chapter thirty-three

Applications of Ohm's Law

MEASUREMENTS. NETWORKS

In this chapter we shall apply Ohm's law to circuits more complicated than those of Chapter 32. We shall see that some special types of circuits can be used to make electrical measurements more accurately than with pointer instruments.

RESISTORS AND THEIR ARRANGEMENTS

Series Resistors

The resistors of an electric circuit may be arranged in series, so that the charges carrying the current flow through each in turn (Fig. 33.1); or they may be arranged in parallel, so that the flow of charge divides between them (Fig. 33.2), p. 808.

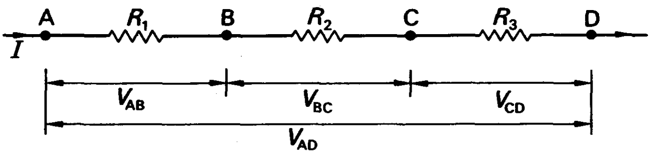


FIG. 33.1. Resistances in series.

Fig. 33.1 shows three passive resistors in series, carrying a current I. If V_{AD} is the potential difference across the whole system, the electrical energy supplied to the system per second is IV_{AD}. This is equal to the electrical energy dissipated per second in all the resistors; therefore

$$IV_{AD} = IV_{AB}+IV_{BC}+IV_{CD},$$

whence

$$V_{AD} = V_{AB}+V_{BC}+V_{CD}, \quad . \quad . \quad . \quad . \quad (1)$$

The individual potential differences are given by Ohm's law:

$$\left.\begin{aligned} V_{AB} &= IR_1 \\ V_{BC} &= IR_2 \\ V_{CD} &= IR_3 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (2)$$

and

Hence, by equation (1),

$$\begin{aligned} V_{AD} &= IR_1+IR_2+IR_3 \\ &= I(R_1+R_2+R_3). \quad . \quad . \quad . \quad (3) \end{aligned}$$

And the effective resistance of the system is

$$R = \frac{V_{AD}}{I} = R_1 + R_2 + R_3. \quad \quad (4)$$

The physical facts are:

(i) *Current same through all resistors.*
(ii) *Total potential difference = sum of individual potential difference* (equation (1)).
(iii) *Individual potential differences directly proportional to individual resistances* (equation (2)).
(iv) *Total resistance greater than greatest individual resistance* (equation (4)).
(v) *Total resistance = sum of individual resistances.*

Resistors in Parallel

Fig. 33.2 shows three passive resistors connected in parallel, between the points A, B. A current I enters the system at A and leaves at B,

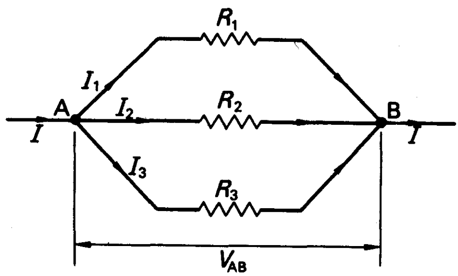


FIG. 33.2. Resistances in parallel.

setting up a potential difference V_{AB} between those points. The current branches into I_1, I_2, I_3, through the three elements, and

$$I = I_1 + I_2 + I_3. \quad \quad (5)$$

Now $$I_1 = \frac{V_{AB}}{R_1}, \quad I_2 = \frac{V_{AB}}{R_2}, \quad I_3 = \frac{V_{AB}}{R_3}.$$

$$\therefore I = V_{AB}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

$$\therefore \frac{I}{V_{AB}} = \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \quad \quad (6)$$

where R is the effective resistance (V_{AB}/I) of the system.

The physical facts about resistors in parallel may be summarized as follows:

(i) *Potential difference same across each resistor.*
(ii) *Total current = sum of individual currents* (equation (5)).

(iii) *Individual currents inversely proportional to individual resistances.*
(iv) *Effective resistance less than least individual resistance* (equation (6)).

Resistance Boxes

In many electrical measurements variable known resistances are required; they are called resistance boxes. As shown in Fig. 33.3 (i) ten coils, each of resistance 1 ohm, for example, are connected in series. A rotary switch with eleven contacts enables any number of these coils to be connected between the terminals AA′. A resistance box contains several sets of coils and switches, the first giving resistances 0–10 ohms in steps of 1 ohm, the next 0–100 ohms in steps of 10 ohms, and so on. These are called decade boxes.

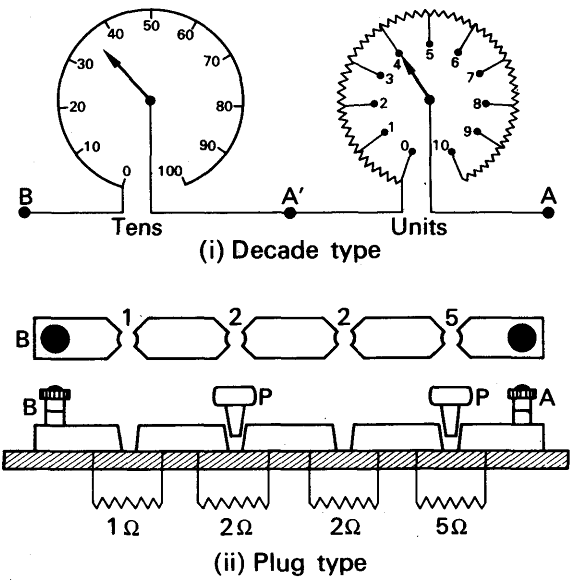


FIG. 33.3. Resistance boxes.

The switches used in a decade box are of very high quality; their contact resistances are negligible compared with the resistances of the coils which they select. Switches of this kind have been developed only in the last twenty years or so: in older boxes no switches are used. Instead, the resistances are varied by means of plugs. As shown in Fig. 33.3 (ii), the resistance coils are joined across gaps in a thick brass bar, and the gaps are formed into tapered sockets to receive short-circuiting plugs P. The resistance between the terminals A and B in Fig. 33.3 (ii) is the sum of the unplugged resistances between them—3 ohms in this example.

The coils of a resistance box are wound in a particular way, which we shall describe and explain later (p. 924). They are not intended to carry large currents, and must not be allowed to dissipate more than one watt. Therefore, since $P = I^2R$, the greatest safe current for a 1-ohm coil is 1 amp, and for a 10-ohm coil about 0·3 amp. If the one-watt limit is exceeded, the insulation will be damaged, or the wire burnt out.

The Potential Divider

Two resistance boxes in series are often used in the laboratory to provide a known fraction of a given potential difference—for example, of one which is too large to measure easily. Fig. 33.4 (i) shows the

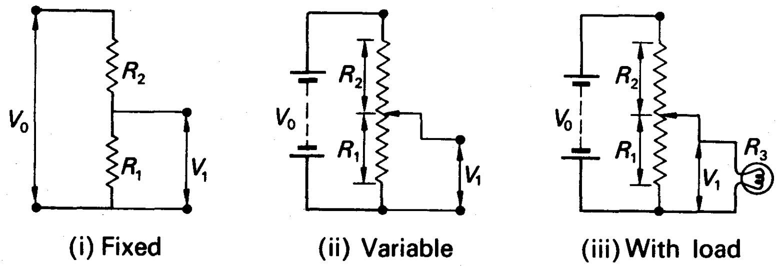


FIG. 33.4. Potential divider.

arrangement, which is called a resistance 'potential divider'. The current flowing, I, is given by

$$I = \frac{V_0}{R_1+R_2},$$

$$\therefore V_1 = IR_1 = \frac{R_1}{R_1+R_2}V_0. \qquad (7)$$

A resistor with a sliding contact can similarly be used, as shown in Fig. 33.4 (ii), to provide a continuously variable potential difference, from zero to the full supply value V_0. This is a convenient way of controlling the voltage applied to a load, such as a lamp (Fig. 33.4 (iii)). The resistance of the load, R_3, however, acts in parallel with the resistance R_1; equation (7) is therefore no longer true, and the voltage V_1 must be measured with a voltmeter. It can be calculated, as in the following example, if R_3 is known; but if the load is a lamp its resistance varies greatly with the current through it, because its temperature varies.

EXAMPLE

A load of 2000 ohms is connected, via a potential divider of resistance 4000 ohms, to a 10-volt supply (Fig. 33.5). What is the potential difference across the load when the slider is (*a*) one-quarter, (*b*) half-way up the divider?

Since $\frac{1}{R} = \frac{1}{2000} + \frac{1}{1000}$,

(*a*) $R_{BC} = \frac{2000 \times 1000}{2000+1000} = \frac{2000}{3}$ ohms.

$\therefore R_{AC} = R_{AB} + R_{BC} = 3000 + \frac{2000}{3}$ ohms,

$\therefore V_{BC} = \frac{R_{BC}}{R_{AC}} V_C$

$= \frac{2000/3}{11000/3} \times 10 = \frac{2}{11} \times 10$

$= 1{\cdot}8$ volts.

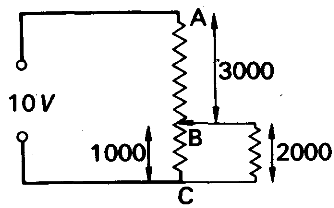


FIG. 33.5. A loaded potential divider.

If the load were removed, V_{BC} would be 2·5 volts.

(*b*) It is left for the reader to show similarly that $V_{BC} = 3{\cdot}3$ volts. Without the load it would be 5 volts.

MEASURING INSTRUMENTS

Conversion of a Milliammeter into a Voltmeter

Ohm's law enables us to use a milliammeter as a voltmeter. Let us suppose that we have a moving-coil instrument which requires 5 milliamperes for full-scale deflection (f.s.d.). And let us suppose that the resistance of its coil, *r*, is 20 ohms (Fig. 33.6). Then, when it is fully deflected, the potential difference across it is

$V = rI$

$= 20 \times 5 \times 10^{-3} = 100 \times 10^{-3}$ volt

$= 0{\cdot}1$ volt.

Since the coil obeys Ohm's law, the current through it is proportional to the potential difference across it; and since the deflection of the pointer is proportional to the current it is therefore also proportional to the potential difference. Thus the instrument can be used as a voltmeter, giving full-scale deflection for a potential difference of 0·1 volt, or 100 millivolts. Its scale could be engraved as shown at the top of Fig. 33.6.

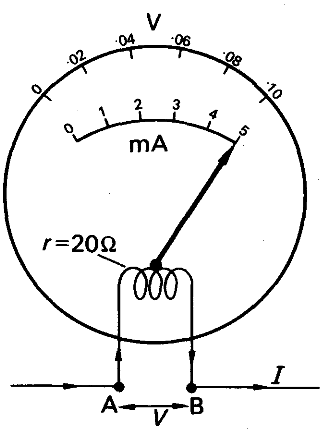


FIG. 33.6. P.D. across moving-coil meter.

The potential differences to be measured in the laboratory are usually greater than 100 millivolts, however. To measure such a potential difference, we insert a resistor *R* in series with the coil, as shown in Fig. 33.7. If we wish to measure up to 10 volts we must choose the resistance *R* so that, when 10 volts are applied between the

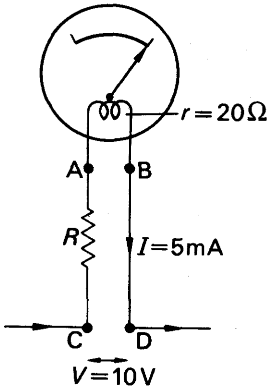


FIG. 33.7. Single-range voltmeter.

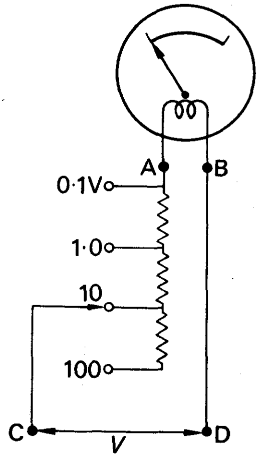


FIG. 33.8. Multi-range voltmeter.

terminals CD, then a current of 5 milliamperes flows through the moving coil. By Ohm's law

$$V = (R+r)I,$$

$$\therefore 10 = (R+20)\times 5\times 10^{-3}$$

or

$$R+20 = \frac{10}{5\times 10^{-3}} = 2\times 10^{3} = 2000 \text{ ohms.}$$

$$\therefore R = 2000-20$$

$$= 1980 \text{ ohms.} \qquad (8)$$

The resistance R is called a *multiplier*. Many voltmeters contain a series of multipliers of different resistances, which can be chosen by a switch or plug-and-socket arrangement (Fig. 33.8).

Conversion of a Milliammeter into an Ammeter

Moving-coil instruments give full-scale deflection for currents smaller than those generally encountered in the laboratory. If we wish to measure a current of the order of an ampere or more we connect a low resistance S, called a shunt, across the terminals of a moving-coil meter (Fig. 33.9). The shunt diverts most of the current to be measured, I, away from the coil—hence its name. Let us suppose that, as before, the coil of the meter has a resistance r of 20 ohms and is fully deflected by a current, I_C, of 5 milliamperes. And let us suppose that we wish to shunt it so that it gives f.s.d. for 5 amperes to be measured. Then the current through the shunt is

$$\begin{aligned} I_S &= I - I_C \\ &= 5 - 0{\cdot}005 \\ &= 4{\cdot}995 \text{ amp.} \end{aligned}$$

The potential difference across the shunt is the same as that across the coil, which is

$$V = rI_C = 20 \times 0{\cdot}005 = 0{\cdot}1 \text{ volt.}$$

The resistance of the shunt must therefore be

$$S = \frac{V}{I_S} = \frac{0{\cdot}1}{4{\cdot}995} = 0{\cdot}02002 \text{ ohms.} \qquad (9)$$

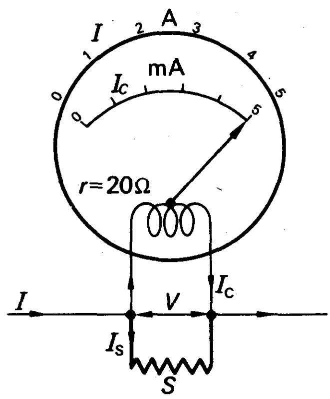


FIG. 33.9. Conversion of milliammeter to ammeter.

The ratio of the current measured to the current through the coil is

$$\frac{I}{I_C} = \frac{5}{5 \times 10^{-3}} = 1000.$$

This ratio is the same whatever the current I, because it depends only on the resistances S and r; the reader may easily show that its value is $(S+r)/S$. The deflection of the coil is therefore proportional to the measured current, as indicated in the figure, and the shunt is said to have a 'power' of 1000 when used with this instrument.

The resistance of shunts and multipliers are always given with four-figure accuracy. The moving-coil instrument itself has an error of the order of 1 per cent.; a similar error in the shunt or multiplier would therefore double the error in the instrument as a whole. On the other hand, there is nothing to be gained by making the error in the shunt less than about 0·1 per cent., because at that value it is swamped by the error of the moving system.

Multimeters

A *multimeter* instrument is one which is adapted for measuring both current and voltage. It has a shunt R as shown, and a series of voltage multipliers R' (Fig. 33.10). The shunt is connected permanently across the coil, and the resistances in R' are adjusted to give the desired full-scale voltages with the shunt in position. A switch or plug enables the various full-scale values of current or voltage to be chosen, but the user does the mental arithmetic. The instrument shown in the figure is reading 1·7 volts; if it were on the 10-volt range, it would be reading 6·4.

The terminals of a meter, multimeter or otherwise, are usually marked + and −; the pointer is deflected to the right when current passes through the meter from + to −.

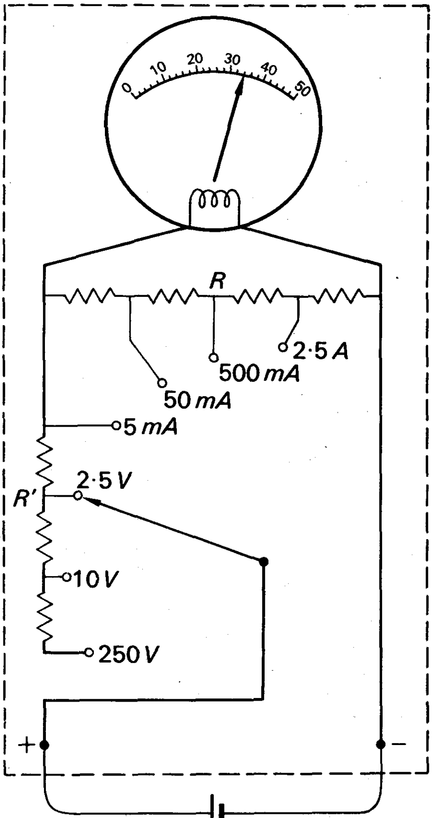


FIG. 33.10. A multimeter.

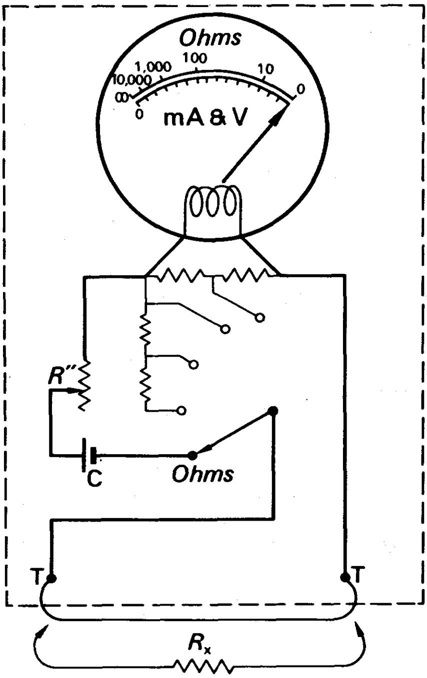


FIG. 33.11. Measurement of resistance with multimeter.

The multimeters are generally arranged to measure resistance as well as current and voltage. An extra position on the switch, marked 'R' or 'ohms', puts a dry cell C and a variable resistor R'' in series with the moving coil (Fig. 33.11). Before the instrument is used to measure a resistance, its terminals TT are short-circuited, and R'' is adjusted until the pointer is fully deflected. As shown in the figure, it is then opposite the zero on the ohms scale. The short-circuit is next removed, and the unknown resistance R_x is connected across the terminals. The current falls, and the pointer moves to the left, indicating on the ohms scale the value of R_x. The ohms scale is calibrated by the makers with known resistances.

Use of Voltmeter and Ammeter

A moving-coil voltmeter is a current-operated instrument. It can be used to measure potential differences only because the current which it draws is proportional to the potential difference applied to it, from Ohm's law. Since its action depends on Ohm's law, a moving-coil voltmeter cannot be used in any experiment to demonstrate that law; that is why, when describing such an experiment on p. 789, we specified measuring instruments whose readings are not dependent on Ohm's law.

Having once established Ohm's law, however, we can use moving-coil voltmeters freely; they are both more sensitive and more accurate than other forms of voltmeters. The current which they take does, however, sometimes complicate their use. To see how it may do so, let us suppose that we wish to measure a resistance R of about 100 ohms. As shown in Fig. 33.12, we connect it in series with a cell, a milliammeter, and a

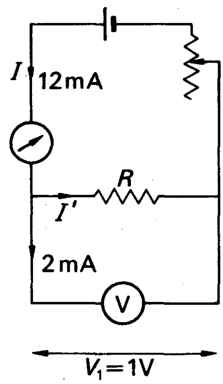


FIG. 33.12

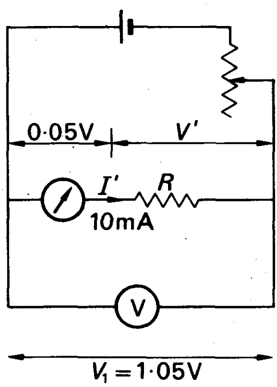


FIG. 33.13.

Use of ammeter and voltmeter.

variable resistance; across it we place the voltmeter. We adjust the current until the voltmeter reads, say, $V_1 = 1$ volt; let us suppose that the milliammeter then reads $I = 12$ mA. The value of the resistance then appears to be

$$R = \frac{V_1}{I} = \frac{1}{12 \times 10^{-3}} = \frac{10^3}{12}$$

$$= 83 \text{ ohms (approx.).}$$

But the milliammeter reading includes the current drawn by the voltmeter. If that is 2 mA, then the current through the resistor, I', is only 10 mA and its resistance is actually

$$R = \frac{V_1}{I'} = \frac{1}{10 \times 10^{-3}} = \frac{1}{10^{-2}}$$

$$= 100 \text{ ohms.}$$

The current drawn by the voltmeter has made the resistance appear 17 per cent. lower than its true value.

In an attempt to avoid this error, we might connect the voltmeter as shown in Fig. 33.13: across both the resistor and the milliammeter. But its reading would then include the potential difference across the milliammeter. Let us suppose that this is 0·05 volt when the current through the milliammeter is 10 mA. Then the potential difference V' across the resistor would be 1 volt, and the voltmeter would read 1·05 volt. The resistance would appear to be

$$R = \frac{1{\cdot}05}{10 \times 10^{-3}} = \frac{1{\cdot}05}{10^{-2}}$$

$$= 105 \text{ ohms.}$$

Thus the voltage drop across the milliammeter would make the resistance appear 5 per cent. higher than its true value.

Errors of this kind are negligible only when the voltmeter current is much less than the current through the resistor, or when the voltage drop across the ammeter is much less than the potential difference across the resistor. If we were measuring a resistance of about 1 ohm, for example, the current I' in Fig. 33.12 would be 1 amp, and I would be 1·002 amp. The error in measuring R would then be only 0·2 per cent—less than the intrinsic error of the meter. But the circuit of Fig. 33.13 would give the same error as before. It could do so because, as we saw when considering shunts, the shunt across the milliammeter would have been chosen to make the voltage drop still 0·05 volt. Thus V_1 would still be 1·05 volt when V' was 1 volt, and the error would be 5 per cent as before.

In low-resistance circuits, therefore, the voltmeter should be connected as in Fig. 33.12, so that its reading does not include the voltage drop across the ammeter. But in high-resistance circuits the voltmeter should be connected as in Fig. 33.13, so that the ammeter does not carry its current.

If a moving-coil voltmeter is connected across a cell, it will not read its true e.m.f., because the current which it draws will set up a voltage drop across the internal resistance of the cell. The drop will be negligible only if the resistance of the voltmeter is very high compared with the internal resistance. E.m.f.s are thus compared by a potentiometer method, discussed shortly.

Figure of Merit of a Voltmeter

If a milliammeter of 1 mA f.s.d. (full scale deflection) is converted into a voltmeter, then if it is to have 1 volt f.s.d. its total resistance—coil plus multiplier—must be 1000 ohms. (One volt across its terminals will send through it a current of 1/1000 amp = 1 mA.) If it is to have 10 volts f.s.d., then its total resistance must be 10000 ohms; for 20 volts f.s.d., 20000 ohms, and so on. It will have a resistance of 1000 ohms for every volt of its full-scale deflection. Such a meter is said to have a *figure of merit* of 1000 ohms per volt. Similarly, a voltmeter which takes 10 mA, or 1/100 amp, for full-scale deflection has a figure of merit of 100 ohms per volt. The greater the figure of merit of a voltmeter, expressed in this way, the less will it disturb any circuit to which it is connected, and the less error will its current cause in any measurements made with it. On the other hand, the greater the figure of merit, the more delicate the moving system of the meter and the greater its intrinsic error. First-grade, and particularly 'sub-standard', meters therefore have medium or low figures of merit: from 500 to 66·7 ohms per volt.

When a voltmeter of low figure of merit is being used, it may be necessary to allow for the current which it draws. The allowance is made in the way indicated on p. 815, where the use of a voltmeter and ammeter together was discussed.

THE POTENTIOMETER

Pointer instruments are useless for very accurate measurements: the best of them have an intrinsic error of abour 1 per cent of full scale. Where greater accuracy than this is required, elaborate measuring circuits are used.

One of the most versatile of these, due to Poggendorf, is the potentiometer. It consists of a uniform wire, AB in Fig. 33.13(i), about a metre long; through it an accumulator X maintains a steady current I. Since the wire is uniform, its resistance per centimetre, R, is constant; the voltage drop across 1 cm of the wire, RI, is therefore also constant.

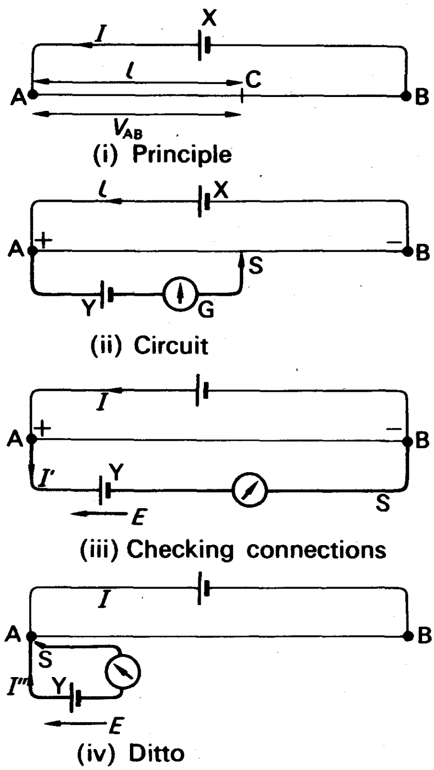


FIG. 33.14. The potentiometer.

The potential difference between the end A of the wire, and any point C upon it, is thus proportional to the length of wire l between A and C:

$$V_{AC} \propto l. \qquad . \quad . \quad . \quad . \quad . \quad (10)$$

Comparison of E.M.F.s

To illustrate the use of the potentiometer, let us suppose that we take a cell, Y in Fig. 33.14(ii), and join its positive terminal to the point A (to which the positive terminal of X is also joined). We connect the negative terminal of Y, via a sensitive galvanometer, to a slider S, which we can press on to any point in the wire. Let us suppose that the cell Y has an e.m.f. E, which is less than the potential difference V_{AB} across the whole of the wire. Then if we press the slider on B, a current I' will flow through Y in opposition to its e.m.f. (Fig. 33.14(iii)). This current will deflect the galvanometer G—let us say to the right. If we now press the slider on A, the cell Y will be connected straight across the galvanometer, and will deliver a current I'' in the direction of its e.m.f. (Fig. 33.14(iv)). The galvanometer will therefore show a deflection to the left. If the deflections at A and B are not opposite, then either the e.m.f. of Y is greater than the potential difference across the whole wire, or we have connected the circuit wrongly. The commonest mistake in connecting up is not joining both positives to A.

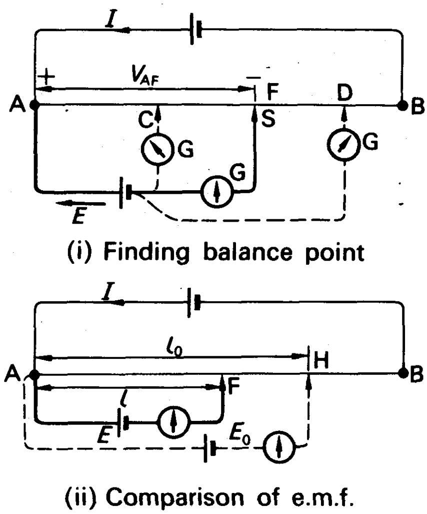


FIG. 33.15. Use of potentiometer.

Now let us suppose that we place the slider on to the wire at a point a few centimetres from A, then at a point a few centimetres farther on, and so forth. (We do not run the slider continuously along the wire, because the scraping would destroy the uniformity.) When the slider is at a point C near A (Fig. 33.15(i)) the potential difference V_{AC} is less than the e.m.f. E of Y; current therefore flows through G in the direction of E, and G may deflect to the left. When the slider is at D near B, V_{AD} is greater than E, current flows through G in opposition to E, and G deflects to the right. By trial and error (but no scraping of the slider) we can find a point F such that, when the slider is pressed upon it, the galvanometer shows no deflection. The potential difference V_{AF} is then equal to the e.m.f. E; no current flows through the galvanometer because E and V_{AF} act in opposite directions in the galvanometer circuit (Fig. 33.15(i)). Because no current flows, the resistance of the galvanometer, and the internal resistance of the cell, cause no voltage drop; the full e.m.f. E therefore appears, between the points, A and S, and is balanced by V_{AF}:

$$E = V_{AF}.$$

If we now take another cell of e.m.f. E_0, and balance it in the same way, at a point H (Fig. 33.15(ii)), then

$$E_0 = V_{AH}.$$

Therefore $$\frac{E}{E_0} = \frac{V_{AF}}{V_{AH}}.$$

The potential differences V_{AF}, V_{AH} are proportional to the lengths l, l_0 from A to F, and from A to H, respectively. Therefore

$$\frac{E}{E_0} = \frac{l}{l_0}. \qquad (11)$$

Accuracy

When the potentiometer is used to compare the e.m.f.s of cells, no errors are introduced by the internal resistances, because no current flows at the balance-points.

The potentiometer is more accurate than an electrometer instrument, which, like a moving-coil voltmeter, has an intrinsic error of about 1 per cent of full-scale. The accuracy of a potentiometer is limited by the non-uniformity of the slide-wire, the uncertainty of the balance-point, and the error in measuring the length l of wire from the balance-point to the end A. With even crude apparatus, the balance-point can be located to within about 0·5 mm; if the length l is 50 cm, or 500 mm, then the error in locating the balance-point is 1:1000. If the wire has been carefully treated, its non-uniformity may introduce an error of about the same magnitude. The overall error is then about ten times less than that of a pointer instrument. A refined potentiometer has a still smaller error.

The precision with which the balance-point of a potentiometer can be found depends on the sensitivity of the galvanometer—the smallness of the current which will give a just-discernible deflection. A moving-coil galvanometer must be protected by a series resistance R of several thousand ohms, which is shorted out when the balance is nearly reached (Fig. 33.16). A series resistance is preferable to a shunt, because it re-

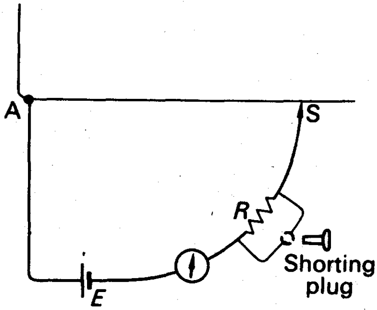


FIG. 33.16. Use of protective resistance with galvanometer.

duces the current drawn from the cell under test, when the potentiometer is unbalanced. The process of seeking the balance-point then causes less change in the chemical condition of the cell, and therefore in its e.m.f.

It is important to realize that the accuracy of a potentiometer does not depend on the accuracy of the galvanometer, but only on its sensitivity. The galvanometer is used not to measure a current but merely to show one when the potentiometer is off balance. It is said to be used as a null-indicator, and the potentiometer method of measurement, like the bridge methods which we shall describe shortly, is called a null method.

The current through the potentiometer wire must be steady—it must not change appreciably between the finding of one balance-point and the next. The accumulator which provides it should therefore be neither freshly charged nor nearly run-down; when an accumulator is in either of those conditions its e.m.f. falls with time. Errors in potentiometer measurements may be caused by non-uniformity of the wire, and by the resistance of its connexion to the terminal at A. This resistance is added to the resistance of the length l of the wire between A and the balance-point, and if it is appreciable it makes equation (11) invalid. Both these sources of error are eliminated in the Rayleigh potentiometer, which we shall describe later (p. 825).

Uses of the Potentiometer. E.M.F. and Internal Resistance

All the uses of the potentiometer depend on the fact that it can measure potential difference accurately, and without drawing current from the circuit under test.

If one of the cells in Fig. 33.15 (ii) has a known e.m.f., say E_0 then the e.m.f. of the other, E, is given by equation:

$$\frac{E}{E_0} = \frac{l}{l_0}. \qquad (12)$$

A cell of known e.m.f. is called a standard cell. The e.m.f.s of standard cells are determined absolutely—that is to say, without reference to the e.m.f.s of any other cell—by methods which depend, in principle, on the definition of e.m.f. (power/current, p. 797). Standard cells are described on p. 865, along with the precautions which must be taken in their use. For simple experiments a Daniell cell (p. 860), whose e.m.f. is about 1·1 volt, may be used as a standard.

Equation (12) is true only if the current I through the potentiometer wire has remained constant. The easiest way to check that it has done so is to balance the standard cell against the wire before and after balancing the unknown cell. If the lengths to the balance-point are equal—within the limits of experimental error— then the current I may be taken as constant. A check of this kind should be made in each of the experiments to be described.

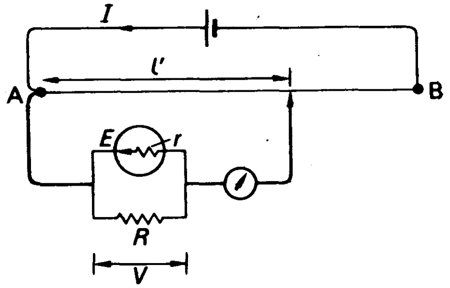


FIG. 33.17. Measurement of internal resistance.

The internal resistance of a cell, r, can be found with a potentiometer by balancing first its e.m.f., E, and the its terminal potential difference, V, when a known resistance R is connected across it (Fig. 33.17). Ohm's law for the complete circuit gives

$$\frac{V}{E}=\frac{R}{R+r}. \qquad (13)$$

But

$$\frac{V}{E}=\frac{l'}{l}, \qquad (14)$$

where l and l' are the lengths of potentiometer wire required to balance E and V. From equations (13) and (14), r can be found from

$$r=\left(\frac{l}{l'}-1\right)R.$$

Calibration of Voltmeter

Fig. 33.18 shows how a potentiometer can be used to calibrate a voltmeter. A standard cell is first used to find the p.d. per cm or volts

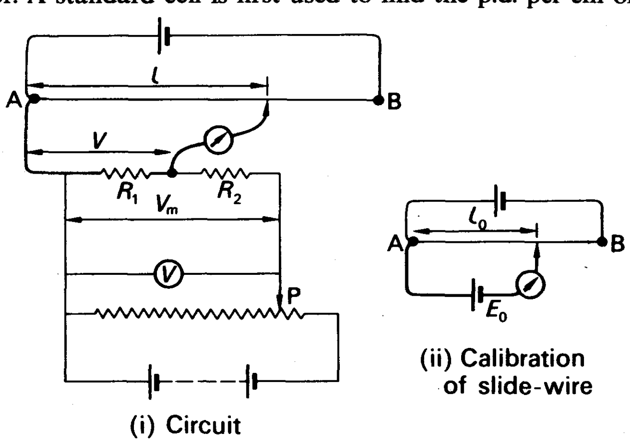


FIG. 33.18. Calibration of voltmeter with potentiometer.

per cm of the wire (Fig. 33.18(ii)): if its e.m.f. E_0 is balanced by a length l_0, then

$$\text{volts per cm} = \frac{E_0}{l_0}. \qquad (15)$$

Different voltages V_m are now applied to the voltmeter by the adjustable potential divider P (Fig. 33.18(i)). The fixed potential divider, comprising R_1R_2, gives a known fraction V of each value of V_m, which is then balanced on the potentiometer:

$$\frac{V}{V_m} = \frac{R_1}{R_1+R_2}. \qquad (16)$$

(The resistances R_1 and R_2 are high—of the order of 1000 to 10000 ohms, so that the voltage adjustment by P is fairly uniform. Their ratio is chosen so that the greatest value of V is measurable on the potentiometer—about 1·5 volts.) If l is the lengths of potentiometer wire which balances a given value of V, then

$$V = l \times (\text{volt/cm of wire})$$
$$= l\frac{E_0}{l_0}.$$

From each value of V, the value of V_m is calculated by equation (16). If the voltmeter reading is V_{obs}, then the correction to be added to it is $V_m - V_{obs}$. This is plotted against V_{obs}, as in Fig. 33.19.

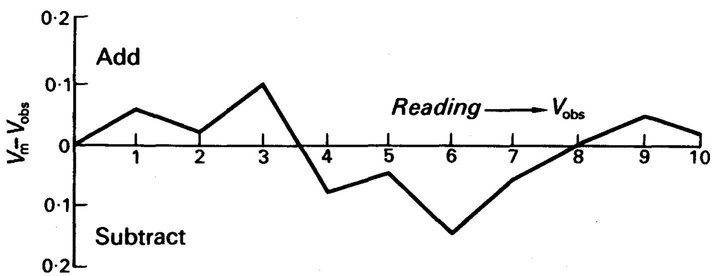


FIG. 33.19. Correction curve of voltmeter.

Measurement of Current

A current can be measured on a potentiometer by means of the potential difference which it sets up across a known resistance, R in Fig. 33.20(i). The resistance is low, being chosen so that the potential difference across it is of the order of 0·1 volt. (A higher value is not chosen, because the voltage drop across the resistor disturbs the circuit in which it is inserted.) Fig. 33.20(ii) shows in detail the kind of resistor used, which is often called a standard shunt. It consists of a broad strip of alloy, such as manganin, whose resistance varies very little with temperature (p. 837). The current is led in and out as the terminals i, i. The terminals v, v are connected to fine wires soldered to points PP on the strip; they are called the potential terminals. The marked value R of the resistance is the value between the points PP; it is adjusted by making hack-saw cuts into the edges of the strip.

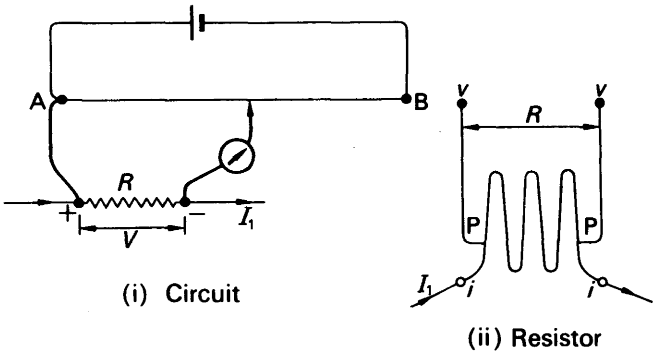


FIG. 33.20. Measurement of current with potentiometer.

As shown in Fig. 33.20(i), the current to be measured, I_1, is passed through the shunt, and the potential difference between its potential terminals, V, is balanced on the potentiometer wire. If l is the length of wire to the balance-point, then

$$\frac{V}{E_0} = \frac{l}{l_0}, \qquad (17)$$

where E_0 and l_0 refer to a standard cell as before. Equation (17) enables the current to be found in terms of E_0, l_0, l, and R, since

$$V = I_1R.$$

The resistance of the wires connecting the potential terminals to the points PP, and to the potentiometer circuit, do not affect the result, because at the balance-point the current through them is zero.

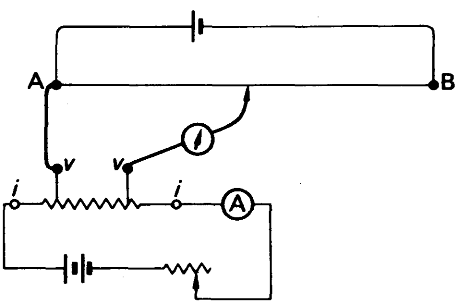


FIG. 33.21. Calibration of ammeter with potentiometer.

This method of measuring a current can be used to calibrate an ammeter, A. The circuit is shown in Fig. 33.21; its principle and use should explain themselves. The results are treated in the same way as in the calibration of a voltmeter.

Comparison of Resistances

A potentiometer can be used to compare resistances, by comparing the potential differences across them when they are carrying the same current I_1 (Fig. 33.22). This method is particularly useful for very low resistances, because, as we have just seen, the resistances of the connecting wires do not affect the result of the experiment. It can, however, be used for higher resistances if desired. With low resistances the ammeter A′ and rheostat P are necessary to adjust the current to a value which will neither exhaust the accumulator Y, nor overheat the resistors. No standard cell is required. The potential difference across the first resistor, $V_1 = R_1I_1$, is balanced against a length l_1 of the potentiometer wire, as shown by the full lines in the figure. Both potential terminals

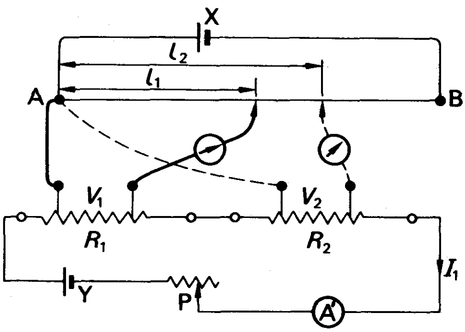


FIG. 33.22.
Comparison of resistances
with potentiometer.

of R_1 are then disconnected from the potentiometer, and those of R_2 are connected in their place. If l_2 is the length to the new balance-point, then

$$\frac{l_1}{l_2} = \frac{V_1}{V_2} = \frac{R_1I_1}{R_2I_1} = \frac{R_1}{R_2}.$$

This result is true only if the current I_1 is constant; as well as the potentiometer current. The accumulator Y, as well as X, must therefore be in good condition. To check the constancy of the current I_1, the ammeter A′ is not accurate enough. The reliability of the experiment as a whole can be checked by balancing the potential V_1 a second time, after V_2. If the new value of l_1 differs from the original, then at least one of the accumulators is running down and must be replaced.

The Rayleigh Potentiometer

Fig. 33.23 shows a potentiometer devised by Lord Rayleigh (1842–1919), which is free from errors due to non-uniformity of the wire and to contact resistance at the end A (p. 820). It consists of two plug-type resistance boxes, R_1, R_2, joined in series. (These boxes may well be the R-sections of two similar Post Office boxes (p. 834). At the start of a measurement all the plugs of R_1 are inserted, and all of R_2 taken out. Then R_1 is zero, and the main current I sets up no potential difference across it; but when the key K is pressed, the unknown e.m.f. E deflects the galvanometer. R_1 is now increased by, say, 100 ohms, and R_2 is

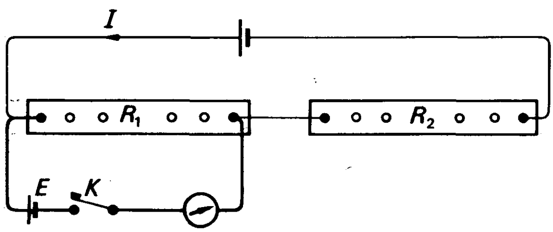


FIG. 33.23. Rayleigh potentiometer.

decreased by the same amount. In this way R_1+R_2 is kept constant, and the current I does not change. But there is now a potential difference across R_1, which opposes E. Plugs are taken out of R_1 and put into R_2, so as to keep R_1+R_2 constant, until the galvanometer shows no deflection when K is pressed. If R_1' is the value of R_1 at this point, then

$$E = R_1'I.$$

The procedure is now repeated with a standard cell of e.m.f. E_0, in place of E. Since R_1+R_2 has been kept constant, the current I is the same as before; hence, if R_1'' is the new value of R_1 at balance,

$$E_0 = R_1''I.$$

Consequently,

$$\frac{E}{E_0} = \frac{R_1'I}{R_1''I} = \frac{R_1'}{R_1''}.$$

Measurement of Thermal E.M.F.

The e.m.f.s of thermojunctions (p. 803) are small—of the order of a millivolt. If we attempted to measure such an e.m.f. on a simple potentiometer we should find the balance-point very near one end of the wire, so that the end-error would be serious. The Rayleigh potentiometer, although it is free from end-errors, is not suitable for measuring small e.m.f.s; if, in Fig. 33.23, $R_1+R_2 = 10000\ \Omega$, and the e.m.f.

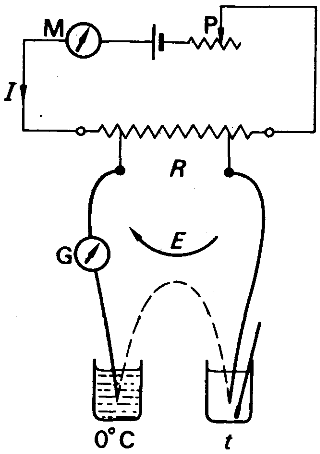


FIG. 33.24. Measurement of thermal e.m.f.

of the accumulator = 2 volts, then $I = 2/10000 = 2\times10^{-4}$ amp = 0·2 mA. To balance a thermal e.m.f. of 2 mV, R_1 would therefore have to be 10 Ω; and since R_1 cannot be adjusted in steps smaller than 1 Ω, the e.m.f. cannot be measured to a greater accuracy than 10 per cent.

For accurate measurement of thermal e.m.f.s special potentiometers have been devised, but the simple circuit of Fig. 33.24 will do for a laboratory experiment. The e.m.f. E is applied via a sensitive galvanometer G across a standard shunt R of about 1 ohm. A current I, of a few milliamperes, is passed through the shunt, and measured on the milliammeter M. Its value is adjusted by the rheostat P until G shows no deflection. The potential difference RI is then equal and opposite to the thermal e.m.f.

$$E = RI.$$

If a balance cannot be found, the connexions of the junction to R should be reversed.

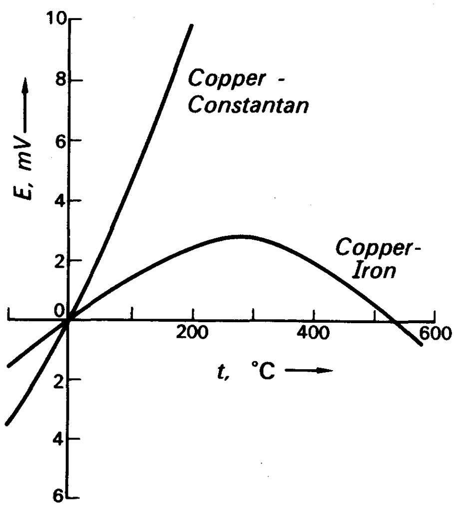


FIG. 33.25. E.m.f.s of thermocouples.
(Reckoned positive when into copper at the cold junction.)

Fig. 33.25 shows the results of measuring the e.m.f. E when the cold junction is at 0°C and the hot is at various temperatures t. The curves approximate to parabolas:

$$E = at + bt^2. \qquad . \quad . \quad . \quad (18)$$

THERMO-ELECTRIC E.M.F.s

(E in micro-volts when t is in °C and cold junction at 0°C)

Junction	a	b	Range for a and b, °C	Limits of use, °C
Cu/Fe	14	−0·02	0–100	*See* 1
Cu/Constantan[2]	41	0·04	−50 to +300	−200 to +300
Pt/Pt−Rh[3]	6·4	0·006	0–200	0–1700
Chromel[4]/Alumel[5]	41	0·00	0–900	0–1300

[1] Simple demonstrations.
[2] See p. 788.
[3] 10 per cent Rh; used only for accurate work or very high temperatures.
[4] 90 per cent Ni, 10 per cent Cr.
[5] 94 per cent Ni, 3 per cent Mn, 2 per cent Al, 1 per cent Si.

NETWORKS

Kirchhoff's Laws

A 'network' is usually a complicated system of electrical conductors. Kirchhoff (1824–87) extended Ohm's law to networks, and gave two laws, which together enabled the current in any part of the network to be calculated.

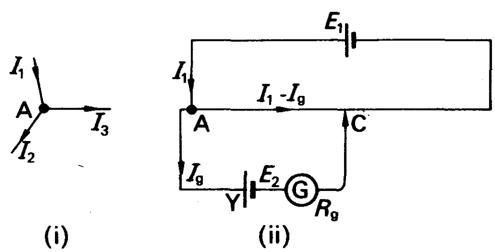


FIG. 33.26. Kirchhoff's laws.

The *first law* refers to any point in the network, such as A in Fig. 33.26 (*a*); it states that the total current flowing into the point is equal to the total current flowing out of it:

$$I_1 = I_2 + I_3.$$

The law follows from the fact that electric charges do not accumulate at the points of a network. It is often put in the form that *the algebraic sum of the currents at a junction of a circuit is zero*, or

$$\Sigma I = 0,$$

where a current, I, is reckoned positive if it flows towards the point, and negative if it flows away from it. Thus at A in Fig. 33.26 (i),

$$I_1 - I_2 - I_3 = 0.$$

Kirchhoff's first law gives a set of equations which contribute towards the solving of the network; in practice, however, we can shorten the work by putting the first law straight into the diagram, as shown in Fig. 33.26 (ii) for example, since

$$\text{current along AC} = I_1 - I_g.$$

Kirchhoff's *second law* is a generalization of Ohm's law for the complete circuit. It refers to any closed loop, such as AYCA in Fig. 33.26 (ii); and it states that, round such a loop, the algebraic sum of the voltage drops is equal to the algebraic sum of the e.m.f.s:

$$\Sigma RI = \Sigma E.$$

Thus, clockwise round the loop,

$$R_{AC}(I_1 - I_g) - R_g I_g = E_2.$$

We have used the potentiometer to illustrate Kirchhoff's laws merely because it is already familiar to us; we shall not go on and solve it as a network, because we have already dealt with as much of the theory of it as we need.

EXAMPLE

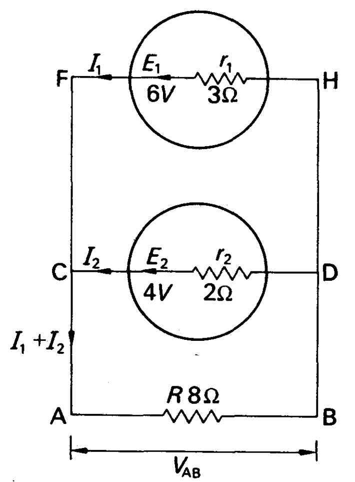


FIG. 33.27. Load across cells in parallel.

Fig. 33.27 shows a network which can be solved by Kirchhoff's laws. From the first law, the current in the 8-ohm wire is $(I_1 + I_2)$, assuming I_1, I_2 are the currents through the cells. Taking closed circuits formed by each cell with the 8-ohm wire, we have, from the second law,

$$E_1 = 6 = 3I_1 + 8(I_1 + I_2) = 11I_1 + 8I_2$$

and

$$E_2 = 4 = 2I_2 + 8(I_1 + I_2) = 8I_1 + 10I_2.$$

Solving the two equations, we find $I_1 = \frac{14}{23} = 0{\cdot}61$ amp, $I_2 = -\frac{2}{23} = -0{\cdot}09$ amp.

The minus sign indicates that the current I_2 flows in the sense opposite to that shown in the diagram; i.e. it flows against the e.m.f. of the generator E_2. It does so because the potential difference V_{AB} is greater than E_2:

$$V_{AB} = \left(R\ I_1 + I_2\right) = 8\left(\frac{14}{23} - \frac{2}{23}\right)$$

$$= 8 \times \frac{12}{23} = \frac{96}{23} = 4{\cdot}2 \text{ volts.}$$

This is equal to the e.m.f. E_2 *plus* the drop across the internal resistance r_2 (p. 828):

$$V_{CD} = 4+2\times\frac{2}{23} = 4+\frac{4}{23}$$

$$= \frac{96}{23}\text{ volts} = V_{AB}.$$

It is also equal to the e.m.f. E_1 *minus* the drop across r_1, because the current flows through the upper generator in the direction of its e.m.f.:

$$V_{FH} = 6-3\times\frac{14}{23} = 6-\frac{42}{23} = \frac{138-42}{23}$$

$$= \frac{96}{23}\text{ volts} = V_{AB}.$$

WHEATSTONE BRIDGE
MEASUREMENT OF RESISTANCE

Wheatstone Bridge Circuit

About 1843 Wheatstone designed a circuit called a 'bridge circuit' which gave an accurate method for measuring resistance. We shall deal later with the practical aspects. In Fig. 33.28, X is the unknown resistance, and P, Q, R are resistance boxes. One of these—usually R— is adjusted until the galvanometer between A, C, represented by its resistance R_g, shows no deflection: that is to say,

$$I_g = 0.$$

Then, as we shall show,

$$\frac{P}{Q} = \frac{R}{X},$$

whence

$$X = \frac{Q}{P}R.$$

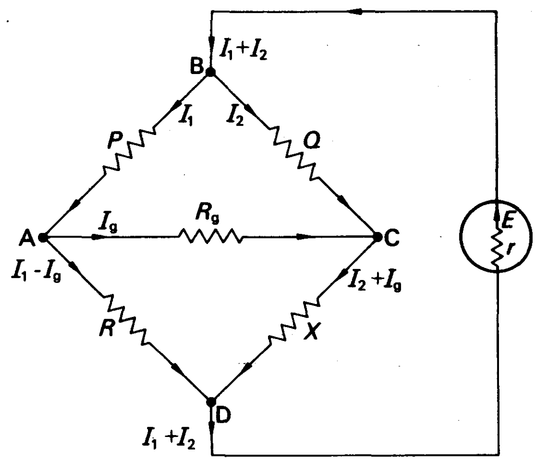


FIG. 33.28. Analysis of Wheatstone bridge.

Fig 33.28 shows Kirchhoff's first law applied to the circuit. From the second law, we have:

$$\text{loop ACBA:} \quad R_g I_g - QI_2 + PI_1 = 0, \quad . \quad . \quad \text{(i)}$$

$$\text{loop ACDA:} \quad R_g I_g + X(I_2 + I_g) - R(I_1 - I_g) = 0,$$

or

$$I_g(R_g + X + R) + XI_2 + RI_1 = 0. \quad . \quad . \quad \text{(ii)}$$

If we wished to find I_g, we would have to set up a third equation, by going round one of the loops, including the battery (p. 831). But if we wish only to find the condition for no deflection of the galvanometer, we have merely to put $I_g = 0$ in equations (i) and (ii). Then

$$-QI_2 + PI_1 = 0, \quad \text{or} \quad PI_1 = QI_2,$$

whence

$$\frac{P}{Q} = \frac{I_2}{I_1};$$

and

$$IX_2 - RI_1 = 0, \quad \text{or} \quad XI_2 = RI_1,$$

whence

$$\frac{R}{X} = \frac{I_2}{I_1}$$

Therefore, as already stated,

$$\frac{P}{Q} = \frac{R}{X}. \quad . \quad . \quad . \quad . \quad (19)$$

This is the condition for balance of the bridge. It is the same, as the reader may easily show, if the battery and galvanometer are interchanged in the citcuit.

Alternative Wheatstone Bridge Proof

Equation (19) for the balance condition can be got without the use of Kirchhoff's laws. At balance, since no current flows through the galvanometer, the points A and C must be at the same potential (Fig. 33.29). Therefore

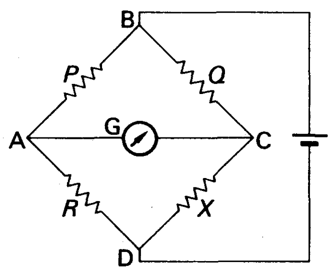


FIG. 33.29. Wheatstone bridge.

$$V_{AB} = V_{CB}$$

and

$$V_{AD} = V_{CD},$$

whence

$$\frac{V_{AB}}{V_{AD}} = \frac{V_{CB}}{V_{CD}}. \quad . \quad . \quad . \quad . \quad \text{(i)}$$

Also, since $I_g = 0$, P and R carry the same current, I_1, and X and Q carry the same current, I_2. Therefore

$$\frac{V_{AB}}{V_{AD}} = \frac{I_1 P}{I_1 R} = \frac{P}{R}$$

and

$$\frac{V_{CB}}{V_{CD}} = \frac{I_2 Q_2}{I_2 X} = \frac{Q}{X} \quad . \quad . \quad . \quad \text{(ii)}$$

Hence by equations (i) and (ii),

$$\frac{P}{R} = \frac{Q}{X}.$$

or

$$\frac{P}{Q} = \frac{R}{X}.$$

Galvanometer Position

We shall now show, by taking a numerical example, how the galvanometer in a bridge circuit can best be positioned.

Fig. 33.30 shows an unbalanced Wheatstone bridge, fed from a cell

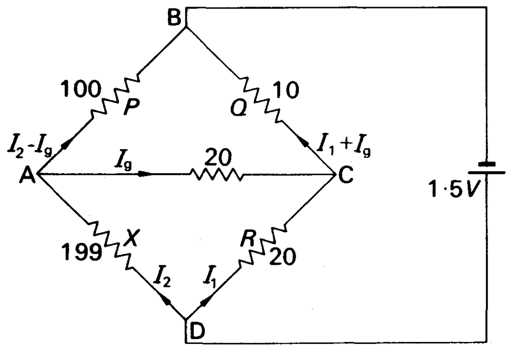


FIG. 33.30. An unbalanced Wheatstone bridge.

of negligible internal resistance. The figures give the resistance in ohms, and I_g is to be found. Applying Kirchhoff's laws:

$$\text{Loop ACBA:} \quad 20I_g + 10(I_1 + I_g) - 100(I_2 - I_g) = 0$$

or

$$130I_g + 10I_1 - 100I_2 = 0,$$

whence

$$I_1 = 10I_2 - 13I_g. \qquad \text{(i)}$$

$$\text{Loop ADCA:} \quad -139I_2 + 20I_1 - 20I_g = 0.$$

$$\text{Substituting for } I_1\text{:} \quad -199I_2 + 200I_2 - 260I_g - 20^9{}_g = 0,$$

or

$$I_2 - 280I_g = 0,$$

whence

$$I_2 = 280I_g.$$

$$\therefore \text{ by (i),} \quad I_1 = 2800I_g - 13I_g = 2787I_g. \qquad \text{(ii)}$$

$$\text{Loop DCBXD:} \quad 20I_1 + 10(I_1 + I_g) = 1{\cdot}5,$$

or

$$30I_1 + 10I_g = 1{\cdot}5.$$

$$\text{Substituting from (ii) for } I_1\text{:} \quad 30 \times 2787I_g + 10I_g = 1{\cdot}5$$

or

$$83620I_g = 1{\cdot}5,$$

whence

$$I_g = \frac{1{\cdot}5}{83620} = 1{\cdot}79 \times 10^{-5}\ \text{A}$$

$$= 17{\cdot}9 \text{ microamperes.}$$

The reader should now show that, if the battery and galvanometer were interchanged, the current I_g would be 13·2 microamperes. This result illustrates an important point in the use of the Wheatstone bridge: with a given unbalance, the galvanometer current is greatest when the galvanometer is connected from the junction of the highest resistances to the junction of the lowest. Therefore, unless $P = Q$, which is unusual, the galvanometer should be connected across PQ.

Practical Arrangement

A practical form of Wheatstone bridge is shown in Fig. 33.31. The resistances P and Q can be given values of 10, 100, or 1000 ohms by three-point switches. The resistance R has four decade dials by which it can be varied from 1 ohm to more than 10000 ohms. Pairs of terminals are provided for connecting the unknown resistance, the battery, and the galvanometer, X, B, G; and keys K_1 and K_2 are fitted in the battery and galvanometer circuits.

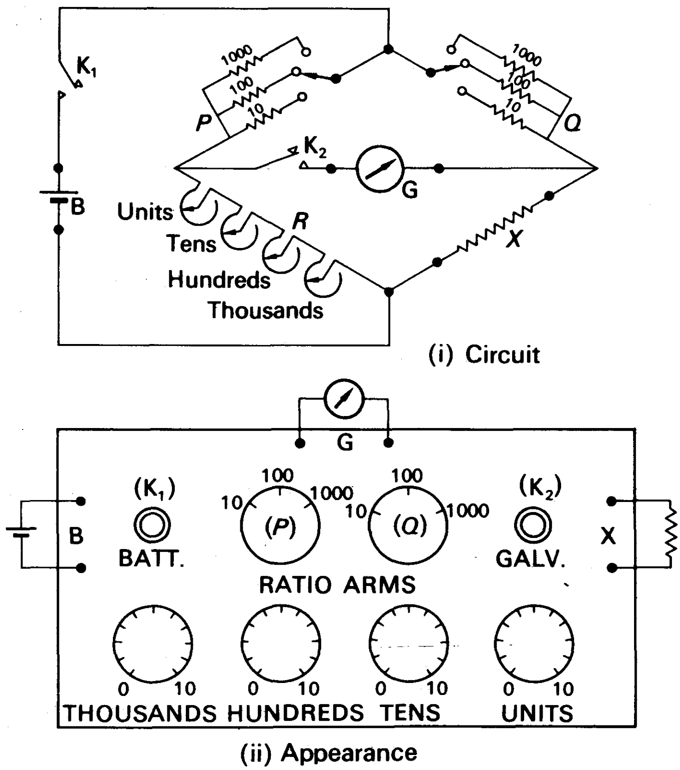


FIG. 33.31. Practical form of Wheatstone bridge.

To measure a resistance, we first set $P = Q = 10\,\Omega$. We set $R = 0$ and press first K_1 then K_2; the small interval between pressing K_1 and K_2 gives time for the currents in the bridge to become steady (Chapter

31). Let us suppose that, when we press K_2, the galvanometer deflects to the right. We then set $R = 10000\ \Omega$ and again press K_1, K_2. If the galvanometer deflects to the left we can proceed; if it deflects again to the right, then either we have made a wrong connexion —which with this form of bridge is almost impossible— or X is greater than 10000 ohms. If the galvanometer deflects to the left, we try again with $R = 1000\ \Omega$; and so on with 100 Ω and 10 Ω, if necessary. Let us suppose that the galvanometer deflects to the left with $R = 100\ \Omega$, but to the right with 10 Ω. We then adjust the 10's dial until we get, say, a leftward deflection with 40 Ω and a rightward with 30 Ω. With the unit's dial we now narrow the limits to, let us say, 36 Ω (left) and 35 Ω (right). We have

$$\frac{X}{R} = \frac{Q}{P} = \frac{10}{10} = 1.$$

$$\therefore X = R.$$

It follows that X lies between 35 and 36 Ω.

We now set $P = 100\ \Omega$, so that

$$\frac{X}{R} = \frac{Q}{P} = \frac{10}{100} = \frac{1}{10}$$

or

$$X = \frac{R}{10}.$$

The balance-point now lies between $R = 350\ \Omega$ (right) and $R = 360\ \Omega$ (left); by using the unit's dial we can now locate it between, say, 353 and 354. Then, from the equation, X lies between 35·3 and 35·4 ohms. If we finally make $P = 1000\ \Omega$, we have

$$\frac{X}{R} = \frac{Q}{P} = \frac{10}{1000} = \frac{1}{100}$$

or

$$X = \frac{R}{100}.$$

Only a sensitive galvanometer will give considerable deflections near the balance-point in this condition; if it locates the balance-point between $R = 3536\ \Omega$ and 3537 Ω then X lies between 35·36 and 35·37 ohms. If a moving-coil galvanometer is used, it must be protected by a high series resistance while balance is being sought.

Range of Measurable Resistance

The resistors P and Q are often called the ratio arms of the bridge, because their resistances determine the ratio of R to X. If X is greater than the greatest value of R, it can be measured by making $Q = 100$, $P = 10$. Then

$$\frac{X}{R} = \frac{Q}{P} = \frac{100}{10} = 10$$

and

$$X = 10R.$$

A balance-point between $R = 14620$ and 14630, say, would mean that R lay between 146 200 and 146 300 ohms. Similarly, by making $Q = 1000$, $P = 10$, resistances can be measured up to 100 times the greatest value of R: that is to say, up to a little more than 1 000 000 ohms. With these high resistances, however, the near-balance currents are very small, and a sensitive galvanometer is necessary.

The lowest resistance which a bridge of this type can measure with reasonable accuracy is about 1 ohm; R can be adjusted in steps of 1 ohm, and P/Q can be made 1/100, so that measurements can be made to within 1/100 ohm. Resistances lower than about 1 ohm cannot be measured accurately on a Wheatstone bridge, whatever the ratios available, or the smallest steps in R. They cannot because of the resistances of the wires connecting them to the X terminals, and of the contacts between those wires and the terminals to which they are, at each end, attached. This is the reason why the potentiometer method is more satisfactory for low resistances.

The Post Office Box

An old-fashioned type of Wheatstone bridge, with plugs instead of switches, is called the Post Office box, and is illustrated in Fig. 33.32. It is connected up and used in the same way as the dial type of bridge,

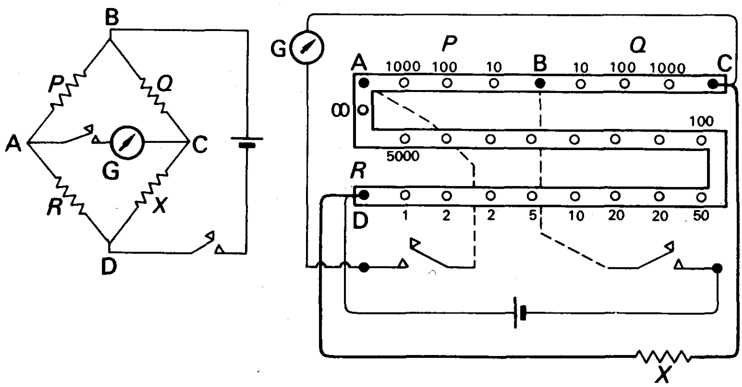


FIG. 33.32. Post Office box.

but requires far more skill by its operator. Anyone who has to use a Post Office box should observe the following rules:

(i) do not attempt to memorize the wiring-up; the circuit should be worked out from the Wheatstone bridge diagram (Fig. 33.29);
(ii) take the 10 Ω plugs out of each ratio arm P, Q before testing the circuit;
(iii) test for correct connexions by seeing whether the galvanometer gives opposite deflections with $R = 0$ and $R = \infty$ (for the latter an 'infinity' plug is provided, whose gap is not bridged by any resistor—Fig. 33.32);

(iv) press plugs home firmly, with a half-turn to the right;
(v) never mix the plugs from different boxes—always put loose plugs in the lid of their box, never on the bench.

The Slide-wire (Metre) Bridge

Fig. 33.33 shows a simple and cheap form of Wheatstone bridge; it is sometimes called a metre bridge, for no better reason than that the wire AB is often a metre long. The wire is uniform, as in a potentiometer, and can be explored by a slider S. The unknown resistance X and a known resistance R are connected as shown in the figure; heavy brass or copper strip is used for the connexions AD, FH, KB, whose resistances are generally negligible. When the slider is at a point C in

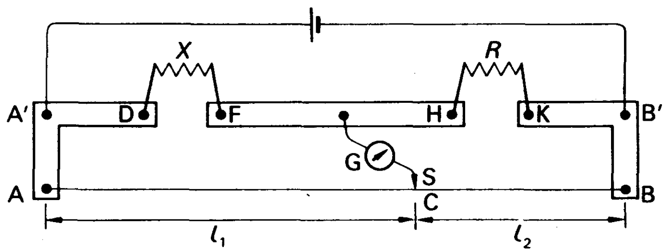


FIG. 33.33. Slide-wire (metre) bridge.

the wire it divides the wire into two parts, of resistances R_{AC} and R_{CB}; these, with X and R, form a Wheatstone bridge. (The galvanometer and battery are interchanged relative to the circuits we have given earlier; that enables the slider S to be used as the galvanometer key. We have already seen that the interchange does not affect the condition for balance (p. 830).) The connexions are checked by placing S first on A, then on B. The balance-point is found by trial and error—not by scraping S along AB. At balance,

$$\frac{X}{R} = \frac{R_{AC}}{R_{CB}}.$$

Since the wire is uniform, the resistances R_{AC} and R_{CB} are proportional to the lengths of wire, l_1 and l_2. Therefore

$$\frac{X}{R} = \frac{l_1}{l_2}. \quad . \quad . \quad . \quad . \quad (20)$$

The resistance R should be chosen so that the balance-point C comes fairly near to the centre of the wire—within, say, its middle third. If either l_1 or l_2 is small, the resistance of its end connexion AA′ or BB′ in Fig. 33.33 is not negligible in comparison with its own resistance; equation (20) then does not hold. Some idea of the accuracy of a particular measurement can be got by interchanging R and X, and balancing again. If the new ratio agrees with the old within about 1 per cent, then their average may be taken as the value of X.

Resistance by Substitution

Fig. 33.34 illustrates a simple way of measuring a resistance X. It is connected in series with a rheostat S, an ammeter A, and a cell. S is adjusted until A gives a large deflection. X is then replaced by a box of known resistances, R, which can be selected by plugs or dials. R is varied until the ammeter gives the same reading as before. Then, if the e.m.f. of the cell has not fallen, $R = X$. The accuracy of this method is limited, by the inherent error of the ammeter, to about 1 per cent. It does not depend on the accuracy of *calibration* of the ammeter, but on the accuracy with which it reproduces a given deflection for a given current. The ammeter is used simple as a 'transfer instrument'—to indicate when the current in the second part of the experiment is the same as in the first. This principle is very useful in measurements more difficult than that of resistance by direct method. For example, the power output of a small radio transmitter can be measured by making it light a lamp, which is placed near to a lightmeter (p. 571). The lamp is then connected to a source of direct current, and the current through it is adjusted until the light-meter gives the same reading. Simple measurements of the current, and the voltage across the lamp, then give the power supplied to it—which is equal to the power output of the radio transmitter.

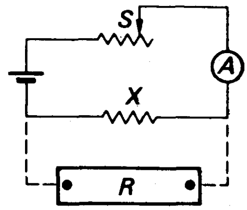
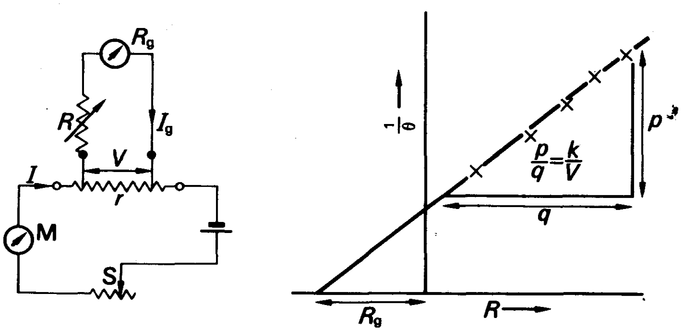


FIG. 33.34. Measurement of resistance by substitution.

Resistance and Sensitivity of a Galvanometer

It is often necessary to know the resistance, R_g, and sensitivity of a suspended-coil galvanometer. To find them, we may use the circuit of Fig. 33.35 (i). S is a rheostat of about 1000 ohms maximum, r is a standard shunt of about 0·01 ohm, M is a milliammeter, and R is a resistance box. The current in the main circuit, I, is adjusted to a value which can be accurately read on M—say 10 milliamperes. Since r is very



(i) Circuit (ii) Results

FIG. 33.35. Galvanometer calibration.

small compared with $R+R_g$, the galvanometer current I_g is negligible compared with I, and we may say that the potential difference across r is

$$V = rI.$$

In this example it would be $0{\cdot}01 \times 10 \times 10^{-3} = 10^{-4}$ volt $= 0{\cdot}1$ millivolt. The galvanometer current is therefore

$$I_g = \frac{V}{R+R_g}.$$

If $R+R_g$ is 1000 ohms, then $I_g = 10^{-4}/10^3 = 10^{-7}$ amp $= 0{\cdot}1$ microampere, which is a reasonable value for a galvanometer of moderate sensitivity. If θ is the deflection of the galvanometer, then

$$I_g = k\theta,$$

where k is a constant (the reduction factor) which we wish to find.

From the equation for I_g above.

$$\frac{V}{R+R_g} = k\theta,$$

whence $$\frac{1}{\theta} = \frac{k}{V}(R+R_g).$$

Therefore, if we vary R and plot $1/\theta$ against it, we get a straight line, as shown in Fig. 33.35 (ii). The line makes an intercept on the R axis, which gives the value of R_g. And its slope p/q is k/V. Since we know V from above, we can hence find k; and $1/k$, the deflection per unit current, is the sensitivity.

Temperature Coefficient of Resistance

We have seen that the resistance of a given wire increases with its temperature. If we put a coil of fine copper wire into a water bath, and use a Wheatstone bridge to measure its resistance at various moderate temperatures t, we find that the resistance, R, increases

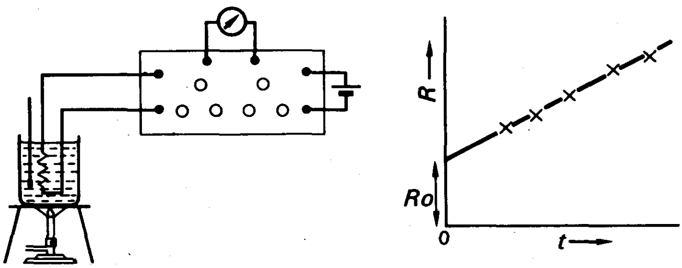


FIG. 33.36. Measurement of temperature coefficient.

uniformly with the temperature (Fig. 33.36). We may therefore define a temperature coefficient of resistance, α, such that

$$R = R_0(1+\alpha t), \qquad . \quad . \quad . \quad (21)$$

where R_0 is the resistance at 0°C. In words,

$$\alpha = \frac{\text{increase of resistance per deg. C rise of temperature}}{\text{resistance at 0°C}}$$

If R_1 and R_2 are the resistances at t_1°C and t_2°C, then

$$\frac{R_1}{R_2} = \frac{1+\alpha t_1}{1+\alpha t_2}. \qquad . \quad . \quad . \quad (22)$$

Values of α for pure metals are of the order of 0·004 per deg. C. They are much less for alloys than for pure metals, a fact which enhances the value of alloys as materials for resistance boxes and shunts.

Equation (21) represents the change of resistance with temperature fairly well, but not as accurately as it can be measured. More accurate equations are given on p. 370 in the Heat section of this book, where resistance thermometers are discussed.

EXAMPLES

1. How would you compare the resistances of two wires A and B, using (*a*) a Wheatstone bridge method and (*b*) a potentiometer? For each case draw a circuit diagram and indicate the method of calculating the result.

In an experiment carried out at 0°C, A was 120 cm of Nichrome wire of resistivity 100×10^{-6} ohm cm and diameter 1·20 mm, and B a German silver wire 0·80 mm diameter and resistivity 28×10^{-6} ohm cm. The ratio of the resistances A/B was 1·20. What was the length of the wire B?

If the temperature coefficient of resistance of Nichrome is 0·00040 per deg. C and of German silver is 0·00030 per deg. C, what would the ratio of the resistances become if the temperature were raised by 100 deg. C? (*L.*)

First part (see pp. 830, 824).

Second part. With usual notation,

for A, $$R_1 = \frac{\rho_1 l_1}{a_1},$$

and for B, $$R_2 = \frac{\rho_2 l_2}{a_2}.$$

$$\therefore \frac{R_1}{R_2} = \frac{\rho_1}{\rho_2}.\frac{l_1}{l_2}.\frac{a_2}{a_1} = \frac{\rho_1}{\rho_2}.\frac{l_1}{l_2}.\frac{d_2^2}{d_1^2},$$

where d_2, d_1 are the respective diameters of B and A.

$$\therefore 1{\cdot}20 = \frac{100}{28}\times\frac{120}{l_2}\times\frac{0{\cdot}8^2}{10{\cdot}2^2}.$$

$$\therefore l_2 = \frac{100\times120\times0{\cdot}64}{1{\cdot}20\times28\times1{\cdot}44} = 159 \text{ cm} \qquad . \quad . \quad (i)$$

When the temperature is raised by 100°C, the resistance increases according to the relation $R_t = R_0(1+\alpha t)$. Thus

$$\text{new Nichrome resistance, } R_A = R_1(1+\alpha\,.\,100) = R_1 \times 1{\cdot}04,$$

$$\text{and new German silver resistance, } R_B = R_2(1+\alpha'\,.\,100) = R_2 \times 1{\cdot}03.$$

$$\therefore \frac{R_A}{R_B} = \frac{R_1}{R_2} \times \frac{1{\cdot}04}{1{\cdot}03} = 1{\cdot}20 \times \frac{1{\cdot}04}{1{\cdot}03} = 1{\cdot}21 \quad . \quad . \quad . \quad \text{(ii)}$$

2. State Kirchhoff's laws relating to the currents in network of conductors. Two cells of e.m.f. 1·5 volts and 2 volts respectively and internal resistances of 1 ohm and 2 ohms respectively are connected in parallel to an external resistance of 5 ohms. Calculate the currents in each of the three branches of the network. (*N.*)

First part (see p. 828).

Second part. Suppose x, y amp are the respective currents through the cells (Fig. 33.37). Then, from Kirchhoff's first law, the current through the 5-ohm wire is $(x+y)$ amp.

Applying Kirchhoff's second law to the complete circuit with the cell of e.m.f. 1·5 volts and external resistance 5 ohms, we have

$$1{\cdot}5 = x+5(x+y) = 6x+5y \quad . \quad . \quad . \quad \text{(i)}$$

Applying the law to the complete circuit with the cell of e.m.f. 2 volts and external resistance 5 ohms, then

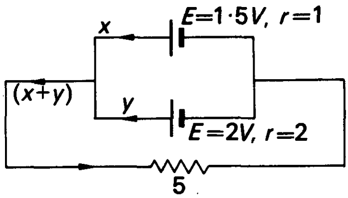


FIG. 33.37. Example.

$$2 = 2y+5(x+y) = 5x+7y. \quad . \quad . \quad . \quad \text{(ii)}$$

Solving (i) and (ii), we find $y = 9/34$ amp, $x = 1/34$ amp. Thus the current through the 5-ohm resistor $= x+y = 10/34 = 5/17$ amp.

EXERCISES 33

Circuit Calculations

1. State Ohm's law and describe an experiment to verify it.

A resistance of 1000 ohms and one of 2000 ohms are placed in series with a 100-volt mains supply. What will be the reading on a voltmeter of internal resistance 2000 ohms when placed across (*a*) the 1000 ohms resistance, (*b*) the 2000 ohms resistance? (*L.*)

2. Define the practical unit of potential difference and hence show that the rate of production of heat in a wire of constant resistance is proportional to the square of the current passing through it.

A cell A, e.m.f. 1·1 volts and internal resistance 3 ohms, is joined in parallel with another cell B, e.m.f. 1·4 volts and internal resistance 1 ohm, similar poles being connected together. The ends of a wire, of resistance 4 ohms, are joined to the terminals of A. Find (*a*) the current through the wire, (*b*) the rate of dissipation of energy in watts in each of the cells A and B. (*L.*)

3. Twelve cells each of e.m.f. 2 volts and of internal resistance $\frac{1}{2}$ ohm are arranged in a battery of n rows and an external resistance of $\frac{3}{8}$ ohm is connected to the poles of the battery. Determine the current flowing through the resistance in terms of n.

Obtain numerical values of the current for the possible values which n may take and draw a graph of current against n by drawing a smooth curve through the points. Give the value of the current corresponding to the maximum of the curve and find the internal resistance of the battery when the maximum current is produced. (*L.*)

4. Describe with full experimental details an experiment to test the validity of Ohm's law for a metallic conductor.

An accumulator of e.m.f. 2 volts and of negligible internal resistance is joined in series with a resistance of 500 ohms and an unknown resistance X ohms. The readings of a voltmeter successively across the 500-ohm resistance and X are 2/7 and 8/7 volts respectively. Comment on this and calculate the value of X and the resistance of the voltmeter. (*N.*)

5. State with reasons the essential requirement for the resistance of (*a*) an ammeter, (*b*) a voltmeter.

A voltmeter having a resistance of 1800 ohms is used to measure the potential difference across a 200 ohm resistance which is connected to the terminals of a d.c. power supply having an e.m.f. of 50 volts and an internal resistance of 20 ohms. Determine the percentage change in the potential difference across the 200 ohm resistor as a result of connecting the voltmeter across it. (*N.*)

6. State Ohm's law and describe how you would test its validity. Why would an experiment involving the use of a moving-coil ammeter and a moving-coil voltmeter be unsatisfactory?

In order to calibrate a galvanometer an accumulator of e.m.f. 200 volts and negligible resistance is connected in series with two resistances P and Q. A resistance R and the galvanometer, resistance G, are joined in series and then connected to the ends of P. The galvanometer is shunted by a resistance S. If $P = 200$ ohms, $Q = 1880$ ohms, $R = 291$ ohms, $S = 10$ ohms, $G = 90$ ohms, and the deflection is 20 divisions, calculate the current sensitivity in micro-amperes per division.

7. State Kirchhoff's laws for flow of electricity through a network containing sources of e.m.f.

Wheatstone bridge with slide wire of 0·5 ohm and length 50 cm is used to compare two resistances, each of 2 ohms. The cell has an e.m.f. of 2 volts and no internal resistance, and the galvanometer has resistance of 100 ohms. Find the current through the galvanometer when the bridge is 1 cm off balance. Compare the result with that of approximate calculation. (*L.*)

8. State Ohm's law, and describe the experiments you would make in order to verify it. The positive poles A and C of two cells are connected by a uniform wire of resistance 4 ohms and their negative poles B and D by a uniform wire of resistance 6 ohms. The middle point of BD is connected to earth. The e.m.f.s of the cells AB and CD are 2 volts and 1 volt respectively, their resistances 1 ohm and 2 ohms respectively. Find the potential at the middle point of AC. (*O. & C.*)

9. From the adoption of either the fundamental units cm, g, second, or the fundamental units m, kg, second, trace the steps necessary to define the *volt* and the *ohm* in terms of the ampere.

Discuss the suitability of (*a*) a moving-coil voltmeter, and (*b*) a slide-wire potentiometer for determining the potential differences in an experiment designed to verify Ohm's law.

Four resistors AB, BC, CD and DA of resistance 4 ohms, 8 ohms, 4 ohms and 8 ohms respectively are connected to form a closed loop, and a 6-volt battery of negligible resistance is connected between A and C. Calculate (i) the potential difference between B and D, and (ii) the value of the additional resistance which must be connected between A and D so that no current flows through a galvanometer connected between B and D. (*O. & C.*)

Measurements

10. Describe and explain how you would use a potentiometer (*a*) to compare the electromotive forces of two cells, (*b*) to test the accuracy of the 1-amp reading of an ammeter.

The e.m.f. of a cell is balanced by the fall in potential along 150 cm of a potentiometer wire. When the cell is shunted by a resistance of 14 ohms the length required is 140 cm. What is the internal resistance of the cell? (*N.*)

11. Describe a metre bridge and the method of using it to compare the resistances of two conductors, giving the theory of the method. Why is the method unsatisfactory if the two resistances (*a*) differ widely from one another, (*b*) are very small?

A metre bridge is balanced with a piece of aluminium wire of resistance 7·30 ohm in the left-hand gap, the slide contact being 42·6 cm from the left-hand end of the bridge wire and the temperature 17°C. If the temperature of the aluminium wire is raised to 57°C, how may the balance be restored (*a*) by adjusting the slide contact, (*b*) by keeping the contact at 42·6 cm and connecting a conductor in parallel with the aluminium wire? (The temperature coefficient of resistance for aluminium may be taken as $3{\cdot}8 \times 10^{-3}\ K^{-1}$) (*L.*)

12. Describe and explain how a potentiometer is used to test the accuracy of the 1 volt reading of a voltmeter.

A potentiometer consists of a fixed resistance of 2030 ohms in series with a slide wire of resistance 4 ohm $metre^{-1}$. When a constant current flows in the potentiometer circuit a balance is obtained when (*a*) a Weston cell of e.m.f. 1·018 volt is connected across the fixed resistance and 150 cm of the slide wire and also when (*b*) a thermocouple is connected across 125 cm of the slide wire only. Find the current in the potentiometer circuit and the e.m.f. of the thermocouple.

Find the value of the additional resistance which must be present in the above potentiometer circuit in order that the constant current shall flow through it, given that the driver cell is a lead accumulator of e.m.f. 2 volt and of negligible resistance and the length of the slide wire is 2 metres. (*L.*)

13. Describe the Wheatstone bridge circuit and deduce the condition for 'balance'. State clearly the fundamental electrical principles on which you base your argument. Upon what factors do (*a*) the sensitivity of the bridge, (*b*) the accuracy of the measurement made with it, depend?

Using such a circuit, a coil of wire was found to have a resistance of 5 ohms in melting ice. When the coil was heated to 100°C, a 100 ohm resistor had to be connected in parallel with the coil in order to keep the bridge balanced at the same point. Calculate the temperature coefficient of resistance of the coil. (*C.*)

14. Explain the use of the Weston cell and the precautions to be taken when using it.

A steady current is passed through a manganin potentiometer wire AB of length 10 metres and diameter 0·56 mm connected to a resistance box BC with a resistance of 1001 ohms. A Weston cell of e.m.f. 1·018 volts, with a sensitive galvanometer in series with it, is connected in parallel between the points A and C. It is seen that no deflection is produced in the galvanometer. The Weston cell is removed and a thermocouple is connected *via* the galvanometer to points on the potentiometer wire 524 cm apart. Again there is no deflection. Draw the two circuits and calculate the e.m.f. of the thermocouple. (Resistivity of manganin $= 41 \cdot 87 \times 10^{-6}$ ohm cm.) (*L.*)

15. Describe, giving full experimental details, how you would compare the values of two unknown resistances each of the order of 0·1 ohm using a potentiometer. Draw a circuit diagram and give the theory of the method.

Suppose that, having set up the circuit to carry out this experiment, you found that no balance point could be obtained along the potentiometer. Discuss three possible reasons for this and the procedure you would adopt in order to trace it. (*N.*)

16. A two-metre potentiometer wire is used in an experiment to determine the internal resistance of a voltaic cell. The e.m.f. of the cell is balanced by the fall of potential along 90·6 cm of wire. When a standard resistance of 10 ohms is connected across the cell the balance length is found to be 75·5 cm. Draw a labelled circuit diagram and calculate, from first principles, the internal reistance of the cell.

How may the accuracy of this determination be improved? Assume that other electrical components are available if required. (*N.*)

17. Define *resistivity* and *temperature coefficient of resistance.*

Explain, with the help of a clear circuit diagram, how you would use a Post Office box to determine the resistance of an electric lamp filament at room temperature.

A carbon lamp filament was found to have a resistance 375 ohms at the laboratory temperature of 20°C. The lamp was them connected in series with an ammeter and a d.c. supply, and a voltmeter of resistance 1050 ohms was connected in parallel with the lamp. The ammeter and voltmeter indicated 0·76 amp and 100 volt respectively. The temperature of the carbon filament was estimated to be 1200°C. Estimate the mean value of the temperature coefficient of resistance of carbon between 20°C and 1200°C, and comment on your result. (*L.*)

18. Explain the principle of the potentiometer.

If a slide wire potentiometer of total resistance 5 ohms and a Weston standard cell of e.m.f. 1·0187 volt were available, together with the necessary auxiliary apparatus, describe in detail how you would (*a*) determine the variation of the e.m.f. of a thermocouple with the temperature difference between its junctions (maximum e.m.f. 3 millivolt), (*b*) compare the values of two resistances nominally 0·010 ohm and 0·005 ohm. (*L.*)

19. State Ohm's law. Deduce a formula for the resistance of a number of resistors connected in parallel.

If you were given a cell of constant internal resistance, an ammeter of negligible resistance and various wires of different lengths but otherwise identical, describe how you would determine how many cm of wire had the same resistance as the internal resistance of the cell.

Describe how you would compare the resistances of two resistors each about 10^6 ohms using only an accumulator of e.m.f. of about 2 volts, a low-resistance uncalibrated galvanometer with a linear scale and full-scale deflection for about 10^{-7} amp and a three-terminal slider rheostat of about 20 ohms resistance. Justify the calculation of the resistance ratio from the readings you would take. (*C.*)

20. Describe how you would calibrate an ammeter using a standard resistor, a rheostat, accumulators, potentiometer slide wire with usual accessories, and a standard cell.

A standard low resistor is accidentally connected across a 100 volt d.c. main. It emits a momentary flash of light, vapourizes, and immediately breaks the circuit without further sparking. Estimate the duration of the flash if the wire becomes incandescent at 550°C, melts at 900°C, has specific heat 0·42 kJ kg^{-1} K^{-1}, density 5500 kg m^{-3}, is 10 cm long and has constant resistivity of 40 microhm cm. Assume that the rate of energy loss by the wire is small compared with the rate of heat development within it.

Give a physical explanation of the fact that this time is independent of the cross-sectional area of the wire. (*C.*)