chapter thirty-six

Electromagnetic Induction

Faraday's Discovery

AFTER Ampere and others had investigated the magnetic effect of a current Faraday attempted to find its converse: he tried to produce a current by means of a magnetic field. He began work on the problem in 1825 but did not succeed until 1831.

The apparatus with which he worked is represented in Fig. 36.1; it consists of two coils of insulated wire, A, B, wound on a wooden core.

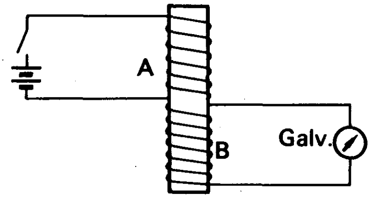


FIG. 36.1. Faraday's experiment on induction.

One coil was connected to a galvanometer, and the other to a battery. No current flowed through the galvanometer, as in all Faraday's previous attempts. But when he disconnected the battery Faraday happened to notice that the galvanometer needle gave a kick. And when he connected the battery back again, he noticed a kick in the opposite direction. However often he disconnected and reconnected the battery, he got the same results. The 'kicks' could hardly be all accidental—they must indicate momentary currents. Faraday had been looking for a steady current, but the effect he sought turned out to be a transient one—that was why it took him six years to find it.

Conditions for Generation of Induced Current

The results of Faraday's experiments showed that a current flowed in coil B of Fig. 36.1 only while the magnetic field due to coil A was changing—the field building up as the current in A was switched on, decaying as the current in A was switched off. And the current which flowed in B while the field was decaying was in the opposite direction to the current which flowed while the field was building up. Faraday called the current in B an induced current. He found that it could be made much greater by winding the two coils on an iron core, instead of a wooden one.

Once he had realized that an induced current was produced only by a change in the magnetic field inducing it, Faraday was able to find induced currents wherever he had previously sought them. In place of the coil A he used a magnet, and showed that as long as the coil and the magnet were at rest, there was no induced current (Fig. 36.2 (i)). But when he moved either the coil or the magnet an induced current flowed as long as the motion continued (Fig. 36.2 (ii)). If the current flowed one way when the north pole of the magnet was approaching

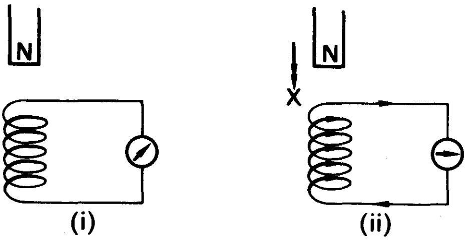


FIG. 36.2. Induction of current by moving magnet.

the end X of the coil, it flowed the other way when the north pole was retreating from X, or the south pole approached X.

Since a flow of current implies the presence of an e.m.f., Faraday's experiments showed that an e.m.f. could be induced in a coil by moving it relatively to a magnetic field. In discussing induction it is more fundamental to deal with the e.m.f. than the current, because the current depends on both the e.m.f. and the resistance.

Direction of E.M.F.; Lenz's Law

Before considering the magnitude of an induced e.m.f., let us investigate its direction. To do so we must first see which way the galvanometer deflects when a current passes through it in a known direction: we can find this out with a battery and a megohm resistor (Fig. 36.3 (i)). We then take a coil whose direction of winding we know, and connect this to the galvanometer. In turn we plunge each pole of a magnet into and out of the coil; and we get the results shown in Fig. 36.3 (ii), (iii), (iv). These results were generalized most elegantly into a rule by Lenz in 1835. He said that *the induced current flows always in such a direction as to oppose the change which is giving rise to it.* If the reader will sketch with a pencil on Fig. 36.3 the magnetic fields of the induced currents, then he will see what Lenz meant: when the magnet is approaching the coil, the coil repels it; when the magnet is retreating from the coil, the coil attracts it.

Lenz's law is a beautiful example of the conservation of energy: the induced current sets up a force on the magnet, which the mover of the magnet must overcome: the work done in overcoming this force provides the electrical energy of the current. (This energy is dissipated as heat in the coil.) If the induced current flowed in the opposite direction to

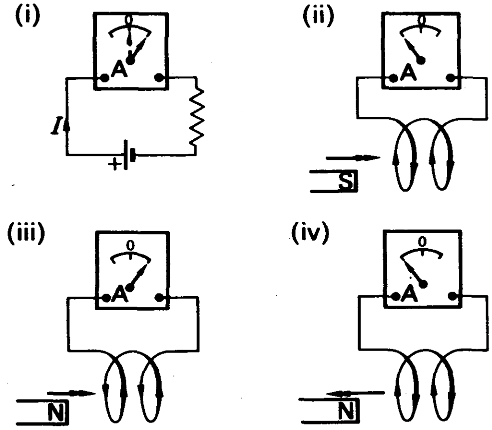


FIG. 36.3. Direction of induced currents.

that which it actually takes, then it would aid—it would speed up—the motion of the magnet. It would enhance its own cause, and grow indefinitely; at the same time, it would continuously increase the kinetic energy of the magnet. Thus both mechanical and electrical energy would be produced, without any agent having to do work. The system would be a perpetual motion machine.

The direction of the induced e.m.f., E, is specified by that of the current, as in Fig. 36.4. If we wished to reword Lenz's law, substituting e.m.f. for current, we would have to speak of the e.m.f.s *tending* to oppose the change . . . etc., because there can be no opposing force unless the circuit is closed and a current can flow.

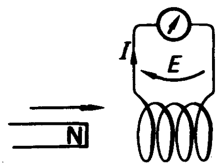


FIG. 36.4. Direction of induced e.m.f.

Magnitude of E.M.F.

Accurate experiments on induction are difficult to contrive with simple apparatus; but rough-and-ready experiments will show on what factors the magnitude depends. We require coils of the same diameter but different numbers of turns, coils of the same number of turns but different diameters, and two similar magnets, which we can use singly or together. If we use a high-resistance galvanometer, the current will not vary much with the resistance of the coil in which the e.m.f. is induced, and we can take the deflection as a measure of the e.m.f. There is no need to plunge the magnet into and out of the coil: we can

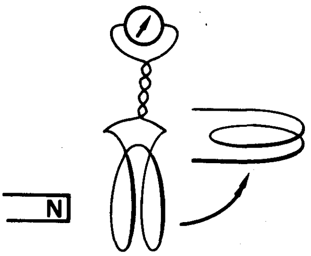


FIG. 36.5. E.m.f. induced by turning coil.

get just as great a deflection by simply turning the coil through a right angle, so that its plane changes from parallel to perpendicular to the magnet, or vice versa (Fig. 36.5). We find that the induced e.m.f. increases with:

(i) the speed with which we turn the coil;
(ii) the area of the coil;
(iii) the strength of the magnetic field (two magnets give a greater e.m.f. than one);
(iv) the number of turns in the coil.

To generalize these results and to build up useful formulae, we use the idea of *magnetic flux*, or field lines passing through a coil. Fig. 36.6 shows a coil, of area A, whose normal makes an angle θ with a uniform magnetic field of induction B. The component of the field at right angles to the plane of the coil is $B\cos\theta$, and we say that the magnetic flux Φ through the coil is

$$\Phi = AB\cos\theta \quad . \quad . \quad . \quad . \quad (1)$$

(We get the same result if we multiply the field-strength B by the area projected at right angles to the field, $A\cos\theta$.) If either the strength of the field is changed, or the coil is turned so as to change the angle θ, then the flux through the coil changes.

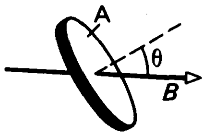


FIG. 36.6. Magnetic flux.

Results (i) to (iii) above, therefore, show that the e.m.f. induced in a coil increases with the *rate of change of the magnetic flux* through it. More accurate experiments show that the induced e.m.f. is actually proportional to the rate of change of flux through the coil; this result is sometimes called *Faraday's*, or *Neumann's, law*.

The unit of magnetic flux Φ is called the *weber* (Wb). Hence the unit of B is the *weber per metre*^{2} (Wb m^{-2}) or *tesla* (T).

Flux Linkage

If a coil has more than one turn, then the flux through the whole coil is the sum of the fluxes through the individual turns. We call this the *flux linkage* through the whole coil. If the magnetic field is uniform, the flux through one turn is given, from (1), by $AB\cos\theta$. If the coil has N turns, the total flux linkage Φ is given by

$$\Phi = NAB\cos\theta \quad . \quad . \quad . \quad . \quad (2)$$

From Faraday's or Neumann's law, the e.m.f. induced in a coil is proportional to the rate of change of the flux linkage, Φ. Hence

$$E \propto \frac{d\Phi}{dt},$$

or

$$E = -k\frac{d\Phi}{dt}, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where k is a positive constant. The minus sign expresses Lenz's law. It means that the induced e.m.f. is in such a direction that, if the circuit is closed, the induced current *opposes* the change of flux. Note that an induced e.m.f. exists across the terminals of a coil when the flux linkage changes, even though the coil is on 'open circuit'. A current, of course, does not flow in the latter case.

On p. 901, it is shown that $E = -kd\Phi/dt$ is consistent with the expression $F = BIl$ for the force on a conductor only if $k = 1$. We may therefore say that

$$E = -\frac{d\Phi}{dt}, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where Φ is the flux linkage in webers, t is in seconds, and E is in volts.

From (4), it follows that one weber is the flux linking a circuit if the induced e.m.f. is one volt when the flux is reduced uniformly to zero in one second.

E.M.F. Induced in Moving Rod

Generators at power stations produce high induced voltages by rotating long *straight conductors*. Fig. 36.7 (i) shows a simple apparatus

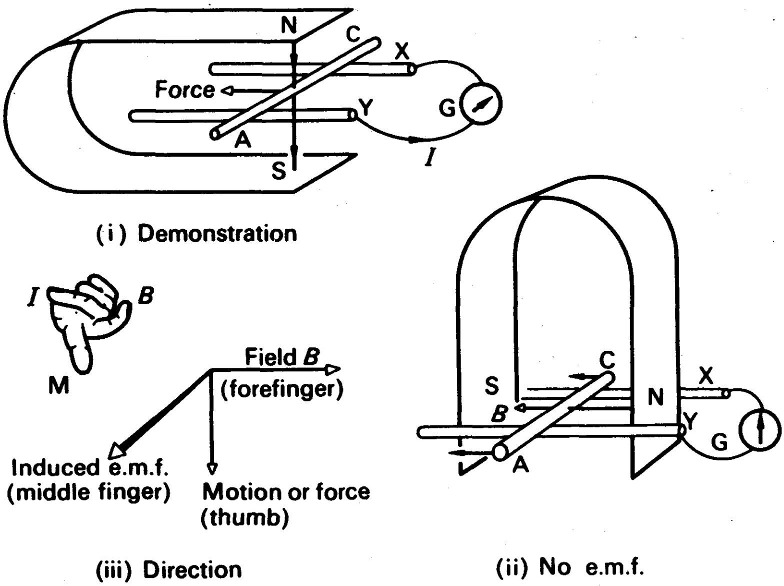


FIG. 36.7. E.m.f. induced in moving rod.

for demonstrating that an e.m.f. may be induced in a straight rod or wire, when it is moved across a magnetic field. The apparatus consists of a rod AC resting on rails XY, and lying between the poles NS of a permanent magnet. The rails are connected to a galvanometer G.

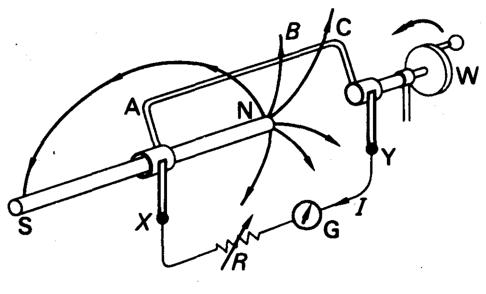
If we move the rod to the left, so that it cuts across the field B of the magnet, a current I flows as shown. If we move the rod to the right, the current reverses. We notice that the current flows only while the rod is moving, and we conclude that the motion of the rod AC induces an e.m.f. in it.

By turning the magnet into a vertical position (Fig. 36.7 (ii)) we can show that no e.m.f. is induced in the rod when it moves parallel to the field B. We conclude that an e.m.f. is induced in the rod only when it *cuts across* the field. And, whatever the direction of the field, no e.m.f. is induced when we slide the rod parallel to its own length. The induced e.m.f. is greatest when we move the rod at right angles, both to its own length and to the magnetic field. These results may be summarized in *Fleming's right-hand rule*:

If we extend the thumb and first two fingers of the right hand, so that they are all at right angles to one another, then the directions of field, motion, and induced e.m.f. are related as in Fig. 36.7 (iii).

To show E.M.F. $\propto$ Rate of Change

The variation of the magnitude of the e.m.f. in a rod with the speed of 'cutting' magnetic flux can be demonstrated with the apparatus in Fig. 36.8 (i).



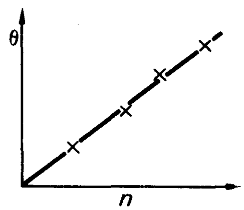


FIG. 36.8. Induced e.m.f.

Here AC is a copper rod, which can be rotated by a wheel W around one pole N of a long magnet. Brushing contacts at X and Y connect the rod to a galvanometer G and a series resistance R. When we turn the wheel, the rod AC cuts across the field B of the magnet, and an e.m.f. is induced in it. If we turn the wheel steadily, the galvanometer gives a steady deflection, showing that a steady current is flowing round the circuit.

To find how the current and e.m.f. depends on the speed of the rod, we keep the circuit resistance constant, and vary the rate at which we turn the wheel. We time the revolutions with a stop-watch, and find that the deflection θ is proportional to the number of revolutions per second, n (Fig. 36.8 (ii)). It follows that the induced e.m.f. is proportional to the speed of the rod.

Calculation of E.M.F. in Rod

Consider the circuit shown in Fig. 36.9. PQ is a straight wire touching the two connected parallel wires QR, PS and free to move over them. All the conductors are situated in a uniform magnetic field of induction B, perpendicular to the plane of PQRS.

Suppose the rod PQ is pulled with a uniform velocity v by an external force F. There will then be a change of flux linkage in the area PQRS and so an e.m.f. will be induced in the circuit. This produces a current I which flows round the circuit. A *force* will now act on the wire PQ

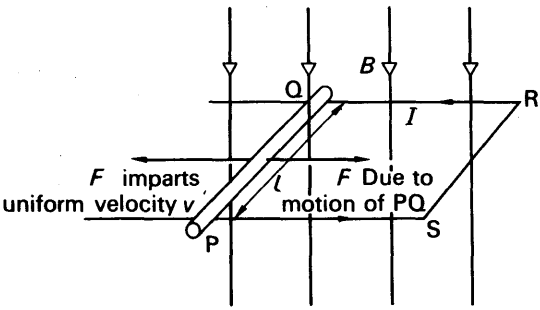


FIG. 36.9 Calculation of induced e.m.f.

due to the current flowing and to the presence of the magnetic field (p. 878). By Lenz's law, the direction of this force will oppose the movement of PQ. If the current flowing is I, and the length of PQ is l, the force on PQ is BIl. This is equal to the external force F, since PQ is not accelerating.

Because energy is conserved, the rate of working by the external force is equal to the rate at which energy is supplied to the electrical circuit. Now in one second, PQ moves a distance v. Hence

$$\text{work done per second} = \text{force} \times \text{distance moved per second.}$$
$$= BIlv$$

If the induced e.m.f. is E, the electrical energy used in one second, or power, $= EI$.

$$\therefore EI = BIlv$$
$$\therefore E = Blv. \qquad . \quad . \quad . \quad . \quad (6)$$

This result has been derived without using the relation $E = -d\Phi/dt$. To see if the same result as (6) can be obtained, consider the flux changes. In one second the area of PQRS changes by vl. Hence the change in flux linkage per second, $d\Phi/dt = B \times$ area change per second $= Blv$. Hence, numerically,

$$\therefore E = Blv.$$

This means that the relation $E = -d\Phi/dt$ may be used to find the induced e.m.f. in a straight wire.

Induced E.M.F. and Force on Moving Electrons

We have seen that an electron moving across a magnetic field experiences a mechanical force (p. 881). This explains neatly the e.m.f. induced in a wire: when we move the wire across the field, we move each free electron in it, likewise across the field. As Fig. 36.10 shows, the force on the electrons, F, is at right angles to the plane containing the velocity v of the wire, and the magnetic field B. Thus it tends to drive the electrons along the wire. The direction in which it does so agrees with the direction of the conventional e.m.f., which is the direction of the force on a positive charge.

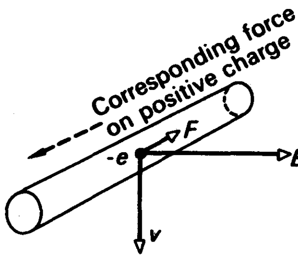


FIG. 36.10. Force on a moving electron.

When a wire AC is swept, as shown in Fig. 36.11, across a magnetic field B, the force on the electrons in it acts from A to C. Therefore, if the wire is not connected to a closed circuit, electrons will pile up at C: the end C will gain a negative

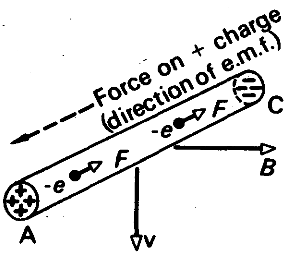
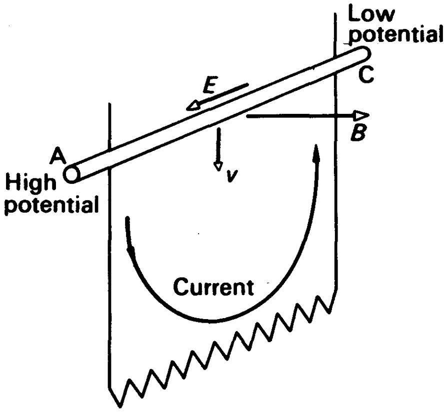


FIG. 36.11. Induced e.m.f. arising from force on moving electrons.

charge, and A will be left with a positive charge. The end A will therefore be at a higher potential than C.

If we now clear our minds of electrons, we see that the conventional e.m.f. acts from C to A. If positive electricity were free to move, it would accumulate at A;

in other words, the tendency of the e.m.f., acting from C to A, is to give A a higher potential than C. The *potential difference* between A and C tends to drive positive electricity the other way. Equilibrium is reached when the potential difference V_{AC} is equal to the e.m.f. acting from C to A.



FɪG. 36.12. E.m.f. and potential difference.

When the wire is connected to a closed circuit, current flows from A to C round the external circuit (Fig. 36.12). Within the source of current—the wire AC—the e.m.f. drives the current (of positive charge) from C to A: from low potential to high. *This is the essential function of an e.m.f.*; an e.m.f. is an agency which can drive an electric current *against* a potential difference. When the e.m.f. arises in a wire moving across a magnetic field, this agency is the force on the electrons moving with the wire.

The e.m.f. induced in a wire can easily be calculated from the force on a moving electron. If the wire moves with a velocity v at right angles to a field B, then so do the electrons in it. Each of them therefore experiences a force

$$F = Bev,$$

(equation (1), p. 881), where e is the electronic charge. The work which this force does in carrying the electron along the length l of the wire is Fl. But it is also, by definition, equal to the product of the e.m.f. E, and the charge e. Therefore

$$Ee = Fl = Bevl,$$

whence

$$E = Blv.$$

APPLICATIONS OF INDUCTION

The Induction Coil

The induction coil is a device for getting a high voltage from a low one. It was at one time used for X-ray tubes (p. 1067), and is nowadays used in car radios. It consists of a core of iron wires, around which is wrapped a coil of about a hundred turns of thick insulated wire, called the primary (Fig. 36.13 (i)). Around the primary is wound the secondary coil, which has many thousands of turns of fine insulated wire. The primary is connected to a battery of accumulators, via a make-and-break M, which works in the same way as the contact-breaker of an electric bell: it switches the current on and off many times a second, thus varying the magnetic flux.

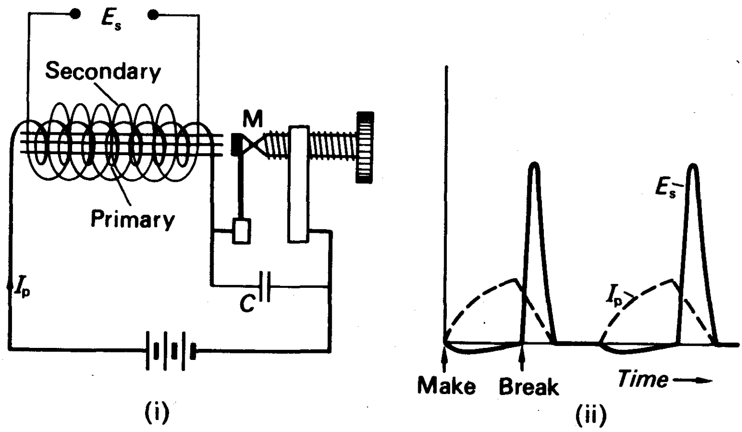


FIG. 36.13. Induction coil.

When the primary current I_p is switched on, the rise of its magnetic field induces an e.m.f. E_s in the secondary. A similar e.m.f., but in the opposite sense, is induced in the secondary when the primary current is switched off, by the collapse of the magnetic field. The secondary e.m.f.s are determined by the number of turns in the secondary coil, and by the rate of change of the magnetic flux through the iron core. Because of the great number of secondary turns, the secondary e.m.f.s may be high and of the order of thousands of volts (Fig. 36.13 (ii)).

In practice, an induction coil such as we have described—consisting simply of primary, secondary, and contact-breaker—would not give high secondary e.m.f.s. For, at the make of the primary current, the current would rise slowly, because of the self-inductance (see p. 924) of the primary winding. The rate of change of flux linked with the secondary would therefore be small, and the secondary e.m.f. low. And at the break of the primary current a spark would pass between the contacts of the make-and-break. The spark would allow primary current to continue to flow, and the primary current would fall slowly. At the instant of break, before the spark began, the primary current would be falling rapidly and the secondary e.m.f. would be high; but the e.m.f. would remain high for only a very short time: as soon as the spark passed the secondary e.m.f. would fall to a value about as low as at make.

Nothing can be done about the low secondary e.m.f. at make. But the secondary e.m.f. at break can be made high, by preventing sparking at the contact-breaker. To prevent sparking, a capacitor, C in Fig. 36.13, is connected across the contacts.

As we shall see on p. 926, the capacitor actually *slows down* the fall of the primary current at the instant of break; but in doing so it prevents the induced e.m.f. in the primary from rising high enough to start a spark. And the rate at which the primary current falls, in charging the capacitor, is greater than the rate at which it would fall if a spark were passing. Thus, with a capacitor, the secondary e.m.f. is less at the instant of break than without one, but it is greater throughout the rest of the fall of the primary current. Consequently the average secondary voltage at break is higher with a capacitor than without; in practice it is much higher. To get the greatest possible secondary voltage, the capacitance of the capacitor is chosen so that it just suppresses sparking at the contacts. The secondary voltage is then a series of almost unidirectional pulses, as shown in Fig. 36.13 (ii).

The iron core of an induction coil is made from a bundle of wires, to minimize eddy-currents (p. 913). If eddy-currents were to flow they would, by Lenz's law, set up a flux opposing the change of primary current. Thus they would reduce the secondary e.m.f.

The Dynamo and Generator

Faraday's discovery of electromagnetic induction was the beginning of electrical engineering. Nearly all the electric current used today is generated by induction, in machines which contain coils moving continuously in a magnetic field. Fig. 36.14 illustrates the principle of such a machine, which is called a dynamo, or generator. A coil DEFG, shown for simplicity as having only one turn, rotates on a shaft, which is not shown, between the poles NS of a horseshoe magnet. The ends of the coil are connected to flat brass rings R, which are supported on the shaft by discs of insulating material, also not shown. Contact with the rings is made by small blocks of carbon B, supported on springs, and shown connected to a lamp L. As the coil rotates, the flux linking it changes, and a current is induced in it which flows, via the carbon blocks, through the lamp.

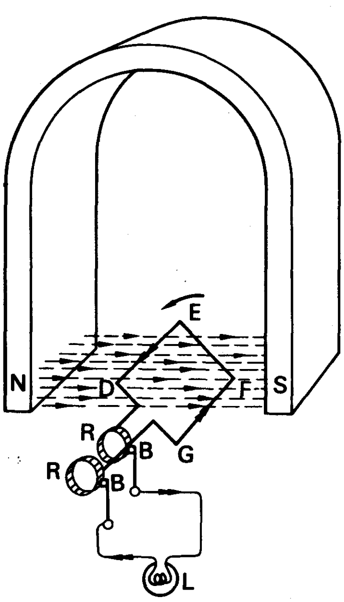


FIG. 36.14. A simple dynamo.

The magnitude (which we study shortly) and the direction of the current are not constant. Thus when the coil is in the position shown, the limb ED is moving downwards through the lines of force, and GF is moving upwards. Half a revolution later, ED and GF will have interchanged their positions, and ED will be moving upwards. Consequently, applying Fleming's right-hand rule, the current round the coil must reverse as ED changes from downward to upward motion. The actual direction of the current at the instant shown on the diagram is indicated by the double arrows, from Fleming's rule. By applying this rule, it can be seen that the current reverses every time the plane of the coil passes the vertical position.

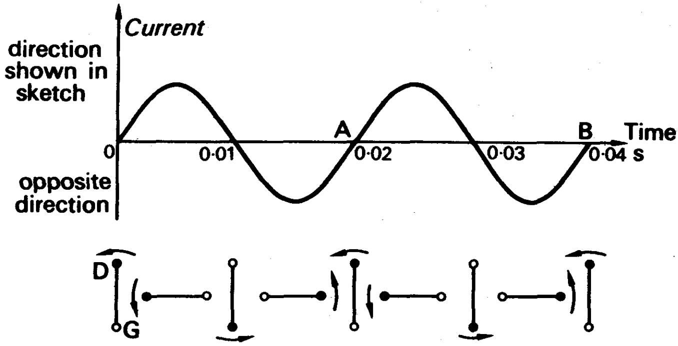


FIG. 36.15. Current generated by dynamo of Fig. 36.14, plotted against time and coil position.

We shall see shortly that the magnitude of the e.m.f. and current varies with time as shown in Fig. 36.15; this diagram also shows the corresponding position of DG. This type of current is called an *alternating current* (*A.C.*). A complete alternation, such as from A to B in the figure, is called a 'cycle'; and the number of cycles which the current goes through in one second is called its 'frequency'. The frequency of the current represented in the figure is that of most domestic supplies in Britain—50 Hz (cycles per second).

E.M.F. in Dynamo

We can now calculate the e.m.f. in the rotating coil. If the coil has an area A, and its normal makes an angle θ with the magnetic field B, as in Fig. 36.16, then the flux through the coil

$$= AB\cos\theta \text{ (see p. 898).}$$

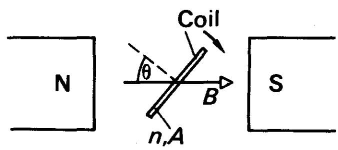


FIG. 36.16. Coil inclined to magnetic field.

The flux linkages with the coil, if it has N turns, are

$$\Phi = NAB\cos\theta.$$

If the coil turns with a steady angular velocity ω or $d\theta/dt$, then the e.m.f. induced in volts in the coil is

$$E = -\frac{d\Phi}{dt}$$

$$= -NAB\frac{d}{dt}(\cos\theta)$$

$$= NAB\sin\theta\frac{d\theta}{dt} \quad . \quad . \quad . \quad . \quad (1)$$

In terms of the number of revolutions per second, f, which the coil makes, we have

$$\frac{d\theta}{dt} = 2\pi f$$

and $$\theta = 2\pi ft,$$

$$\therefore E = 2\pi f NAB \sin 2\pi f t. \qquad (2)$$

Thus the e.m.f. varies sinusoidally with time, like the pressure in a sound-wave, the frequency being f cycles per second.

The maximum (peak) value or amplitude of E occurs when $\sin 2\pi f t$ reaches the value 1. If the maximum value is denoted by E_0, it follows that

$$E_0 = 2\pi f NAB,$$

and $$E = E_0 \sin 2\pi f t. \qquad (3)$$

The e.m.f. E sends an alternating current, of a similar sine equation, through a resistor connected across the coil.

Alternators

Generators of alternating current are often called *alternators*. In all but the smallest, the magnetic field of an alternator is provided by an electromagnet called a field-magnet or *field*, as shown in Fig. 36.17; it has a core of cast steel, and is fed with direct current from a separate d.c.

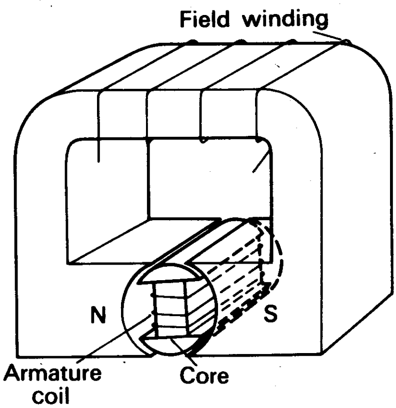


FIG. 36.17. Field magnet and armature.

generator. The rotating coil, called the armature, is wound on an iron core, which is shaped so that it can turn within the pole-pieces of the field-magnet. With the field-magnet, the armature core forms a system which is almost wholly iron, and can be strongly magnetized by a small current through the field winding. The field in which the armature turns is much stronger than if the coil had no iron core, and the e.m.f. is proportionately greater. In the small alternators used for bicycle lighting the armature is stationary, and the field is provided by permanent magnets, which rotate around it. In this way rubbing contacts, for leading the current into and out of the armature, are avoided.

When no current is being drawn from a generator, the horse-power required to turn its armature is merely that needed to overcome friction, since no electrical energy is produced. But when a current is drawn, the horse-power required increases, to provide the electrical power. The current, flowing through the armature winding, causes the magnetic field to set up a couple which opposes the rotation of the armature, and so demands the extra horse-power. The reader should check the truth of this statement by marking the direction of the e.m.f., current, and force on the limbs of the coil in Fig. 36.17.

The Transformer

A transformer is a device for stepping up—or down—an alternating voltage. It has primary and secondary windings, as in an induction coil, but no make-and-break (Fig. 36.18). It has an iron core, which is made from E-shaped laminations, interleaved so that the magnetic flux does not pass through air at all; in this way the greatest flux is obtained

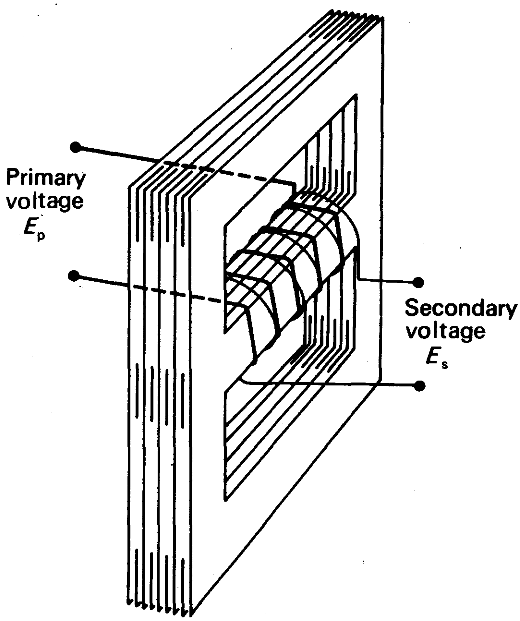


FIG. 36.18. Transformer.

with a given current. When an alternating e.m.f. E_p is impressed on the primary winding, it sends an alternating current through it, which sets up an alternating flux in the core of magnitude BA, where B is the induction and A is the cross-sectional area. This induces an alternating e.m.f. in the secondary E_s. If N_p, N_s are the number of turns in the primary and secondary coils, their linkages with the flux Φ are:

$$\Phi_p = N_p AB \qquad \Phi_s = N_s AB$$

The magnitude of the e.m.f. induced in the secondary is, from the formula on p. 899:

$$E_s = \frac{d\Phi_s}{dt} = N_s A\frac{dB}{dt}$$

The changing flux also induces a back-e.m.f. in the primary, whose magnitude is

$$E_p = \frac{d\Phi_p}{dt} = N_p A\frac{dB}{dt}$$

Because the primary winding has inevitably some resistance, the current flowing through it sets up a voltage drop across the resistance. But in practice this is negligible compared with the back-e.m.f. due to the changing flux. Consequently we may say that the voltage applied to the primary, from the source of current, is used simply in overcoming the back-e.m.f. E_p. Therefore it is equal in magnitude to E_p. (This is analogous to saying, in mechanics, that action and reaction are equal and opposite.) Consequently we have

$$\frac{\textit{e.m.f. induced in secondary}}{\textit{voltage applied to primary}} = \frac{E_s}{E_p} = \frac{N_s}{N_p} \quad . \quad . \quad (1)$$

Thus the transformer steps voltage up or down according to its '*turns-ratio*':

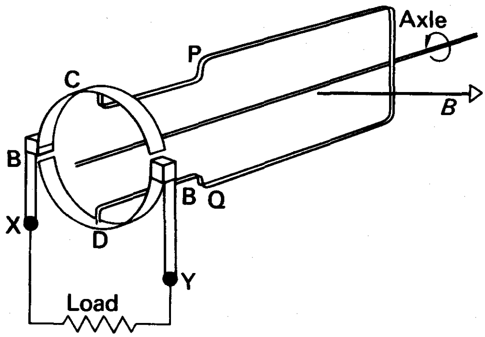
$$\frac{\text{secondary voltage}}{\text{primary voltage}} = \frac{\text{secondary turns}}{\text{primary turns}}.$$

When a load is connected to the secondary winding, a current flows in it. This current flows in such a direction as to reduce the flux in the core. At the instant that the load is connected, therefore, the back-e.m.f. in the primary falls. The primary current then increases. The increase in primary current increases the flux through the core, and continues until the flux is restored to its original value. The back-e.m.f. in the primary is then again equal to the applied voltage, and equilibrium is restored. But now a greater primary current is flowing than before the secondary was loaded. Thus the power drawn from the secondary is drawn, in turn, from the supply to which the primary is connected.

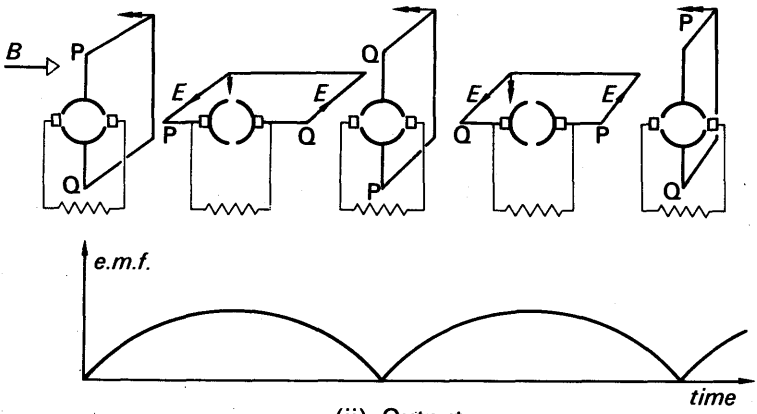
Transformers are used to step up the voltage generated at a power station, from 11000 to 132 000 volts for high-tension transmission (p. 793). After transmission they are used to step it down again to a value safer for distribution (240 volts in houses). Inside a house a transformer may be used to step the voltage down from 240 to 4, for ringing bells. Transformers with several secondaries are used in, for example, radio-receivers, where several different voltages are required.

D.C. Generators

Fig. 36.19 (i) is a diagram of a direct-current generator or dynamo. Its essential difference from an alternator is that the armature winding is connected to a *commutator* instead of slip-rings. The commutator consists of two half-rings of copper C, D, insulated from one another, and turning with the coil. Brushes BB, with carbon tips, press against the commutator and are connected to the external circuit. The commutator



(i) Principle



(ii) Output

FIG. 36.19. D.C. generator.

is oriented so that it reverses the connexions from the coil to the circuit at the instant when the e.m.f. reverses in the coil. Fig. 36.19 (ii) shows several positions of the coil and commutator, and the e.m.f. observed at the terminals XY. This e.m.f. pulsates in magnitude, but it acts always in the same sense round the circuit connected to XY. It is a pulsating direct e.m.f. The average value in this case can be shown to be $2/\pi$ of the maximum e.m.f. E_0, given in equation (3), p. 907.

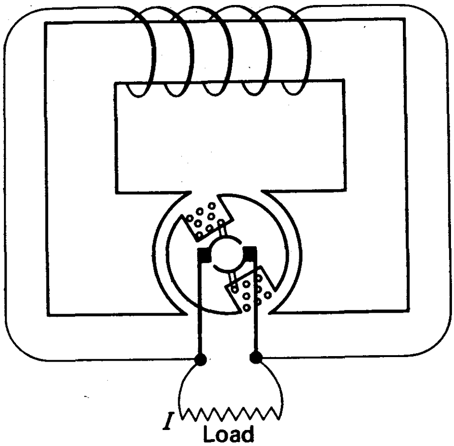


FIG. 36.20. D.C. generator with energized field.

In practice, as in an alternator, the armature coil is wound with insulated wire on a soft iron core, and the field-magnet is energized by a current (Fig. 36.20). This current is provided by the dynamo itself. The steel of the field-magnet has always a small residual magnetism, so that as soon as the armature is turned an e.m.f. is induced in it. This then sends a current through the field winding, which increases the field and the e.m.f.; the e.m.f. rapidly builds up to its working value.

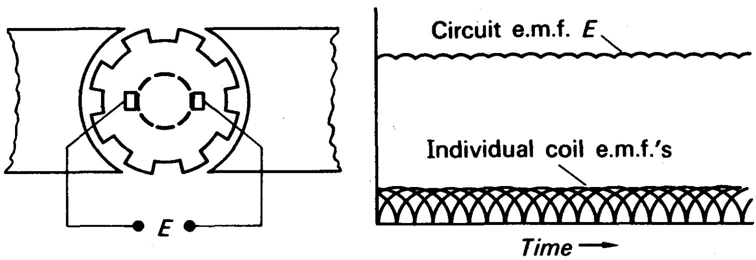


FIG. 36.21. D.C. generator with segment commutator.

Most consumers of direct current wish it to be steady, not pulsating as in Fig. 36.19. A reasonably steady e.m.f. is given by an armature with many coils, inclined to one another, and a commutator with a correspondingly large number of segments. The coils are connected to the commutator in such a way that their e.m.f.s add round the external circuit (Fig. 36.21).

Homopolar Generator

Another type of generator, which gives a very steady e.m.f., is illustrated in Fig. 36.22. It consists of a copper disc which rotates between the poles of a magnet; connexions are made to its axle and circumference. If we assume (as is not true) that the magnetic field B is uniform over the radius XY, then we can calculate the induced e.m.f. E. In one revolution the radius XY sweeps out an area $(\pi(r_1^2 - r_2^2)$, where r_1 and r_2 are the radii of the wheel and the axle. If T is the time for one revolution, then the rate at which XY sweeps out are is $\pi(r_1^2 - r_2^2)/T$.

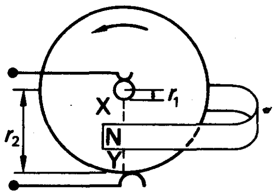


FIG. 36.22.
E.m.f. induced in a disc.

The rate at which it sweeps out flux is therefore

$$\frac{\pi(r_1^2 - r_2^2)}{T}B = \pi(r_1^2 - r_2^2)Bf,$$

where f denotes the revolutions of the wheel per second. Thus

$$E = \pi(r_1^2 - r_2^2)Bf$$
$$= \pi(r_1^2 - r_2^2)Bf$$

Generators of this kind are called *homopolar* because the e.m.f. induced in the moving conductor is always in the same sense. They are sometimes used for electroplating, where only a small voltage is required, but they are not useful for most purposes, because they give too small an e.m.f. The e.m.f. of a commutator dynamo can be made large by having many turns in the coil; but the e.m.f. of a homopolar dynamo is limited to that induced in one radius of the disc.

Applications of Alternating and Direct Currents

Direct currents are less easy to generate than alternating currents, and alternating e.m.f.s are more convenient to step up and to step down, and to distribute over a wide area. The national grid system, which supplies electricity to the whole country, is therefore fed with alternating current. Alternating current is just as suitable for heating as is direct current, because the heating effect of a current is independent of its direction. It is also equally suitable for lighting, because filament lamps depend on the heating effect, and gas-discharge lamps —neon, sodium, mercury—run as well on alternating current as on direct. Small motors, of the size used in vacuum-cleaners and common machine-tools, run satisfactorily on alternating current, but large ones, as a general rule, do not. Direct current is therefore used on most electric railway and tramway systems. These systems either have their own generating stations, or convert alternating current from the grid into direct current. One way of conveying alternating current into direct is to use a valve rectifier, whose principle we shall describe later.

For electro-chemical processes alternating current is useless. The chemical effect of a current reverses with its direction, and if, therefore, we tried to deposit a metal by alternating current, we would merely cause a small amount of the metal to be alternately deposited and dissolved. For electroplating, and for battery charging, alternating current must be rectified.

Eddy-currents

The core of the armature of a dynamo is built up from thin sheets of soft iron insulated from one another by an even thinner film of oxide, as shown in Fig. 36.23 (i). These are called laminations, and the armature is said to be laminated. If the armature were solid, then, since iron is a conductor, currents would be induced in it by its motion across the magnetic field (Fig. 36.23 (ii)). These currents would absorb power by opposing the rotation of the armature, and they would dissipate that power as heat, which would damage the insulation of the winding; but

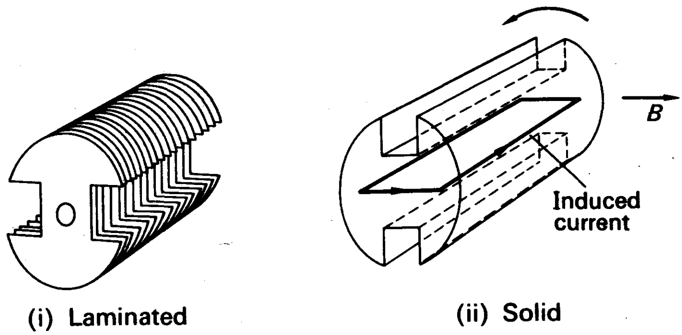


FIG. 36.23. Armature cores.

when the armature is laminated, these currents cannot flow, because the induced e.m.f. acts at right angles to the laminations, and therefore to the insulation between them. The magnetization of the core, however, is not affected, because it acts along the laminations. Thus the eddy-currents are suppressed, while the desired e.m.f.—in the armature coil—is not.

Eddy-currents, by Lenz's law, always tend to oppose the motion of a solid conductor in a magnetic field. The opposition can be shown in many ways. One of the most impressive is to make a chopper with a thick copper blade, and to try to slash it between the poles of a strong electromagnet; then to hold it delicately and allow it to drop between them. The resistance to the motion in the former case can be felt.

Sometimes eddy-currents can be made use of—for example, in damping a galvanometer. When a current is passed through the coil of a galvanometer, it applies a couple to the coil which sets it swinging. If the swings are opposed only by the viscosity of the air, they decay very slowly and are said to be naturally damped (Fig. 36.24). The pointer or light-spot takes a long time to come to its final steady deflection θ. To bring the spot or pointer more rapidly to rest, the damping must be increased. One way of increasing the damping is to wind the coil on a metal former. Then, as the coil swings, the field of the permanent magnet induces eddy-currents in it; and these, by Lenz's law, oppose its motion. They therefore slow down the turning of the coil towards its eventual position, but they also suppress its swings about that position; in the end the coil comes to rest sooner than if it

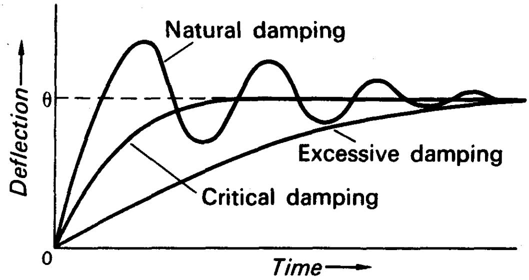


FIG. 36.24. Damping of galvanometer.

were not damped. Galvanometer coils which are wound on insulating formers can be damped by short-circuiting a few of their turns, or by connecting an external shunt across the whole coil. With a shunt the eddy-currents circulate round the coil and shunt, independently of the current to be measured. The smaller the shunt, the greater the eddy-currents and the damping; if the coil is overdamped, as shown in Fig. 36.24, it may take almost as long to come to rest as when it is undamped. The damping which is just sufficient to prevent overshoot is called 'critical' damping.

Electric Motors

If a simple direct-current dynamo, of the kind described on p. 910, is connected to a battery it will run as a motor (Fig. 36.25). Current flows round the armature coil, and the magnetic field exerts a couple on this, as in a moving-coil galvanometer. The commutator reverses the current just as the limbs of the coil are changing from upward to

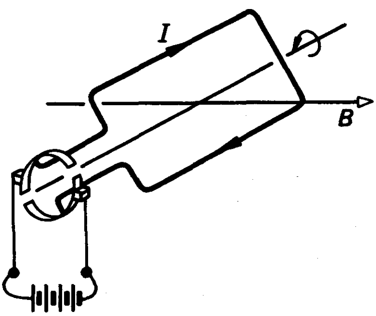


FIG. 36.25. Principle of D.C. motor.

downward movement and vice versa. Thus the couple on the armature is always in the same sense, and the shaft turns continuously. (The reader should verify these statements with the help of Fig. 36.25.)

The armature of a motor is laminated, in the same way and for the same reason, as the armature of a dynamo.

Back-e.m.f. in Motor

When the armature of a motor rotates, an e.m.f. is induced in its windings; by Lenz's law this e.m.f. opposes the current which is making the coil turn. It is therefore called a back-e.m.f. If its magnitude is E, and V is the potential difference applied to the armature by the supply, then the armature current is

$$I_a = \frac{V-E}{R_a}. \qquad (1)$$

Here R_a is the resistance of the armature, which is generally small—of the order of 1 ohm.

The back-e.m.f. E is proportional to the strength of the magnetic field, and the speed of rotation of the armature. When the motor is first switched on, the back-e.m.f. is zero: it rises as the motor speeds up. In a large motor the starting current would be ruinously great; to limit it, a variable resistance is inserted in series with the armature, which is gradually reduced to zero as the motor gains speed.

When a motor is running, the back-e.m.f. in its armature E is not much less than the supply voltage V. For example, a motor running off the mains ($V = 230$ volts) might develop a back-e.m.f. $E = 220$ volts. If the armature had a resistance of 1 ohm, the armature current would then be 10 amp (equation (1)). When the motor was switched on, the armature current would be 230 amp if no starting resistor were used.

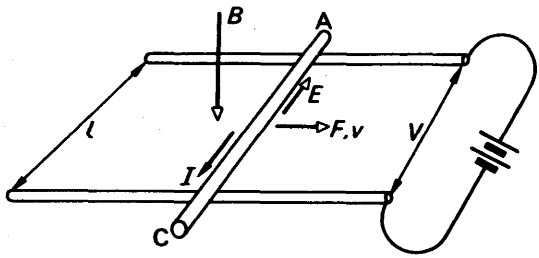


FIG. 36.26. Back-e.m.f. and mechanical power.

Back-e.m.f. and Power

The back-e.m.f. in the armature of a motor represents the mechanical power which it develops. To see that this is so, we use an argument similar to that which we used in finding an expression for the e.m.f. induced in a conductor. We consider a rod AC, able to slide along rails, in a plane at right angles to a magnetic field B (Fig. 36.26). But we now suppose that a current I is maintained in the rod by a battery, which sets up a potential difference V between the rails. The magnetic field then exerts a force F on the rod, given by

$$F = BIl.$$

The force F makes the rod move; if its velocity is v, the mechanical power developed by the force F is

$$P_m = Fv = BIlv \quad . \quad . \quad . \quad . \quad (1)$$

As the rod moves, a back-e.m.f. is induced in it, whose magnitude E is given by the expression for the e.m.f. in a moving rod (p. 901):

$$E = Blv. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Equations (1) and (2) together give

$$P_m = EI. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Thus the mechanical power developed is equal to the product of the back-e.m.f. and the current.

Before returning to consider motors, we may complete the analysis of the action represented in Fig. 36.26. If R is the resistance of the rails and rod, the heat developed in them is I^2R. The power supplied by the battery is IV, and the battery is the only source of power in the whole system. Therefore

$$IV = I^2R + P_m; \quad . \quad . \quad . \quad . \quad (4)$$

the power supplied by the battery goes partly into heat, and partly into useful mechanical power. Also, by equation (3),

$$IV = I^2R + EI, \quad . \quad . \quad . \quad . \quad (5)$$

whence

$$V = IR + E$$

or

$$I = \frac{V - E}{R}.$$

This is equation (1), p. 915, which we previously obtained simply from Ohm's law.

Let us apply this theory to the example which we were considering. We had:

supply voltage, V, = 230 volts;
back-e.m.f., E, = 220 volts;
armature resistance, R_a, = 1 ohm;
armature current, I_a, = 10 amp.

The power dissipated as heat in the armature is $I_a^2R_a = 100 \times 1 = 100$ watts. The power supplied to the armature is $I_aV = 10 \times 230 = 2300$ watts, and the mechanical power is $I_aE = 10 \times 220 = 2200$ watts. Of the power supplied to the armature, the fraction which appears as mechanical power is 2200/2300 = 96 per cent. This is not, however, the efficiency of the motor as a whole, because current is taken by the winding on the field magnet.

The Field Winding

The field winding of a motor may be connected in series or in parallel with the armature. If it is connected in series, it carries the armature current, which is large (Fig. 36.27). The field winding therefore has few turns of thick wire, to keep down its resistance, and so the power wasted in it as heat. The few turns are enough to magnetize the iron, because the current is large. If the field coil is connected in parallel with the armature, as in Fig. 36.28, the motor is said to be 'shunt-wound'. The field winding has many turns of fine wire to keep down

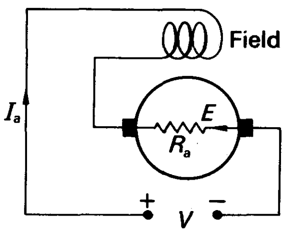


FIG. 36.27. Series-wound motor.

the current which it consumes. In the above example, if the motor is shunt-wound and the field current is 0·5 A, then the power dissipated as heat in the field is $0{\cdot}5 \times 230 = 115$ watts. The power consumption of the motor is therefore $2300 + 115 = 2415$ watts, and its efficiency is

$$\frac{\text{mechanical power developed}}{\text{electrical power consumed}} = \frac{2220}{2415}$$

$$= 92 \text{ per cent.}$$

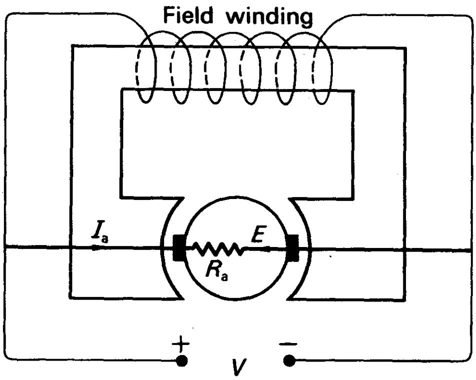


FIG. 36.28. Current and voltages in shunt-wound motor.

The working efficiency of the motor will be a little less than this, because some of the mechanical power will be used in overcoming friction in the bearings.

Shunt Field

Shunt-wound motors are used for driving machine-tools, and in other jobs where a steady speed is required. A shunt motor keeps a nearly steady speed for the following reason. If the load is increased, the speed falls a little; the back-e.m.f. then falls in proportion to the speed, and the current rises, enabling the motor to develop more power to overcome the increased load. In the example, p. 916, if the speed falls by 1 part in 220, the back-e.m.f. falls from 220 to 219 V. The current then rises from $\frac{230-220}{1} = 10$ A to $\frac{230-219}{1} = 11$ A. And the mechanical power increases from $220 \times 10 = 2200$ watts to $219 \times 11 = 2409$ watts ($\simeq 2400$). Thus an increase in load of $\frac{2400-2200}{2200} = 9$ per cent causes a fall in speed of 1 part in 220—less than $\frac{1}{2}$ per cent.

Series Field

Series motors are used where great torque is required in starting—for example, in cranes.

They develop a great starting torque because the armature current flows through the field coil. At the start the armature back-e.m.f. is small, and the current is great—as great as the starting resistance will allow. The field-magnet is therefore very strongly magnetized. The torque on the armature is proportional to the field and to the armature current; since both are great at the start, the torque is very great.

A series motor does not keep such a steady speed as a shunt motor. Just as in a shunt motor, when the load increases the speed falls; and the fall in speed decreases the back-e.m.f., and allows more current to flow. But, as we will see in a moment, the back-e.m.f., in a series motor, does not fall with the speed as sharply as it does in a shunt motor. To meet a given increase in load, the armature current must increase by a definite amount. And therefore the back-e.m.f. must fall by a definite amount. But it falls less with the speed than it does in a shunt motor. Consequently, to meet a given increase in load, the speed of a series motor must fall more than that of a shunt motor.

We now show that the back-e.m.f. in a series motor falls less with the speed than in a shunt one. The argument is best given in steps:

(i) when the speed falls, the back-e.m.f. falls;
(ii) the current through both armature and field winding increases;
(iii) the field becomes stronger;
(iv) the increase in the field tends to *increase* the back-e.m.f., i.e. to offset its initial fall;
(v) thus the very fall of the back-e.m.f., by permitting a greater current and strengthening the field, tends to offset itself;
(vi) therefore the back-e.m.f. falls slowly with the speed—more slowly than in a shunt motor, where the field is constant;
(vii) as we have already seen, this means that the speed must fall further, to meet a given increase in load.

CHARGE AND FLUX LINKAGE

Flux and Charge relation

We have already seen that an electromotive force is induced in a circuit when the magnetic flux linked with it changes. If the circuit is closed, a current flows, and electric charge is carried round the circuit. As we shall now show, there is a simple relationship between the charge and the change of flux.

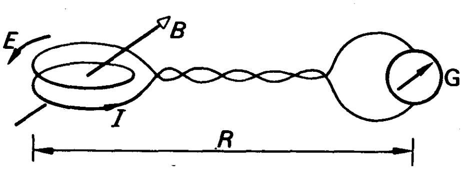


FIG. 36.29. Coil with changing flux.

Consider a closed circuit of total resistance R ohm, which has a total flux linkage Φ with a magnetic field (Fig. 36.29). If the flux linkages start to change,

$$\text{induced e.m.f., } E = -\frac{d\Phi}{dt}.$$

$$\therefore \text{ current, } I = \frac{E}{R} = -\frac{1}{R}\frac{d\Phi}{dt} \qquad . \quad . \quad (1)$$

In general, the flux linkage will not change at a steady rate, and the current will not be constant. But, throughout its change, charge is being carried round the circuit. If a time t seconds is taken to reach a new constant value, the charge carried round the circuit in that time is

$$Q = \int_0^t I dt.$$

From (1),

$$\therefore Q = -\frac{1}{R}\int_0^t \frac{d\Phi}{dt} . dt$$

$$= -\frac{1}{R}\int_{\Phi_0}^{\Phi_t} d\Phi,$$

where Φ_0 is the number of linkages at $t = 0$, and Φ_t is the number of linkages at time t. Thus

$$Q = -\frac{\Phi_t - \Phi_0}{R} = \frac{\Phi_0 - \Phi_t}{R}.$$

The quantity $\Phi_0 - \Phi_t$ is positive if the linkages Φ have decreased, and negative if they have increased. But as a rule we are interested only in the magnitude of the charge, and we may write

$$Q = \frac{\textit{change of flux linkage}}{R}. \qquad . \quad . \quad (2)$$

Equation (2) shows that the charge circulated is proportional to the change of flux-linkages, and independent of the time.

Ballistic Galvanometer

It can be seen from the last section that the charge which flows round a given circuit is directly proportional to the change of flux linkage. If the charge flowing is measured by a *ballistic galvanometer* G, as shown in Fig. 36.29, then we have a measure of the change in flux linkage, Φ.

Ballistics is the study of the motion of a body, such as a projectile, which is set off by a blow, and then allowed to move freely. By freely, we mean without friction. A ballistic galvanometer is one used to measure an electrical blow, or impulse: for example, the charge Q which circulates when a capacitor is discharged through it. A galvanometer which is intended to be used ballistically has a heavier coil than one which is not; and it has as little damping as possible—an insulating former, no short-circuited turns, no shunt. The mass of its coil makes it swing slowly; in the example above, for instance, the capacitor has discharged, and the charge has finished circulating, while the galvanometer coil is just beginning to turn. The galvanometer coil continues to turn, however; and as it does so it twists the suspension. The coil stops turning when its kinetic energy, which it gained from the forces set up by the current, has been converted into potential energy of the suspending fibre. The coil then swings back, as the suspension untwists itself, and it continues to swing back and forth for some time. Eventually it comes to rest, but only because of the damping due to the viscosity of the air, and to the internal friction of the fibre. Theory shows that, if the damping is negligible, *the first deflection of the galvanometer is proportional to the quantity of electricity, Q, that passed through its coil, as it began to move.* This first deflection, θ, is often called the 'throw' of the galvanometer; we have, then,

$$Q = k\theta, \qquad . \quad . \quad . \quad . \quad (1)$$

where k is a constant of the galvanometer.

Equation (1) is true only if all the energy given to the coil is spent in twisting the suspension. If an appreciable amount of energy is used to overcome damping—i.e. dissipated as heat by eddy currents—then the galvanometer is not ballistic, and θ is not proportional to Q.

To calibrate the ballistic galvanometer, a capacitor of known capacitance, e.g. 2 μF, is charged by a battery of known e.m.f., e.g. 50 volt, and then discharged through the instrument. See p. 768. Suppose the deflection is 200 divisions. The charge $Q = CV = 100$ microcoulomb, and thus the galvanometer sensitivity is 2 divisions per microcoulomb.

Measurement of Induction

Fig. 36.30 illustrates the principle of measuring the induction B in the field between the poles of a powerful magnet. A small coil, called a *search coil*, with a known area and number of turns, is connected to a ballistic galvanometer G. It is positioned at right angles to the field to be measured, as shown, so that the flux enters the coil face normally.

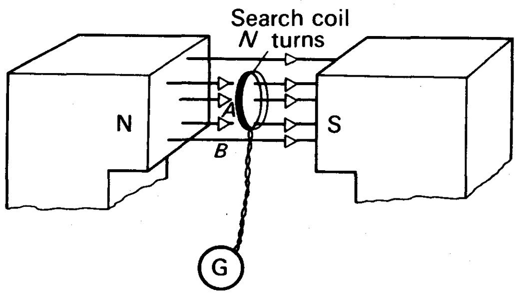


FIG. 36.20. Induction by ballistic galvanometer.

The coil is then pulled completely out of the field by moving it smartly downwards, for example, and the throw θ produced in the galvanometer is observed. The charge q which passes round the circuit is proportional to θ, from above.

Suppose B is the field-strength in Wb m^{-2} or tesla (T), A is the area of the coil in m^2 and N is the number of turns. Then

$$\text{change of flux-linkages} = NAB$$

$$\therefore \text{quantity, } Q, \text{ through galvanometer} = \frac{NAB}{R},$$

where R is the *total* resistance of the galvanometer and search coil. But

$$Q = c\theta,$$

where c is the quantity per unit deflection of the ballistic galvanometer.

$$\therefore \frac{NAB}{R} = c\theta$$

$$\therefore B = \frac{Rc\theta}{NA} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The constant c is found by discharging a capacitor through the galvanometer (see p. 768). If C is the capacitance in farads, V the p.d. in volts of the battery originally charging it, and α the deflection of the galvanometer, then $c = CV/\alpha$ coulomb per unit deflection.

The Earth Inductor

As another example of the use of a search coil and ballistic galvanometer, we describe a method that has been used for measuring the angle of dip of the earth's magnetic field (p. 944). The earth's field is so nearly uniform that the search coil may be large, usually about 30 cm square; but the field is also so weak that even a large coil must have many turns—of the order of 100. The coil, which is called an earth inductor, is pivoted in a wooden frame, and this is fitted with stops so that the coil can be turned rapidly through 180° (Fig. 36.31). To find

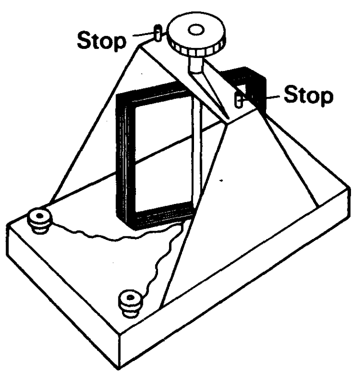


FIG. 36.31. Earth inductor.

the angle of dip, we connect the coil to a ballistic galvanometer, and set it with its plane horizontal, as shown at (i) in Fig. 36.32. The flux linking the coil at N turns is then

$$\Phi = NAB_V,$$

where A is the area of the coil, and B_V is the vertical component of the earth's field. If we were to turn the coil through 90°, the flux would fall to zero; and if we were to turn it through a further 90°, the flux linkage would become NAB_V once more, but it would thread the coil in the opposite direction. Therefore we turn the coil through 180°, and change the flux linkage by $2NAB_V$; at the same time we observe the throw, θ, of the galvanometer.

If R is the total resistance of galvanometer and search coil, the circulated charge is, by equation (2) on p. 919,

$$Q = \frac{\Phi}{R} = \frac{2NAB_V}{R}.$$

But

$$Q = c\theta,$$

where c is the constant of the galvanometer. Therefore

$$\frac{2NAB_V}{R} = c\theta. \quad . \quad . \quad . \quad (1)$$

We now set the frame of the earth inductor so that the axis of the coil is vertical, and so that, when the coil is held by one of the stops, its plane lies East–West. See Fig. 36.32 (ii). The flux threading the coil is now NAB_H, where B_H is the horizontal component of the earth's field. Therefore, when we turn the coil through 180°, the throw θ' of the galvanometer is given by

$$\frac{2NAB_H}{R} = c\theta' \quad . \quad (2)$$

Now the angle of dip, δ, is given by

$$\tan\delta = \frac{B_V}{B_H}.$$

Therefore, from equations (1) and (2),

$$\tan\delta = \frac{\theta}{\theta'}.$$

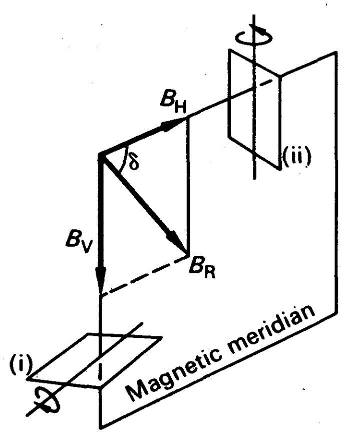


FIG. 36.32. Measurement of dip.

Self-induction

The phenomenon which we call self-induction was discovered by the American, Joseph Henry, in 1832. He was led to it by a theoretical argument, starting from the phenomena of induced e.m.f., which he had discovered at about the same time as Faraday.

When a current flows through a coil, it sets up a magnetic field. And that field threads the coil which produces it. Fig. 36.33 (i). If the current through the coil is changed—by means of a variable resistance, for example—the flux linked with the turn of the coil changes. An e.m.f. is therefore induced in the coil. By Lenz's law the direction of the induced e.m.f. will be such as to oppose the change of curent; the e.m.f. will be against the current if it is increasing, with it if it is decreasing (Fig. 36.33 (ii)).

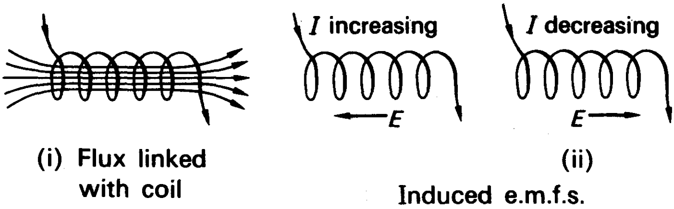


FIG. 36.33. Self-induction.

Back-E.M.F.

When an e.m.f. is induced in a circuit by a change in the current through that circuit, the process of induction is called self-induction. The e.m.f. induced is called a back-e.m.f. Self-induction opposes the growth of current in a coil, and so makes it gradual. This effect can be demonstrated by connecting an iron-cored coil of many turns in series with an ammeter and a few accumulators (Fig. 36.34 (i)). (The ammeter

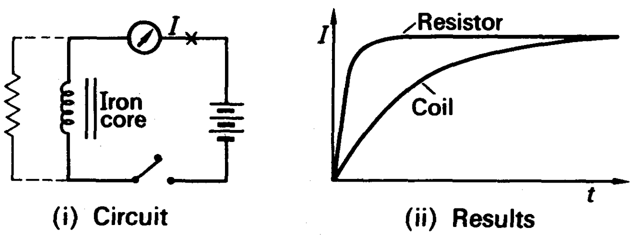


FIG. 36.34. Self-induction experiment.

should be of the 'short-period' type and critically damped.) When the current is switched on, the pointer of the ammeter moves slowly over to its final position. If the coil is now replaced by a rheostat of the same *resistance*, the pointer moves much more swiftly to the same reading (Fig. 36.34 (ii)).

Just as self-induction opposes the rise of an electric current when it is switched on, so also it opposes the decay of the current when it is switched off. When the circuit is broken, the current starts to fall very rapidly, and a correspondingly great e.m.f. is induced, which tends to maintain the current. This e.m.f. is often great enough to break down the insulation of the air between the switch contacts, and produce a spark. To do so, the e.m.f. must be about 350 volts or more, because air will not break down—not over any gaps, narrow or wide—when the voltage is less than that value. The e.m.f. at break may be much greater than the e.m.f. of the supply which maintained the current: a spark can easily be obtained, for example, by breaking a circuit consisting of an iron-cored coil and an accumulator.

Non-inductive Coils

In bridge circuits, such as are used for resistance measurements, self-induction is a nuisance. When the galvanometer key of a bridge is closed, the currents in the arms of the bridge are redistributed, unless the bridge happens to be balanced. While the currents are being redistributed they are changing, and self-induction delays the reaching of a new equilibrium. Thus the galvanometer deflection at the instant of closing the key, does not correspond to the steady state which the bridge will eventually reach. It may therefore be misleading. To minimize their self-inductance, the coils of bridges and resistance boxes are wound so as to set up extremely small magnetic fields: as shown in Fig. 36.35, the wire is doubled-back on itself before being coiled up. Every part of the coil is then traversed by the same current travelling in opposite directions, and its magnetic field is negligible. Such a coil is said to be non-inductive.

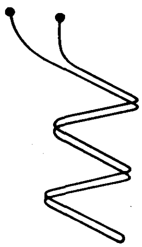


FIG. 36.35. Non-inductive winding.

When describing the use of a bridge, we said that the battery key should be pressed before the galvanometer key. Doing so gives time for the currents in the arms of the bridge to become steady before the galvanometer key is pressed. It therefore minimizes any possible effects of self-induction.

Self-inductance

To discuss the effects of self-induction we must define the property of a coil which gives rise to them. This property is called the self-inductance of the coil, and is defined as follows:

$$\textit{self-inductance} = \frac{\textit{back-e.m.f. induced in coil by a changing current}}{\textit{rate of change of current through coil}}$$

Self-inductance is denoted by the symbol L; we may therefore write its definition as

$$L = \frac{E_{back}}{dI/dt}.$$

or

$$E_{back} = L\frac{dI}{dt}. \qquad (1)$$

Equation (1) is the simplest form in which to remember the definition.

The unit of self-inductance is the henry (H). It is defined by making each term in equation (1) equal to unity; thus *a coil has a self-inductance of* 1 *henry if the back-e.m.f. in it is* 1 *volt, when the current through it is changing at the rate of* 1 *ampere per second.* Equation (1) then becomes:

$$E_{back}\,(\text{volts}) = L\,(\text{henrys}) \times \frac{dI}{dt}\,(\text{ampere/second}).$$

The iron-cored coils used for smoothing the rectified supply current to a radio receiver (p. 1011) are usually very large and have an inductance of about 30 henrys.

L for Coil

Since the induced e.m.f. $E = d\Phi/dt = L = dI/dt$, numerically, it follows by integration from a limit of zero that

$$\Phi = LI.$$

Thus $L = \Phi/I$. Hence the self-inductance may be defined as the *flux linkage per unit current*. When Φ is in webers and I in amperes, then L is in henrys. Thus if a current of 2A produces a flux linkage of 4 Wb in a coil, the inductance $L = 4\ \text{Wb}/2\text{A} = 2\text{H}$.

We shall see later that when a long coil of N turns and length l carries a current I, (i) the magnetizing field $H = NI/l$ and (ii) this produces a flux-density or induction B inside the coil given by $B = \mu H$, where μ is the permeability of the material inside the coil (p. 941). Hence

$$\text{flux linkage } \Phi = NAB = NA\mu H = \frac{\mu N^2 AI}{l}$$

$$\therefore L = \frac{\Phi}{I} = \frac{\mu N^2 A}{l} \quad . \quad . \quad . \quad (1)$$

This formula may be used to find the approximate value of the inductance of a coil. L is in henrys when A is in metre^2, l in metre and μ is in henry metre^{-1}.

Energy Stored; E.M.F. at Break

The spark which passes when the current in a coil is interrupted liberates energy in the form of heat and light. This energy has been stored in the magnetic field of the coil, just as the energy of a charged capacitor is stored in the electrostatic field between its plates (p. 779). When the current in the coil is first switched on, the back-e.m.f. opposes the rise of current; the current flows against the back-e.m.f. and therefore does work against it (p. 795). When the current becomes steady, there is no back-e.m.f. and no more work done against it. The total work done in bringing the current to its final value is stored in the magnetic field of the coil. It is liberated when the current collapses; for then the induced e.m.f. tends to maintain the current, and to do external work of some kind.

To calculate the energy stored in a coil, we suppose that the current through it is rising at a rate dI/dt ampere per second. Then, if L is its self-inductance in henrys, the back-e.m.f. across it is given by

$$E = L\frac{dI}{dt} \text{ volt.}$$

If the value of the current, at the instant concerned, is I amperes, then the rate at which work is being done against the back-e.m.f. is

$$P = EI = LI\frac{dI}{dt} \text{ watt.}$$

The total work done in bringing the current from zero to a steady value I_0 is therefore

$$W = \int P dt = \int_0^{I_0} LI\frac{dI}{dt}dt = \int_0^{I_0} LI dt$$
$$= \tfrac{1}{2}LI_0^2 \text{ joule.}$$

This is the energy stored in the coil.

To calculate the e.m.f. induced at break is, in general, a complicated business. But we can easily do it for one important practical circuit. To prevent sparking

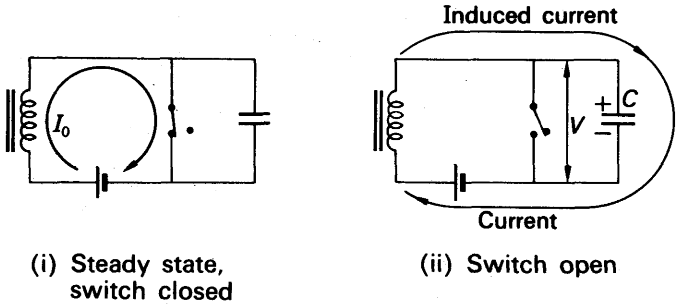


FIG. 36.36. Prevention of sparking by capacitor.

at the contacts of the switch in an inductive circuit, a capacitor is often connected across them (Fig. 36.36 (i)). When the circuit is broken, the collapsing flux through the coil tends to maintain the current; but now the current can continue to flow for a brief time; it can flow by charging the capacitor (Fig. 36.36 (ii)). Consequently the current does not decay as rapidly as it would without the capacitor, and the back-e.m.f. never rises as high. If the capacitance of the capacitor is great enough, the potential difference across it (and therefore across the switch) never rises high enough to cause a spark.

To find the value to which the potential difference does rise, we assume that all the energy originally stored in the magnetic field of the coil is now stored in the electrostatic field of the capacitor.

If C is the capacitance of the capacitor in farad, and V_0 the final value of potential difference across it in volt, then the energy stored in it is $\frac{1}{2}CV_0^2$ joule (p. 779). Equating this to the original value of the energy stored in the coil, we have

$$\tfrac{1}{2}CV_0^2 = \tfrac{1}{2}LI_0^2.$$

Let us suppse that a current of 1 ampere is to be broken, without sparking, in a circuit of self-inductance 1 henry. To prevent sparking, the potential difference across the capacitor must not rise above 350 volt. The least capacitance that must be connected across the switch is therefore given by

$$\tfrac{1}{2}C \times 350^2 = \tfrac{1}{2} \times 1 \times 1^2.$$

Hence $$C = \frac{1}{350^2} = 8 \times 10^{-6} \text{ farad} = 8\ \mu\text{F}.$$

A paper capacitor of capacitance 8 μF, and able to withstand 350 volts, would therefore be required.

Mutual Induction

We have already seen than an e.m.f. may be induced in one circuit by a changing current in another (Fig. 36.1, p. 895). The phenomenon is often called mutual induction, and the pair of circuits which show it are said to have mutual inductance. The mutual inductance, M, between two circuits is defined by the equation:

$$\left.\begin{array}{l}\text{e.m.f. induced in B, by}\\ \text{changing current in A}\end{array}\right\} = M \times \left\{\begin{array}{l}\text{rate of change of}\\ \text{current in A.}\end{array}\right.$$

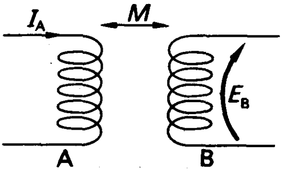


FIG. 36.37. Mutual induction.

See Fig. 36.37. In symbols,

$$E_B = M\frac{dI_A}{dt}.$$

Mutual inductance is truly mutual; it is the same from B to A as A to B. Its unit is the same as that of self-inductance, the henry.

EXERCISES 36

1. State Lenz's law of electromagnetic induction and describe, with explanation, an experiment which illustrates its truth.

Describe the structure of a transformer suitable for supplying 12 volts from 240-volt mains and explain its action. Indicate the energy losses which occur in the transformer and explain how they are reduced to a minimum.

When the primary of a transformer is connected to the a.c. mains the current in it (*a*) is very small if the secondary circuit is open, but (*b*) increases when the secondary circuit is closed. Explain these facts. (*L.*)

2. Describe, with the aid of a large labelled diagram, the structure of a simple form of a.c. generator.

Explain (*a*) how it could be modified to produce direct current;
(*b*) the features that enable is to produce a high e.m.f. (compared with a cell);
(*c*) the features that minimize the heat wasted. (*L.*)

3. Define *electromotive force* and state the *laws of electromagnetic induction.* Using the definition and the laws, derive an expression for the e.m.f. induced in a conductor moving in a magnetic field.

When a wheel with metal spokes 120 cm long is rotated in a magnetic field of flux density $0{\cdot}5 \times 10^{-4}$ Wb m^{-2} normal to the plane of the wheel, an e.m.f. of 10^{-2} volt is induced between the rim and the axle. Find the rate of rotation of the wheel. (*L.*)

4. Describe the differences in structure and action between a non-ballistic and a ballistic moving-coil galvanometer.

A corrected deflection of 24 scale divisions of a ballistic galvanometer is obtained *either* by charging a capacitor of 3 μF capacitance to a potential difference of 2 volt and discharging it through the galvanometer, or by connecting the ballistic galvanometer in series with a flat circular coil of 80 turns each of diameter 1 cm, the combined resistance of coil and galvanometer being 2000 ohms, and quickly thrusting the coil into a strong magnetic field so that the plane of the coil is perpendicular to the direction of the field. State the sensitivity of the galvanometer and calculate the strength of the magnetic field. (The strength of the earth's magnetic field may be neglected.) (*L.*)

5. Explain what is meant by *self inductance* and define the practical unit in which it is measured.

Describe and explain an experiment which demonstrates the phenomenon of self-induction. (*N.*)

6. State the laws relating to (*a*) the direction, (*b*) the magnitude of an electromagnetically induced electromotive force, and describe very briefly an experiment illustrating each.

Deduce the relation between the quantity of electricity flowing through a circuit and the flux change producing it.

A flat coil of 150 turns, each of area 300 cm^2 and of total resistance 50 ohms, is connected to a circuit whose resistance is 40 ohms. Starting with its plane horizontal, the coil is rotated quickly through a half-turn about a diametral axis pointing along the magnetic meridian. If the quantity of electricity which then flows round the circuit is 4 microcoulombs, find the intensity of the vertical component of the earth's magnetic field. (*N.*)

7. What are eddy currents?

Describe and explain an experiment in which eddy currents are produced.

Describe one useful application of eddy currents. (*N.*)

8. Define *self inductance* and *mutual inductance.*

Explain the differences in structure and action between a ballistic and an aperiodic galvanometer.

A ballistic galvanometer of resistance 15 ohms and sensitivity 5 divisions per microcoulomb is connected in series with a resistance of 100 ohms and a secondary coil of 500 turns and of resistance 50 ohms. This coil is wound round the middle of a long solenoid of radius 3 cm having 10 turns cm^{-1} and carrying a current of 0·6 A. Assuming no damping, calculate the deflection produced in the galvanometer when the current in the solenoid is switched off. (*L.*)

9. Describe experiments (one in each case) involving the use of a moving-coil ballistic galvanometer to (*a*) compare two capacitances of approximately the same magnitude, (*b*) compare the magnetic induction (flux density) between the poles of one electromagnet with that between the poles of another. In each case justify the method used to calculate the result.

Explain two special features of a galvanometer suitable for use in these experiments. (*N.*)

10. State Lenz's law and describe how you would demonstrate it using a solenoid with two separate superimposed windings with clearly visible turns, a cell with marked polarity, and a centre-zero galvanometer. Illustrate your answer with diagrams.

A metal aircraft with a wing span of 40 m flies with a ground speed of 1000 km h^{-1} in a direction due east at constant altitude in a region of the northern hemisphere where the horizontal component of the earth's magnetic field is $1·6 \times 10^{-5}$ T (Wb m^{-2}) and the angle of dip is 71·6°. Find the potential difference in volts that exists between the wing tips and state, with reasons, which tip is at the higher potential. (*N.*)

11. State the laws of electromagnetic induction and describe briefly experiments to show their validity.

A coil *A* passes a current of 1·25 A when a steady potential difference of 5 V is maintained across it, and an r.m.s. current of 1 A when it has across it a sinusoidal potential difference of 5 V r.m.s. at a frequency of 60 Hz (cycles per second). Explain why the current is less in the second case, and calculate the resistance and the inductance of the coil.

The same coil A, which has 100 turns, has a second coil B with 500 turns wound on it so that all the magnetic flux produced by A is linked by B. Find the r.m.s. value of the e.m.f. that appears across the open-circuit ends of B when a sinusoidal alternating current of 1 A r.m.s. at a frequency of 50 Hz is passed through A. Why is the ratio of this e.m.f. to the r.m.s. potential difference across A not the same as the ratio of the number of turns in B and A, i.e. 5:1?

Explain why the insertion of an iron core into the coils would decrease the current in A and increase the e.m.f. across B, if the alternating potential difference across A were kept unchanged; the effects of hysteresis and eddy currents in the iron may be neglected. (*O. & C.*)

12. A choke of large self inductance and small resistance, a battery and a switch are connected in series. Sketch and explain a graph illustrating how the current varies with time after the switch is closed. If the self inductance and resistance of the coil are 10 henrys and 5 ohms respectively and the battery has an e.m.f. of 20 volts and negligible resistance, what are the greatest values after the switch is closed of (*a*) the current, (*b*) the rate of change of current? (*N.*)

13. State the laws of electromagnetic induction, and describe an experiment by which one of them can be verified.

A piece of wire 8 cm long, of resistance 0·020 ohm and mass 22 mg, is bent to form a closed square $ABCD$. It is mounted so as to turn without friction about a horizontal axis through AB; a uniform horizontal magnetic field of flux-density 0·50 Wb m^{-2} is applied at right angles to this axis. The side CD is raised until the plane of the square is horizontal and then released. Calculate approximately the time taken for the plane of the square to become vertical. You can assume that, during the falling, the couple due to gravity is equal and opposite to that due to electromagnetic forces. (*C.*)

14. State the laws of electromagnetic induction. Hence derive an expression for the time variation of the electromotive force induced in a single turn of wire rotating about an axis in its plane, the axis being perpendicular to a uniform magnetic field. Explain the action of a simple alternating current generator.

What modification of this generator is required to produce a direct current? Indicate by a sketch how the e.m.f. across the output terminals of a single coil would vary with time in the case of (*a*) an a.c. and (*b*) a d.c. generator. How may a more uniform output e.m.f. be obtained in the latter case?

A rectangular coil of wire having 100 turns, of dimensions 30 cm × 30 cm, is rotated at a constant speed of 600 r.p.m. in a magnetic field of 0·1 Wb m^{-2}, the axis of rotation being in the plane of the coil and perpendicular to the field. Calculate the induced e.m.f. (*L.*)

15. Describe with the aid of diagrams (*a*) a transformer and (*b*) an induction coil. Explain the action of each.

Draw diagrams to show, in a general way, how the voltage output from each of these appliances varies with the time. (*N.*)

16. What are eddy-currents? Give two examples of the practical use of such currents.

A metal disc of diameter 20 cm rotates at a constant speed of 600 r.p.m. about an axis through its centre and perpendicular to its plane in a uniform magnetic field of 5×10^{-3} Wb m^{-2} established parallel to the axis of rotation. Calculate the e.m.f. in volts between the centre and rim of the disc. Show clearly on a diagram the direction of rotation of the disc and the direction of the magnetic field and of the e.m.f. induced. (*L.*)

17. How would you show that a change in the number of lines of magnetic force, however produced, threading through a circuit produces an induced e.m.f.?

A magnet is suspended by a thin wire so that its axis is horizontal and its centre is above the centre of a circular copper disc, mounted horizontally. Explain what happens to the magnet when the disc is rotated.

What would be the effect of replacing the disc by one of identical dimensions but made of a substance of high resistivity? (*L.*)

18. State Lenz's law and describe fully a method by which you could verify this law experimentally.

A horizontal metal disc of radius 10 cm is rotated about a central vertical axis at a region where the value of the earth's magnetic flux density is $5{\cdot}3 \times 10^{-5}$ T (Wb m^{-2}) and the angle of dip is 70°. A sensitive galvanometer of resistance 150 ohms is connected between the centre of the disc and a brush pressing on the rim. Assuming the resistance of the disc to be negligible, what will be the current through the galvanometer when the disc is rotated at 1500 rev. min.^{-1}? If the system is frictionless, calculate the power required to maintain the motion. (*C.*)

19. State the laws of electromagnetic induction and describe experiments you would perform to illustrate the factors which determine the magnitude of the induced current set up in a closed circuit.

A simple electric motor has an armature of 0·1 ohm resistance. When the motor is running on a 50-volt supply the current is found to be 5 amp. Explain this and show what bearing it has on the method of starting large motors. (*L.*)

20. State the laws relating to the electromotive force induced in a conductor which is moving in a magnetic field.

Describe the mode of action of a simple dynamo.

Find in volts the e.m.f. induced in a straight conductor of length 20 cm, on the armature of a dynamo and 10 cm from the axis when the conductor is moving in a uniform radial field of $0{\cdot}5\ \text{Wb m}^{-2}$ and the armature is rotating at 1000 r.p.m. (*L.*)