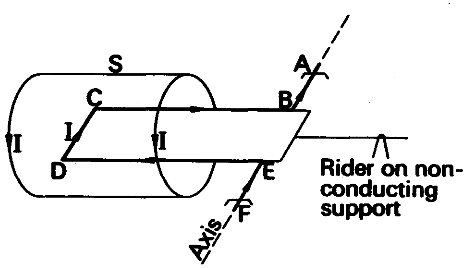


FIG. 37.15.

Fig. 37.15 shows a simple form of current balance. The 'long' solenoid S, which has 2000 turns per metre, is in series with the horizontal rectangular copper loop ABCDEF, where BC = 10 cm and CD = 3 cm. The loop, which is freely pivoted on the axis AF, goes well inside the solenoid, and CD is perpendicular to the axis of the solenoid. When the current is switched on, a rider of mass 0·2 g placed 5 cm from the axis is needed to restore equilibrium. Calculate the value of the current, I. (*O.*)

4. Define *magnetic moment*.

A small magnet, suspended with its axis horizontal so as to be able to rotate freely about a vertical axis, is situated at the centre of a long horizontal solenoid, the axis of which lies at right angles to the magnetic meridian. If the solenoid has 20 turns per cm, determine the value of the current passing through it which would cause the magnet to rotate through 50°. (Horizontal component of earth's magnetic field intensity = 14 A m^{-1}.) (*N.*)

5. Describe with experimental details how you would carry out any *two* of the following in the laboratory:

(*a*) Determine the current sensitivity of a moving-coil galvanometer.
(*b*) Determine the internal resistance of a dry cell using a potentiometer.
(*c*) Investigate, using a vibration magnetometer, how the magnetic field due to a long straight wire carrying a steady current varies with distance from the wire. (*L.*)

6. Define the *ampere.*

Draw a labelled diagram of an instrument suitable for measuring a current absolutely in terms of the ampere, and describe the principle of it.

A very long straight wire PQ of negligible diameter carries a steady current I_1. A square coil ABCD of side l with n turns of wire also of negligible diameter is set up with sides AB and DC parallel to and coplanar with PQ; the side AB is nearest to PQ and is at a distance d from it. Derive an expression for the resultant force on the coil when a steady current I_2 flows in it, and indicate on a diagram the direction of this force when the current flows in the same direction in PQ and AB.

Calculate the magnitude of the force when $I_1 = 5$ A, $I_2 = 3$ A, $d = 3$ cm, $n = 48$ and $l = 5$ cm. (*O. & C.*)

7. (i) Draw a diagram showing the pattern of the magnetic field lines in a horizontal plane passing through the centre of a plane vertical circular coil carrying a steady current. Indicate on your diagram possible positions for neutral points if the plane of the coil were to be set in the magnetic meridian.

(ii) How would you show experimentally that the value of B at the centre of such a coil is proportional to nI/r, where n is the number of turns and r the radius?

8. A long straight wire carries a steady current I. Write down a formula for the magnetic intensity (magnetizing force) H due to the current at a point distant y from the wire. Give a consistent set of units for the quantities in your formula.

Show on a diagram the lines of magnetic force due to the currents when two parallel wires separated by a distance $2a$ carry equal currents in the same direction. Use your formula to derive an expression for the magnetic intensity at a point P in a plane perpendicular to the wires at a distance x from the point midway between the wires and along the right bisector of the line joining them in this plane. Hence derive the condition that the intensity at P shall be a maximum. (*N.*)