PART FOUR

Electricity and Atomic Physics

chapter thirty
Electrostatics

GENERAL PHENOMENA

IF a rod of ebonite is rubbed with fur, or a fountain-pen with a coat-sleeve, it gains the power to attract light bodies, such as pieces of paper or tin-foil or a suspended pith-ball. The discovery that a body could be made attractive by rubbing is attributed to Thales (640–548 B.C.). He seems to have been led to it through the Greeks' practice of spinning silk with an amber spindle; the rubbing of the spindle in its bearings caused the silk to adhere to it. The Greek word for amber is *elektron*, and a body made attractive by rubbing is said to be 'electrified'. This branch of Electricity, the earliest discovered, is called *Electrostatics*.

Conductors and Insulators

Little progress was made in the study of electrification until the sixteenth century A.D. Then Gilbert (1540–1603), who was physician-in-ordinary to Queen Elizabeth, found that other substances besides amber could be electrified: for example, glass when rubbed with silk. He failed to electrify metals, however, and concluded that to do so was impossible.

More than 100 years later—in 1734—he was shown to be wrong, by du Fay; du Fay found that a metal could be electrified by rubbing with fur or silk, but only if it were held in a handle of glass or amber; it could not be electrified if it were held directly in the hand. His experiments followed the discovery, by Gray in 1729, that electric charges could be transmitted through the human body, water, and metals. These are examples of *conductors*; glass and amber are examples of *insulators*.

Positive and Negative Electricity

In the course of his experiments du Fay also discovered that there were two kinds of electrification: he showed that electrified glass and amber tended to oppose one another's attractiveness. To illustrate how he did so, we may use ebonite instead of amber, which has the same electrical properties. We suspend a pith-ball, and attract it with an electrified ebonite rod E (Fig. 30.1(i)); we then bring an electrified glass rod G towards the ebonite rod, and the pith-ball falls away (Fig. 30.1(ii)). Benjamin Franklin, a pioneer of electrostatics, gave the name of 'positive electricity' to the charge on a glass rod rubbed with silk, and 'negative electricity' to that on an ebonite rod rubbed with fur.

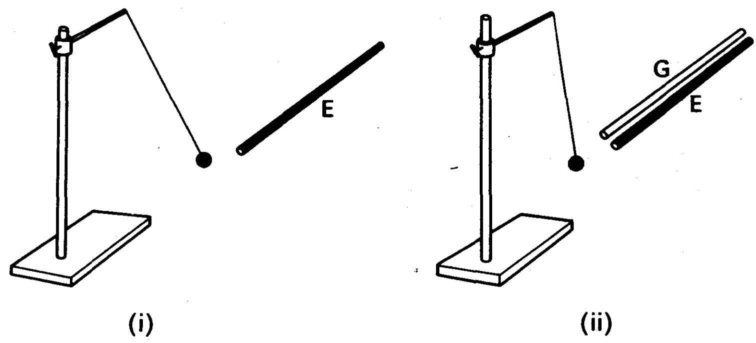


FIG. 30.1. Demonstrating that electrified glass tends to oppose effect of electrified ebonite.

Electrons and Electrostatics

Towards the end of the nineteenth century Sir J. J. Thomson discovered the existence of the *electron* (p. 1002). This is the lightest particle known—it is about 1/1840th of the mass of the hydrogen atom—and experiments show that it carries a tiny quantity of *negative* electricity. Later experiments showed that electrons are present in all atoms.

The detailed structure of atoms is complicated, but, generally, electrons exist round a minute core or nucleus carrying positive electricity. Normally, atoms are electrically neutral, that is, there is no surplus of charge on them. Consequently the total negative charge on the electrons is equal to the positive charge on the nucleus. In insulators, all the electrons appear to be firmly 'bound' to the nucleus under the attraction of the unlike charges. In metals, however, some of the electrons appear to be relatively 'free'. These electrons play an important part in electrical phenomena concerning metals.

The theory of electrons (negatively charged particles) gives simple explanations of electrification by friction, and of the attraction of uncharged bodies by charged ones. If the silk on which a glass rod has been rubbed is brought near to a charged and suspended ebonite rod it repels it; the silk must therefore have a negative charge. We know that the glass has a positive charge, and therefore we suppose that when the two were rubbed together electrons from the surface atoms were transferred from the glass to the silk. Likewise we suppose that when fur and ebonite are rubbed together, electrons go from the fur to the ebonite.

Attraction of Charged Body for Uncharged Bodies

To explain the attraction of a charged body for an uncharged one, we shall suppose that the uncharged body is a conductor—a metal. If it is brought near to a charged ebonite rod, say, then the negative charge on the rod repels the free electrons in the metal to its remote end (Fig. 30.2). A positive charge is thus left on the near end of the metal; this, being nearer than the negative charge on the far end, is attracted more strongly than the negative charge is repelled. On the whole, therefore, the metal is attracted. If the uncharged body is not a conductor, the mechanism by which it is attracted is more complicated; we shall postpone its description to a later chapter.

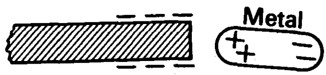


FIG. 30.2. Attraction by charged body.

Electrostatics Today

The discovery of the electron has led, in the last twenty or thirty years, to a great increase in the practical importance of electrostatics. In devices such as radio valves and cathode-ray tubes, for example, electrons are moving under the influence of electrostatic forces. The problems of preventing sparks and the breakdown of insulators are essentially electrostatic. There are also difficulties in making measurements at very high voltages. These problems occur in high-voltage electrical engineering. Later, we shall also describe a modern electrostatic generator, of the type used to provide a million volts or more for X-ray work and nuclear bombardment. Such generators work on principles of electrostatics discovered over a hundred years ago.

Gold-leaf Electroscope

One of the earliest instruments used for testing positive and negative charges consisted of a metal rod A to which gold leaves L were

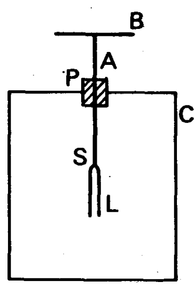


FIG. 30.3. A gold-leaf electroscope.

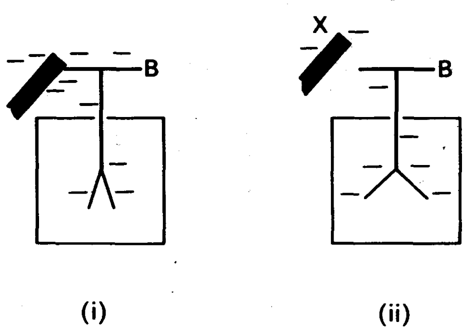


FIG. 30.4. Testing charge with electroscope.

attached (Fig. 30.3). The rod was fitted with a circular disc or cap B, and was insulated with a plug P from a metal case C which screened L from outside influences other than those brought near to B.

When B is touched by an ebonite rod rubbed with fur, some of the negative charge on the rod passes to the cap and L; and since like charges repel, the leaves diverge (Fig. 30.4(i)). If an unknown charge *X* is now brought near to B, an increased divergence implies that *X* is negative (Fig. 30.4(ii)). A positive charge is tested in a similar way; the electroscope is first given a positive charge and an increased divergence indicates a positive charge.

Induction

We shall now show that it is possible to obtain charges, called *induced charges*, without any contact with another charge. An experiment on electrostatic induction, as the phenomenon is called, is shown in Fig. 30.5(i). Two insulated metal spheres A, B are arranged so that they

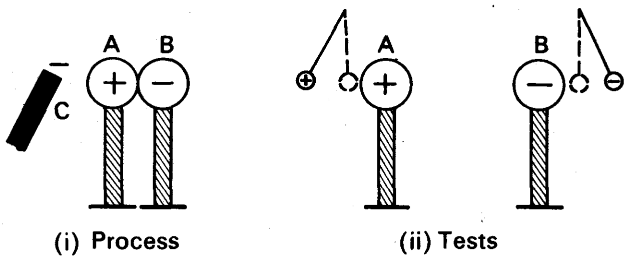


FIG. 30.5. Charges induced on a conductor.

touch one another, and a negatively charged ebonite rod C is brought near to A. The spheres are now separated, and then the rod is taken away. Tests with a charged pith-ball now show that A has a positive charge and B a negative charge (Fig. 30.5(ii)). If the spheres are placed together so that they touch, it is found that they now have no effect on a pith-ball held near. Their charges must therefore have neutralized each other completely, thus showing that the induced positive and negative charges are equal. This is explained by the movement of electrons from A to B when the rod is brought near. B has then a negative charge and A an equal positive charge.

Charging by Induction

Fig. 30.6 shows how a conductor can be given a permanent charge by induction, without dividing it in two. We first bring a charged ebonite rod, say, near to the conductor, (i); next we connect the conductor to earth by touching it momentarily (ii); finally we remove the ebonite. We then find that the conductor is left with a positive charge (iii). If we use a charged glass rod, we find that the conductor is left with a negative charge; the charge left, called the induced charge, has always the opposite sign to the inducing charge.

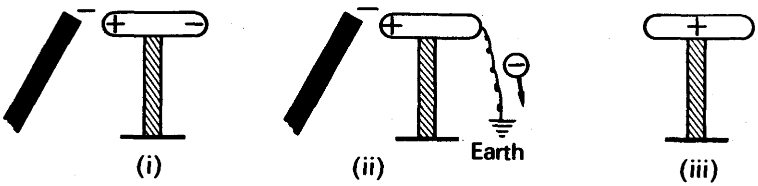


FIG. 30.6. Charging permanently by induction.

This phenomenon of induction can again be explained by the movement of electrons. If the inducing charge is negative, then, when we touch the conductor, electrons are repelled from it to earth, as shown in Fig. 30.6(ii), and a positive charge is left on the conductor. If the inducing charge is positive, then the electrons are attracted up from the earth to the conductor, which then becomes negatively charged.

Induction and the Electroscope

It is always observed that the leaves of an electroscope diverge when a charged body is brought near its cap, without touching it. This we can now easily understand; if, for example, we bring a negatively charged rod near the cap, it induces a positive charge on the cap, and a negative one on the leaves: the leaves then repel each other. Further, the negative charge on the leaves induces a positive one on the inside of the case, the corresponding negative charge running to the earth, on which the case rests. The positive charge on the case attracts the negative charge on the leaves, and makes them diverge further.

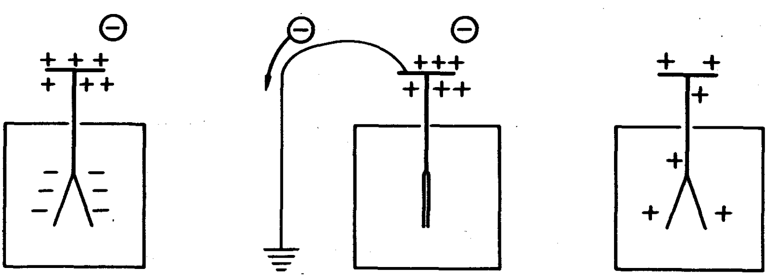


FIG. 30.7. Charging electroscope by induction.

We can use induction to give a permanent charge to the cap and leaves of an electroscope, by momentarily earthing the cap while holding an inducing charge near it. This is illustrated in Fig. 30.7.

The Electrophorus

A device which provides an almost unlimited supply of charge, by induction, was invented by Volta about 1800; it is called an *electrophorus*. It consists of an ebonite or perspex base, E in Fig. 30.8, and a metal disc D on an insulating handle. The ebonite is charged negatively by rubbing it—or, much better, beating it— with fur. The disc is then laid upon it, and acquires induced charges, positive underneath and negative on top, (i). Very little negative charge escapes from the ebonite to the disc, because the natural unevenness of their surfaces prevents them touching at more than a few points; charge escapes from these points only, because the ebonite is a non-conductor. After it has been placed on the ebonite, the disc is earthed with the finger, and the negative charge on its upper surface flows away, (ii). The disc can then be removed, and carries with it the positive charge which was on its underside, (iii).

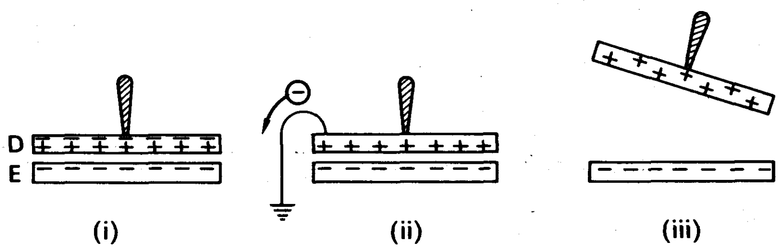


FIG. 30.8. The electrophorus.

An electrophorus produces sufficient charge to give an audible—and sometimes a visible—spark. The disc can be discharged and charged again repeatedly, until the charge on the ebonite has disappeared by leakage. Apparently, therefore, it is in principle an inexhaustible source of energy. However, work is done in raising the disc from the ebonite, against the attraction of their opposite charges, and this work must be done each time the disc is charged; the electrophorus is therefore not a source of energy, but a device for converting it from a mechanical into an electrical form.

The action of the electrophorus illustrates the advantages of charging by induction. First, the supply of charge is almost inexhaustible, because the original charge is not carried away. Second, a great charge —nearly equal to the charge on the whole of the ebonite—can be concentrated on to the conducting disc. As we have seen, only a very small charge could be transferred by contact, because the ebonite is not a conductor.

The Action of Points, Van de Graaff Generator

Sometimes in experiments with an electroscope connected to other apparatus by a wire, the leaves of the electroscope gradually collapse, as though its charge were leaking away. This behaviour can often be traced to a sharp point on the wire—if the point is blunted, the leakage stops. Charge leaks away from a sharp point through the air, being carried by molecules away from the point. This is explained later (p. 749).

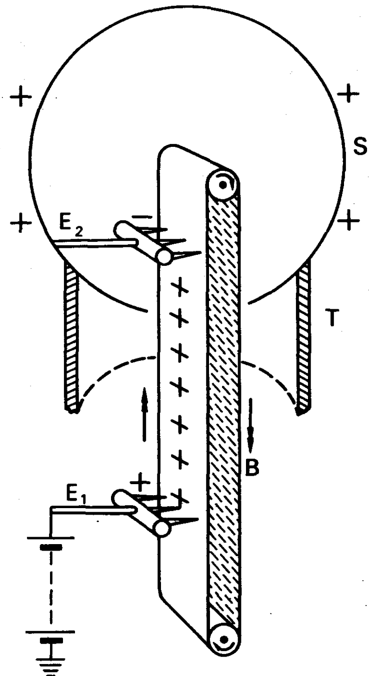


FIG. 30.9. Principle of Van de Graaff generator.

Points are used to collect the charges produced in *electrostatic generators*. These are machines for induction, and thus building up very great charges and potential differences. Fig. 30.9 is a simplified diagram of one such machine, due to Van de Graff. A hollow metal sphere S is supported on an insulating tube T, many feet high. A silk belt B runs over the pulleys shown, of which the lower is driven by an electric motor. Near the bottom and top of its run, the belt passes close to the elctrodes *E*, which are sharply pointed combs, pointing towards the belt. The electrode E_1 is made about 10000 volts positive with respect to the earth by a battery. Its point then sprays the lower part of the belt with positive charge, which is carried up into the sphere. There it induces a negative charge on the points of electrode E_2 and a positive charge on the sphere to which the blunt end of E_2 is connected. The point sprays the belt with negative charge, and discharges it before it passes over the pulley. The sphere gradually charges up positively, until its potential is about a million volts relative to the earth.

Large machines of this type are used with high-voltage X-ray tubes, and for atom-splitting experiments. They have more elaborate electrode systems, stand about 15 m high, and have 4 m spheres. They can produce potential differences up to 5 000 000 volts and currents of about 50 microamperes. The electrical energy which they deliver comes from the work done by the motor in drawing the positively charges belt towards the positively charged sphere, which repels it.

In all types of high-voltage equipment sharp corners and edges must be avoided, except where points are deliberately used as electrodes. Otherwise, corona discharges may break out from the sharp places. All such places are therefore enlarged by metal globes, these are called stress-distributors. See also p. 749.

FIG. 30.10. Van de Graaff Electrostatic Generator at Aldermaston, England. The dome is the high-voltage terminal. The insulated rings are equipotentials, and provide a uniform potential gradient down the column. Beams of protons or deuterons, produced in the dome, are accelerated down the column to bombard different materials at the bottom, thereby producing nuclear reactions which can be studied.

Ice-pail Experiment

A famous experiment on electrostatic induction was made by Faraday in 1843. In it he used the ice-pail from which it takes its name; but it was a modest pail, 27 cm high—not a bucket. He stood the pail on an insulator, and connected it to a gold-leaf electroscope, as in Fig. 30.11(i). He next held a metal ball on the end of a long silk thread, and charged it positively by a spark from an electrophorus. Then he lowered the ball into the pail, without letting it touch the sides or bottom (Fig. 30.11(ii)).

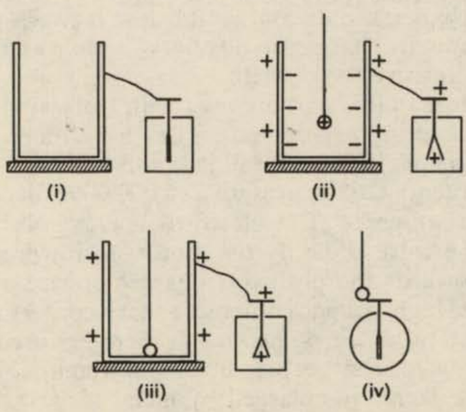


FIG. 30.11. Faraday's ice-pail experiment.

A positive charge was induced on the outside of the pail and the leaves, and made the leaves diverge. Once the ball was well inside the pail, Faraday found that the divergence of the leaves did not change when he moved the ball about—nearer to or farther from the walls or the bottom. This showed that the amount of the induced positive charge did not depend on the position of the ball, once it was well inside the pail. Faraday then allowed the ball to touch the pail, and noticed that the leaves of the electrocope still did not move (Fig. 30.11(iii)). When the ball touched the pail, therefore, no charge was given to, or taken from, the outside of the pail. Faraday next lifted the ball out of the pail, and tested it for charge with another electroscope. He found that the ball had no charge whatever (Fig. 30.11(iv)). The induced negative charge on the inside of the pail, must therefore have been equal in magnitude to the original positive charge on the ball.

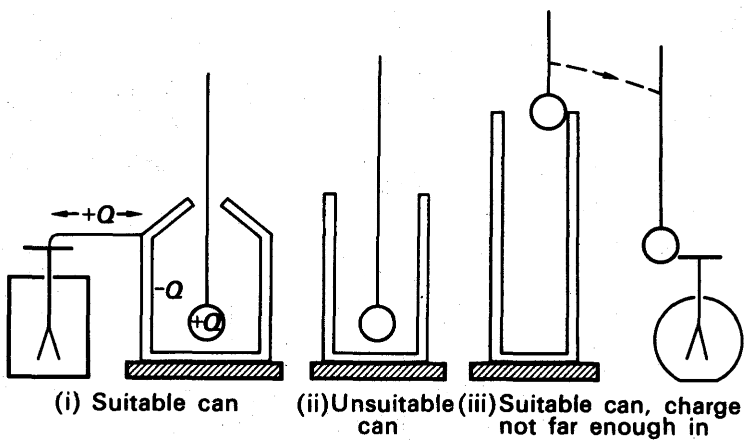


FIG. 30.12. Referring to Faraday's ice-pail experiment.

Faraday's experiment does not give these simple results unless the pail—or whatever is used in place of it—very nearly surrounds the charged ball (Fig. 30.12(i)(ii)). If, for example, the ball is allowed to touch the pail before it is well inside, as in Fig. 30.12(iii), then it does not lose all its charge.

Conclusions

The conclusions to be drawn from the experiment therefore apply, strictly, to a *hollow closed conductor*. They are:

(*i*) When a charged body is enclosed in a hollow conductor it induces on the inside of that conductor a charge equal but opposite to its own; and on the outside a charge equal and similar to its own (Fig. 30.11(i)).

(*ii*) The *total* charge inside a hollow conductor is always zero: either there are equal and opposite charges on the inside walls and within the volume (before the ball touches), or there is no charge at all (after the ball has touched).

Comparison and Collection of Charges

Faraday's ice-pail experiment gives us a method of comparing quantities of electric charges. The experiment shows that if a charged body is lowered well inside a tall, narrow can then it gives to the outside of the can a charge equal to its own. If the can is connected to the cap of an electroscope, the divergence of the leaves is a measure of the charge on the body. Thus we can compare the magnitudes of charges, without removing them from the bodies which carry them: we merely lower those bodies, in turn, into a tall insulated can, connected to an electroscope

Sometimes we may wish to discharge a conductor completely, without letting its charge run to earth. We can do this by letting the conductor touch the bottom of a tall can on an insulating stand. The whole of the body's charge is then transferred to the outside of the can.

Charges Produced by Separation; Lines of Force

The ice-pail experiment suggests that a positive electric charge, for example, is always accompanied by an equal negative charge. Faraday repeated his experiment with a nest of hollow conductors, insulated

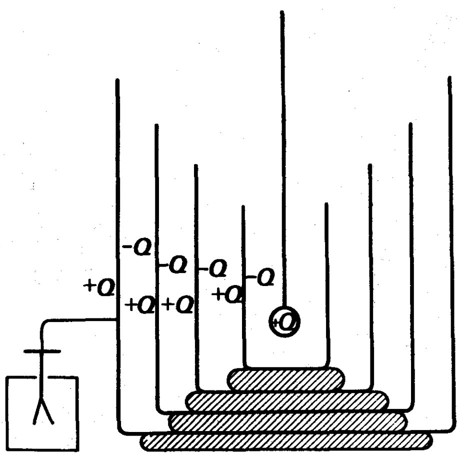


FIG. 30.13. Extension of the ice-pail experiment.

from one another, and showed that equal and opposite charges were induced on the inner and outer walls of each (Fig. 30.13).

Faraday also showed that equal and opposite charges are produced when a body if electrified by rubbing. He fitted an ebonite rod with a fur cap, which he rotated by a silk thread or string wrapped round it (Fig. 30.14(i)); he then compared the charges produced with an ice-pail and electroscope (Fig. 30.14(i)(ii)(iii)(iv)).

In describing the conclusions from this last experiment, we now say, as indeed we have done already, that electrons flow from the fur to the

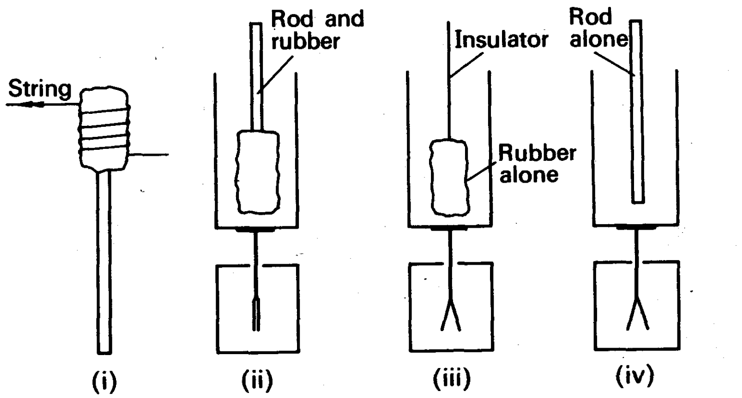


FIG. 30.14. Stages in showing that equal and opposite charges are produced by friction.

ebonite, carrying to it a negative charge, and leaving on the fur a positive charge. It appears, therefore, that free charges are always produced by separating equal amounts of the opposite kinds of electricity.

The idea that charges always occur in equal opposite pairs affects our drawing of lines of force diagrams. Lines of force radiate outwards

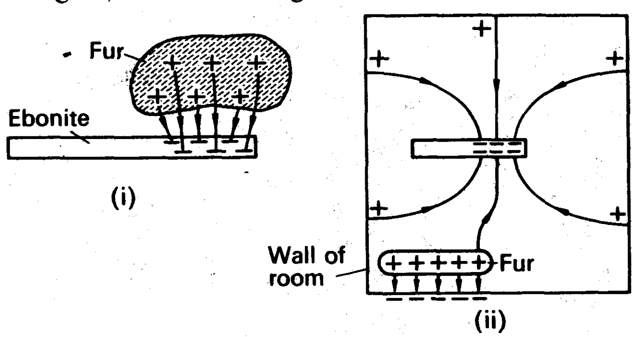


FIG. 30.15. Charging by friction—lines of force.

from a positive charge, and inwards to a negative one; from any positive charge, therefore, we draw lines of force ending on an equal negative charge. Figs. 30.15, 30.16 give some illustrations of this procedure.

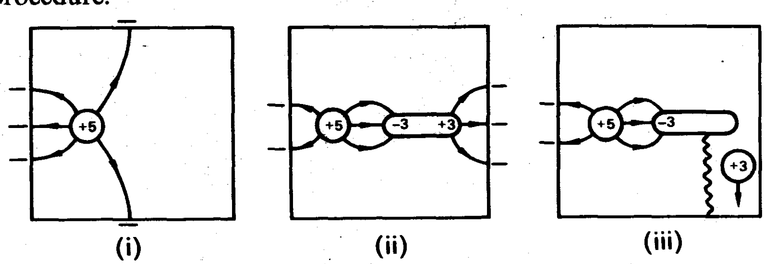


FIG. 30.16. Charging by induction—lines of force.

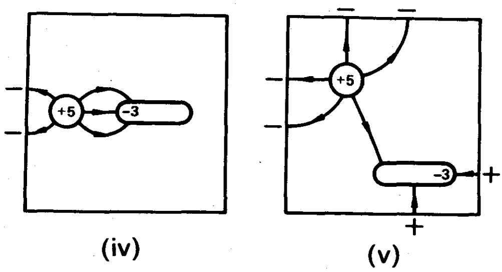


FIG. 30.16. Charging by induction—lines of force.

Distribution of Charge; Surface Density

By using a can connected to an electroscope we can find how electricity is distributed over a charged conductor of any form—pear-shaped, for example. We take a number of small leaves of tin-foil, all of the same area, but differently shaped to fit closely over the different parts of the conductor, and mounted on ebonite handles (Fig. 30.17(i)).

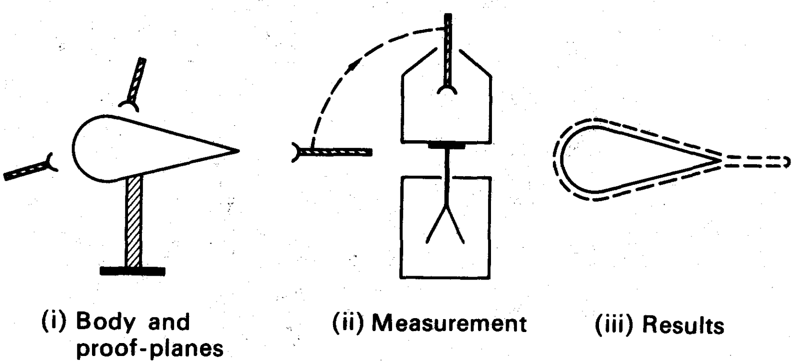


FIG. 30.17. Investigating charge distribution.

These are called *proof-planes*. We charge the body from an electrophorus, press a proof-plane against the part which it fits, and then lower the proof-plane into a can connected to an electroscope (Fig. 30.17(ii)). After noting the divergence of the leaves we discharge the can and electroscope by touching one of them, and repeat the observation with a proof-plane fitting a different part of the body. Since the proof-planes have equal areas, each of them carries away a charge proportional to the charge per unit area of the body, over the region which it touched. The charge per unit area over a region of the body is called the *surface-density* of the charge in that region. We find that the surface-density increases with the curvature of the body, as shown in Fig. 30.17(iii); the distance of the dotted line from the outline of the body is proportional to the surface-density of charge.

THE ELECTROSTATIC FIELD

Law of Force between two Charges

The magnitude of the force between two electrically charged bodies was studied by Coulomb in 1875. He showed that, if the bodies were small compared with the distance between them, then the force F was inversely proportional to the square of the distance r, i.e.

$$F \propto \frac{1}{r^2}. \qquad (1)$$

This result is known as the *inverse square law*, or Coulomb's law.

It is not possible to verify the law accurately by direct measurement of the force between two charged bodies. In 1936 Plimton and Lawton showed, by an indirect method, that the power in the law cannot differ from 2 by more than $\pm 2 \times 10^{-9}$. We have no reason to suppose, therefore, that the inverse square law is other than exactly true.

Quantity of Charge

The SI unit of charge is the *coulomb* (C). The *ampere* (A), the unit of current, is defined later (p. 939). The coulomb is defined as that quantity of charge which passes a section of a conductor in one second when the current flowing is one ampère.

By measuring the force F between two charges when their respective magnitudes Q and Q' are varied, it is found that F is proportional to the product QQ'. Thus

$$F \propto QQ' \qquad (2)$$

Law of Force

Combining (1) and (2), we have

$$F \propto \frac{QQ'}{r^2}$$

$$\therefore F = k\frac{QQ'}{r^2}, \qquad (3)$$

where k is a constant. For reasons explained later, k is written as $1/4\pi\varepsilon_0$, where ε_0 is a constant called the *permittivity* of free space if we suppose the charges are situated in a vacuum. Thus

$$F = \frac{1}{4\pi\varepsilon_0}\frac{QQ'}{r^2} \qquad (4)$$

In this expression, F is measured in newtons (N), Q in coulombs (C) and r in metres (m). Now, from (4),

$$\varepsilon_0 = \frac{QQ'}{4\pi F r^2}.$$

Hence the units of ε_0 are coulomb^2 newton^{-1} metre^{-2} ($C^2 N^{-1} m^{-2}$). Another unit of ε_0, more widely used, is *farad metre*$^{-1}$ ($F m^{-1}$). See p. 774.

We shall see later that ε_0 has the numerical value of $8{\cdot}854 \times 10^{-12}$, and $1/4\pi\varepsilon_0$ then has the value 9×10^9 approximately.

Permittivity

So far we have considered charges in a vacuum. If charges are situated in other media such as water, then the force between the charges is reduced. Equation (4) is true only in a vacuum. In general, we write

$$F = \frac{1}{4\pi\varepsilon}\frac{QQ'}{r^2} \quad . \quad . \quad . \quad . \quad (1)$$

where ε is the *permittivity* of the medium. The permittivity of air at normal pressure is only about 1·005 times that, ε_0, of a vacuum. For most purposes, therefore, we may assume the value of ε_0 for the permittivity of air. The permittivity of water is about eighty times that of a vacuum. Thus the force between charges situated in water is eighty times less than if they were situated the same distance apart in a vacuum.

EXAMPLE

(a) Calculate the value of two equal charges if they repel one another with a force of 0·1 N when situated 50 cm apart in a vacuum.

(b) What would be the size of the charges if they were situated in an insulating liquid whose permittivity was ten times that of a vacuum?

(a) From (4),

$$F = \frac{1}{4\pi\varepsilon_0}\frac{QQ'}{r^2}.$$

Since $Q = Q'$ here,

$$0{\cdot}1 = \frac{9 \times 10^9\, Q^2}{(0{\cdot}5)^2}$$

or

$$Q^2 = \frac{0{\cdot}1 \times (0{\cdot}5)^2}{9 \times 10^9}$$

$$Q = 1{\cdot}7 \times 10^{-6}\ \text{C (coulomb), approx.}$$

$$= 1{\cdot}7\ \mu\text{C (microcoulomb).}$$

(b) The permittivity of the liquid $\varepsilon = 10\,\varepsilon_0$.

$$F = \frac{1}{4\pi\varepsilon}\frac{QQ'}{r^2}$$

$$= \frac{1}{10(4\pi\varepsilon_0)}\frac{Q^2}{r^2}$$

$$Q^2 = \frac{(0{\cdot}1) \times (0{\cdot}5)^2 \times 10}{9 \times 10^9}$$

$$Q = 5{\cdot}3 \times 10^{-6}\ \text{C} = 5{\cdot}3\ \mu\text{C}.$$

Electric Intensity or Field-strength. Lines of Force

An 'electric field' can be defined as a region where an electric force is experienced. As in magnetism, electric fields can be mapped out by electrostatic lines of force, which may be defined as a line such that the tangent to it is in the direction of the force on a small positive charge at that point. Arrows on the lines of force show the direction of the force on a positive charge; the force on a negative charge is in the opposite direction. Fig 30.18 shows the lines of force, also called *electric flux*, in some electrostatic fields.

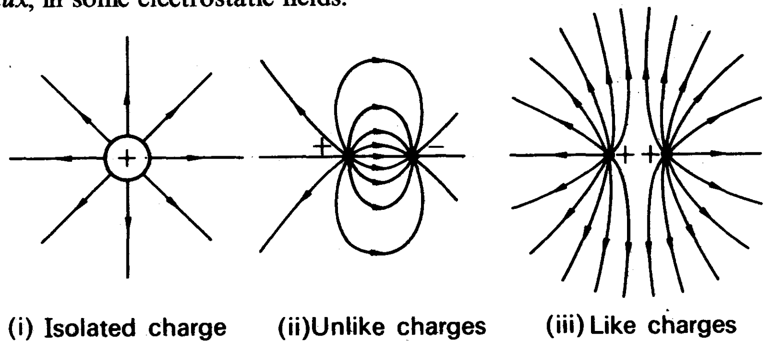


FIG. 30.18. Lines of electrostatic force.

The force exerted on a charged body in an electric field depends on the charge of the body and on the *intensity* or *strength* of the field. If we wish to explore the variation in intensity of an electric field, then we must place a test charge Q' at the point concerned which is small enough not to upset the field by its introduction. The intensity E of an electrostatic field at any point is defined as *the force per unit charge* which it exerts at that point. Its direction is that of the force exerted on a positive charge.

From this definition,

$$E = \frac{F}{Q'}$$

$$F = EQ' \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Since F is measured in newtons and Q' in coulombs, it follows that *intensity E has units of newton per coulomb* ($N\ C^{-1}$). We shall see later that a more practical unit of E is *volt metre*$^{-1}$ ($V\ m^{-1}$) (see p. 755).

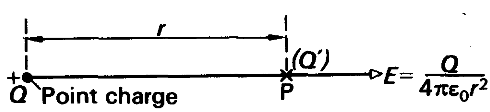


FIG. 30.19. Electric field intensity due to point charge.

We can easily find an expression for the strength E of the electric field due to a point charge Q situated in a vacuum (Fig. 30.19). We start from equation (4), p. 743, for the force between two such charges:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{QQ'}{r^2}$$

If the test charge Q' is situated at the point P in Fig. 30.19, the electric field strength at that point is given by (1).

$$\therefore E = \frac{F}{Q'} = \frac{Q}{4\pi\varepsilon_0 r^2} \quad . \quad . \quad . \quad (2)$$

The direction of the field is radially outward if the charge Q is positive (Fig. 30.18(i)); it is radially inward if the charge Q is negative. If the charge were surrounded by a material of permittivity ε then,

$$E = \frac{Q}{4\pi\varepsilon r^2} \quad . \quad . \quad . \quad . \quad (3)$$

Flux from a Point Charge

We have already shown how electric fields can be described by lines of force. From Fig. 30.18(i) it can be seen that the density of the lines increases near the charge where the field intensity is high. The intensity E at a point can thus be represented by *the number of lines per unit area* through a surface perpendicular to the lines of force at the point considered. The *flux* through an area perpendicular to the lines of force is the name given to the product of $E \times$ *area*, where E is the intensity at that place. This is illustrated in Fig. 30.20(i).

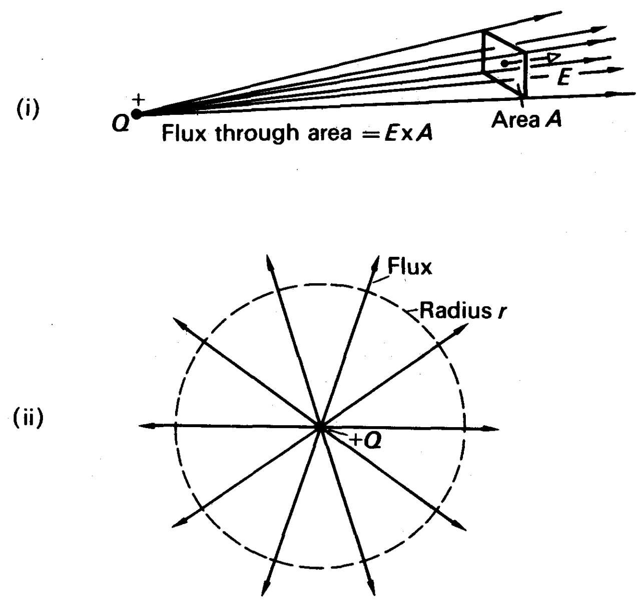


FIG. 30.20. Flux from a point charge.

Consider a sphere of radius r drawn in space concentric with a point charge (Fig. 30.20(ii)). The value of E at this place is given by (3),

p. 746. The total flux through the sphere is,

$$E \times \text{area} = E \times 4\pi r^2$$

$$= \frac{Q}{4\pi\varepsilon r^2} \times 4\pi r^2$$

$$= \frac{Q}{\varepsilon}$$

$$= \frac{\text{charge inside sphere}}{\text{permittivity}} \quad . \quad . \quad (1)$$

This demonstrates the important fact that the total flux crossing any sphere drawn outside and concentrically around a point charge is a constant. It does not depend on the distance from the charged sphere. It should be noted that this result is only true if the inverse square law is true.

To see this, suppose some other force law were valid, i.e. $E = Q/4\pi\varepsilon r^n$. Then the total flux through the area

$$= \frac{Q}{4\pi\varepsilon r^n} \times 4\pi r^2$$

$$= \frac{Q}{\varepsilon} r^{(2-n)}$$

This is only independent of r if $n = 2$.

Field due to Charged Sphere and Plane Conductor

Equation (1) can be shown to be generally true. Thus the flux passing through any *closed* surface whatever its shape, is always equal to Q/ε, where Q is the total charge enclosed by the surface. This relation, called *Gauss's Theorem*, can be used to find the value of E in other common cases.

(1) *Outside a charged sphere*

The flux across a spherical surface of radius r, concentric with a small sphere carrying a charge Q (Fig. 30.21), is given by,

$$\text{Flux} = \frac{Q}{\varepsilon}$$

$$\therefore E \times 4\pi r^2 = \frac{Q}{\varepsilon}$$

$$\therefore E = \frac{Q}{4\pi\varepsilon r^2}$$

This is the same answer as that for a point charge. This means that *outside* a charged sphere, the field behaves as if all the charge on the sphere were concentrated at the centre.

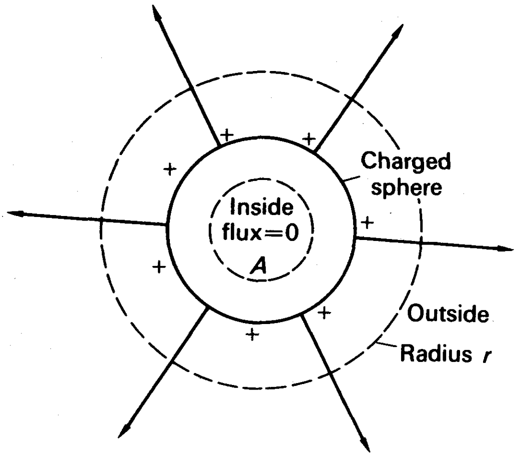


FIG. 30.21. Electric field of a charged sphere.

(2) *Inside a charged empty sphere*

Suppose a spherical surface A is drawn *inside* a charge sphere, as shown in Fig. 30.21. Inside this sphere there are no charges and so Q in equation (1), p. 747, is zero. This result is independent of the radius drawn, provided that it is less than that of the charged sphere. Hence from (1), p. 747, *E must be zero everywhere inside a charged sphere.*

(3) *Outside a Charged Plane Conductor*

Now consider a charged *plane* conductor S, with a surface charge density of σ coulomb metre^{-2}. Fig. 30.22 shows a plane surface P, drawn outside S, which is parallel to S and has an area A metre^2. Applying equation (1),

$$\therefore\ E \times \text{area} = \frac{\text{Charge inside surface}}{\varepsilon}$$

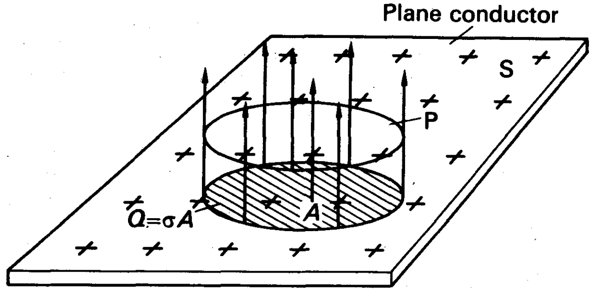


FIG. 30.22. Field of a charged plane conductor.

Now by symmetry, the intensity in the field must be perpendicular to the surface. Further, the charges which produce this field are those in the projection of the area P on the surface S, i.e. those within the shaded area A in Fig. 30.22. The total charge here is thus σA coulomb.

$$\therefore E.A = \frac{\sigma A}{\varepsilon}$$

$$\therefore E = \frac{\sigma}{\varepsilon}$$

Field Round Points

On p. 742 we saw that the surface-density of charge (charge per unit area) round a point of a conductor is very great. Consequently, the strength of the electric field near the point is very great. The intense electric field breaks down the insulation of the air, and sends a stream of charged molecules away from the point. The mechanism of the breakdown, which is called a 'corona discharge', is complicated, and we shall not discuss it here; some of the processes in it are similar to those in conduction through a gas at low pressure, which we shall describe in Chapter 40. Corona breakdown starts when the electric field strength is about 3 million volt metre^{-1}. The corresponding surface-density is about $2{\cdot}7 \times 10^{-5}$ coulomb metre^{-2}.

EXAMPLE

An electron of charge $1{\cdot}6 \times 10^{-19}$ C is situated in a uniform electric field of intensity 1200 volt cm^{-1}. Find the force on it, its acceleration, and the time it takes to travel 2 cm from rest (electronic mass, m, = $9{\cdot}10 \times 10^{-31}$ kg).

Force on electron $F = eE$.
Now E = 1200 volt cm^{-1} = 120 000 volt m^{-1}.

$$\therefore \quad F = 1{\cdot}6 \times 10^{-19} \times 1{\cdot}2 \times 10^{5}$$

$$= 1{\cdot}92 \times 10^{-14} \text{ N (newton).}$$

Acceleration,
$$a = \frac{F}{m} = \frac{1{\cdot}92 \times 10^{-14}}{9{\cdot}1 \times 10^{-31}}$$

$$= 2{\cdot}12 \times 10^{16} \text{ m s}^{-2} \text{ (metre second}^{-2}\text{)}$$

Time for 2 cm travel is given by

$$s = \tfrac{1}{2}at^2$$

$$\therefore \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 0{\cdot}02}{2{\cdot}12 \times 10^{16}}}$$

$$= 1{\cdot}37 \times 10^{-9} \text{ seconds.}$$

The extreme shortness of this time is due to the fact that the ratio of charge-to-mass for an electron is very great:

$$\frac{e}{m} = \frac{1{\cdot}6 \times 10^{-19}}{9{\cdot}1 \times 10^{-31}} = 1{\cdot}8 \times 10^{11} \text{ C kg}^{-1}.$$

In an electric field, the charge e determines the force on an electron, while the mass m determines its inertia. Because of the large ratio e/m, the electron moves almost instantaneously, and requires very little energy to displace it. Also it can respond to changes in an electric field which take place even millions of times per second. Thus it is the large value of e/m for electrons which makes electronic tubes, for example, useful in electrical communication and remote control.

ELECTRIC POTENTIAL

Potential in Fields

When an object is held at a height above the earth it is said to have potential energy. A heavy body tends to move under the force of attraction of the earth from a point of great height to one of less, and we say that points in the earth's gravitational field have potential values depending on their height.

Electric potential is analogous to gravitational potential, but this time we think of points in an electric field. Thus in the field round a positive charge, for example, a positive charge moves from points near the charge to points further away. Points round the charge are said to have an 'electric potential'.

Potential Difference

In mechanics we are always concerned with differences of height; if a point A on a hill is h metre higher than a point B, and our weight is w newton, then we do wh joule of work in climbing from B to A (Fig. 30.23 (i)). Similarly in electricity we are often concerned with differences of potential; and we define these also in terms of work.

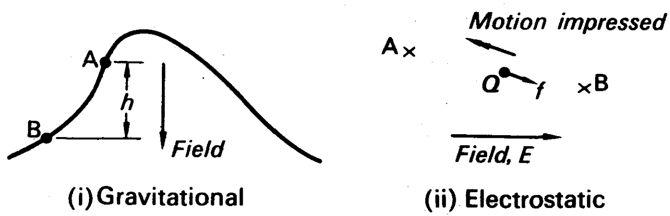


FIG. 30.23. Work done, in gravitational and electrostatic fields.

Let us consider two points A and B in an electrostatic field, and let us suppose that the force on a positive charge Q has a component f in the direction AB (Fig. 30.23 (ii)). Then if we move a positively charged body from B to A, we do work against this component of the field E. *We define the potential difference between A and B as the work done in moving a unit positive charge from B to A.* We denote it by the symbol V_{AB}.

The work done will be measured in joules (J). The unit of potential difference is called the volt and may be defined as follows: *The potential difference between two points A and B is one volt if the work done in taking one coulomb of positive charge from B to A is one joule.*

From this definition, if a charge of Q coulombs is moved through a p.d. of V volt, then the work done W in joules is given by

$$W = QV \quad \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Potential and Energy

Let us consider two points A and B in an electrostatic field, A being at a higher potential than B. The potential difference between A and

B we denote as usual by V_{AB}. If we take a positive charge Q from B to A, we do work on it of amount QV_{AB}: the charge gains this amount of potential energy. If we now let the charge go back from A to B, it loses that potential energy: work is done on it by the electrostatic force, in the same way as work is done on a falling stone by gravity. This work may become kinetic energy, if the charge moves freely, or external work if the charge is attached to some machine, or a mixture of the two.

The work which we must do in first taking the charge from B to A does not depend on the path along which we carry it, just as the work done in climbing a hill does not depend on the route we take. If this were not true, we could devise a perpetual motion machine, in which we did less work in carrying a charge from B to A via X than it did for us in returning from A to B via Y (Fig. 30.24).

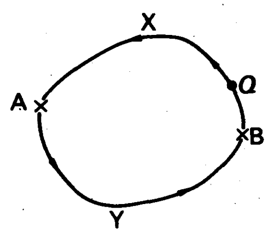


FIG. 30.24. A closed path in an electrostatic field.

The fact that the potential differences between two points is a constant, independent of the path chosen between the points, is the most important property of potential in general; we shall see why later on. This property can be conveniently expressed by saying that the work done in carrying a charge round a closed path in an electrostatic field, such as BXAYB in Fig. 30.24 is zero.

Potential Difference Formula

To obtain a formula for potential difference, let us calculate the potential difference between two points in the field of a single point positive charge, Q in Fig. 30.25. For simplicity we will assume that the points, A and B, lie on a line of force at distances a and b respectively

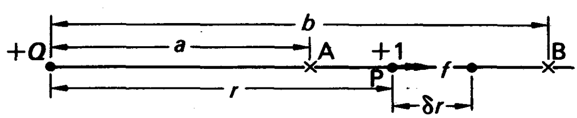


FIG. 30.25. Calculation of potential.

from the charge. When a unit positive charge is at a distance r from the charge Q in free space the force on it is

$$f = \frac{Q \times 1}{4\pi\varepsilon_0 r^2}$$

The work done in taking the charge from B to A, against the force f, is equal to the work which the force f would do if the charge were allowed to go from A to B. Over the short distance δr, the work done by the force f is

$$\delta W = f\delta r.$$

Over the whole distance AB, therefore, the work done by the force on the unit charge is

$$\int_A^B \delta W = \int_{r=a}^{r=b} f dr = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$= -\left[\frac{Q}{4\pi\varepsilon_0 r}\right]_a^b = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$$

This, then, is the value of the work which an external agent must do to carry a unit positive charge from B to A. The work per coulomb is the potential difference V_{AB} between A and B.

$$\therefore V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) \quad . \quad . \quad . \quad (1)$$

V_{AB} will be in volts if Q is in coulombs, a and b are in metres and ε_0 is taken as $8{\cdot}85 \times 10^{-12}$ or $1/4\pi\varepsilon_0$ as 9×10^9 approximately (see p. 744).

EXAMPLE

Two positive point charges, of 12 and 8 microcoulomb respectively, are 10 cm apart. Find the work done in bringing them 4 cm closer. (Assume $1/4\pi\varepsilon_0 = 9 \times 10^9$.)

Suppose the 12 μC (microcoulomb) charge is fixed in position. Since 6 cm = 0·06 m and 10 cm = 0·1 m, then the potential difference between points 6 and 10 cm from it is given by (1).

$$\therefore \quad V = \frac{12 \times 10^{-6}}{4\pi\varepsilon_0}\left(\frac{1}{0{\cdot}06} - \frac{1}{0{\cdot}1}\right)$$

$$= 12 \times 10^{-6} \times 9 \times 10^9 \,(16\tfrac{2}{3} - 10)$$

$$= 720\,000 \text{ V}.$$

(Note the very high potential difference due to quite small charges.)

The work done in moving the 8 μC charge from 10 cm to 6 cm away from the other is given by, using $W = QV$,

$$W = 8 \times 10^6 \times V$$

$$= 8 \times 10^{-6} \times 720\,000$$

$$= 5{\cdot}8 \text{ J}.$$

Zero Potential

Instead of speaking continually of potential differences between pairs of points, we may speak of the potential at a single point—provided we always refer it to some other, agreed, reference point. This procedure is analogous to referring the heights of mountains to sea-level.

For practical purposes we generally choose as our reference point the electric potential of the surface of the earth. Although the earth is large it is all at the same potential, because it is a good conductor of electricity; if one point on it were at a higher potential than another, electrons would flow from the lower to the higher potential. As a result, the higher potential would fall, and the lower would rise; the flow of electricity would cease only when the potentials became equalized.

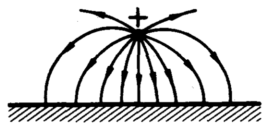


FIG. 30.26. Electric field of positive charge near earth.

In general it is difficult to calculate the potential of a point relative to the earth. This is because the electric field due to a charged body near a conducting surface is complicated, as shown by the lines of force diagram in Fig. 30.26. In theoretical calculations, therefore, we often find it convenient to consider charges so far from the earth that the effect of the earth on their field is negligible; we call these 'isolated' charges.

Thus we define the potential at a point A as V volts if V joules of work is done in bringing one coulomb of positive charge from infinity to A.

Potential Formula

Equation (1), p. 752, gives the potential difference between two points in the field of an isolated point charge Q. If we let the point B retreat to infinity, then $b \gg a$, and the equation gives for the potential at A:

$$V_A = \frac{Q}{4\pi\varepsilon_0 a} \quad . \quad . \quad . \quad . \quad (1)$$

The derivation of this equation shows us what we mean by the word 'infinity': the distance b is infinite if $1/b$ is negligible compared with $1/a$. If a is 1 cm, and b is 1 m, we make an error of only 1 per cent. in ignoring it; if b is 100 m, then for all practical purposes the point B is at infinity. In atomic physics, where the distances concerned have the order of 10^{-8} cm, a fraction of a millimetre is infinite.

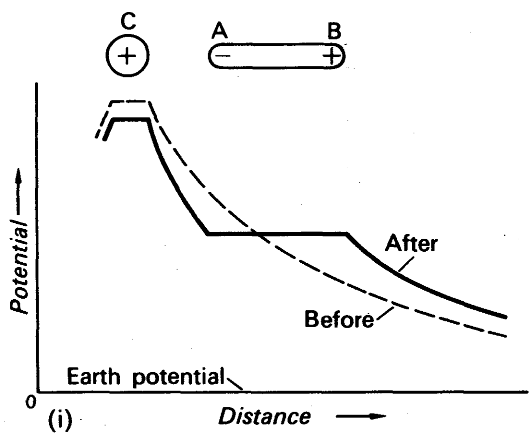


FIG. 30.27 (i). Potential distribution near a positive charge before and after bringing up an uncharged conductor.

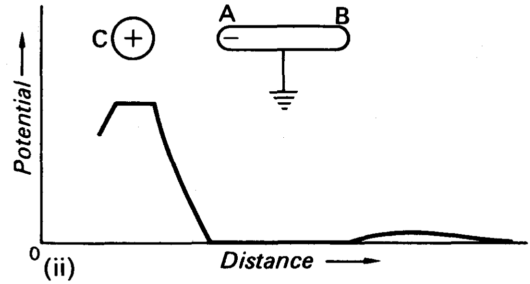


FIG. 30.27 (ii). Potential distribution near a positive charge in the presence of an earthed conductor.

In the neighbourhood of an isolated negative charge, the potential is negative, because Q in equation (1) is negative. The potential is also negative in the neighbourhood of a negative charge near the earth: the earth is at zero potential, and a positive charge will tend to move from it towards the negative charge. A negative potential is analogous to the depth of a mine below sea-level. Fig. 30.27 (i) shows the potential variation near a positive charge C before and after a conductor AB is brought near. Fig. 30.27 (ii) shows the potential variation when AB is earthed.

Potential Difference and Intensity

We shall now see how potential difference is related to intensity or field-strength. Suppose A, B are two neighbouring points on a line of force, so close together that the electric field-intensity between them is constant and equal to E (Fig. 30.28). If V is the potential at A, $V+\delta V$ is that at B, and the respective distances of A, B from the origin are x and $x+\delta x$, then

$$V_{AB} = \text{potential difference between A, B}$$
$$= V_A - V_B = V-(V+\delta V) = -\delta V.$$

The work done in taking a unit charge from B to A

$$= \text{force} \times \text{distance} = E \times \delta x = V_{AB} = -\delta V.$$

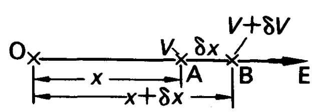


FIG. 30.28. Field strength and potential gradient.

Hence $$E = -\frac{\delta V}{\delta x},$$

or, in the limit,

$$E = -\frac{dV}{dx}. \quad . \quad . \quad . \quad . \quad (1)$$

The quantity dV/dx is the rate at which the potential rises with distance, and is called the potential gradient. Equation (1) shows that the strength of the electric field is equal to the negative of the potential gradient, and strong and weak fields in relation to potential are illustrated in Fig. 30.29.

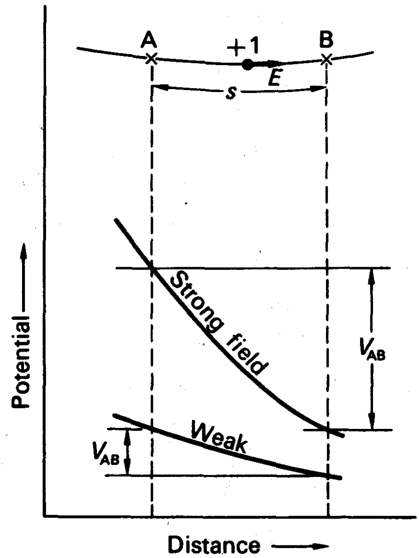


FIG. 30.29. Relationship between potential and field strength.

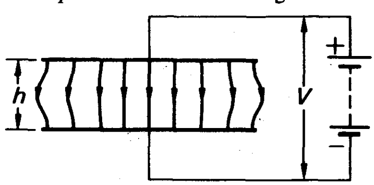


FIG. 30.30. Electric field between parallel plates.

In Fig. 30.30 the electric intensity $= V/h$, the potential gradient, and this is uniform in magnitude in the middle of the plates. At the edge of the plates the field becomes non-uniform.

We can now see why E is usually given in units of 'volt per metre' ($V\,m^{-1}$).

From (1), $E = -(dV/dx)$. Since V is measured in volts and x in metres, then E will be in volt per metre ($V\,m^{-1}$). From the original definition of E, summarized by equation (1) on p. 745, the units of E were newton coulomb^{-1}. To show that these are equivalent, from (1),

$$1 \text{ volt} = 1 \text{ joule coulomb}^{-1}$$
$$= 1 \text{ newton metre coulomb}^{-1}$$

Since
$$1 \text{ joule} = 1 \text{ newton} \times 1 \text{ metre}$$
$$\therefore 1 \text{ volt metre}^{-1} = 1 \text{ newton coulomb}^{-1}$$

EXAMPLES

1 An electron is liberated from the lower of two large parallel metal plates separated by a distance $h = 2$ cm. The upper plate has a potential of 2400 volts relative to the lower. How long does the electron take to reach it?

Between large parallel plates, close together, the electric field is uniform except near the edges of the plates, as shown in Fig. 30.30. Except near the edges, therefore, the potential gradient between the plates is uniform; its magnitude is V/h,

$$\text{electric intensity } E = \text{potential gradient}$$
$$= 2400/0{\cdot}02 \text{ V m}^{-1}$$
$$= 1{\cdot}2 \times 10^5 \text{ V m}^{-1}.$$

The rest of this problem may now be worked out exactly as the example on p. 749.

2 An electron is liberated from a hot filament, and attracted by an anode, of potential 1200 volts positive with respect to the filament. What is the speed of the electron when it strikes the anode?

$$e = \text{electronic charge} = 1{\cdot}6 \times 10^{-19} \text{ C}.$$
$$V = 1200 \text{ V}, m = \text{mass of electron} = 9{\cdot}1 \times 10^{-31} \text{ kg}.$$

The energy which the electron gains from the field $= QV = eV$.

Kinetic energy gained

$$= \tfrac{1}{2}mv^2 = eV,$$

where v is the speed gained from rest.

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1{\cdot}6 \times 10^{-19} \times 1200}{9{\cdot}1 \times 10^{-31}}}$$
$$= 2{\cdot}1 \times 10^7 \text{ m s}^{-1}.$$

The Electron-Volt

The kinetic energy gained by an electron which has been accelerated through a potential difference of 1 volt is called an *electron-volt* (eV). Since the energy gained in moving a charge Q through a p.d. $V = QV$,

$$\therefore 1 \text{ eV} = \text{electronic charge} \times 1 = 1{\cdot}6 \times 10^{-19} \times 1 \text{ joule} = 1{\cdot}6 \times 10^{-19} \text{ J}.$$

The electron-volt is a useful unit of energy in atomic physics. For example, the work necessary to extract a conduction electron from tungsten is 4·52 electron-volt. This quantity determines the magnitude of the thermionic emission from the metal at a given temperature (p. 1026); it is analogous to the latent heat of evaporation of a liquid.

EQUIPOTENTIALS

Equipotentials

We have already said that the earth must have the same potential all over, because it is a conductor. In a conductor there can be no differences of potential, because these would set up a potential gradient or electric field; electrons would then redistribute themselves throughout the conductor, under the influence of the field, until they had destroyed the field. This is true whether the conductor has a net charge, positive or negative, or whether it is uncharged; it is true whatever the actual potential of the conductor, relative to any other body.

Any surface or volume over which the potential is constant is called an *equipotential*. The volume or surface may be that of a material body, or simply a surface or volume in space. For example, as we shall see later, the space inside a hollow charged conductor is an equipotential volume. Equipotential surfaces can be drawn throughout any space in which there is an electric field, as we shall now explain.

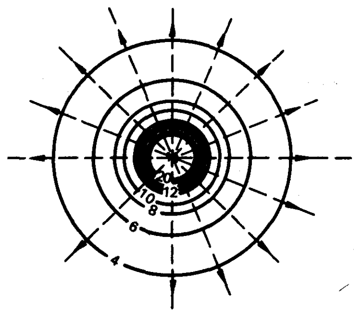


FIG. 30.31. Equipotentials and lines of force around a point charge.

Let us consider the field of an isolated point-charge Q. At a distance a from the charge, the potential is $Q/4\pi\varepsilon_0 a$; a sphere of radius a and centre at Q is therefore an equipotential surface, of potential $Q/4\pi\varepsilon_0 a$. In fact, all spheres centred on the charge are equipotential surfaces, whose potentials are inversely proportional to their radii (Fig. 30.31). An equipotential surface has the property that, along any direction lying in the surface, there is no electric field; for there is no potential gradient. *Equipotential surfaces are therefore always at right angles to lines of force*, as shown in Fig. 30.31. This also shows numerical values proportional to their potentials. Since conductors are always equipotentials, if any conductors appear in an electric-field diagram the lines of force must always be drawn to meet them at right angles.

Potential due to a System of Charges

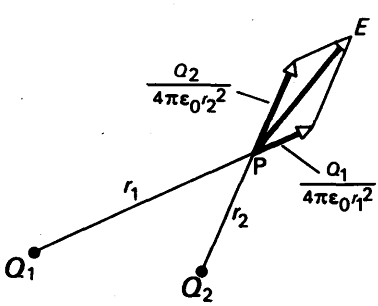
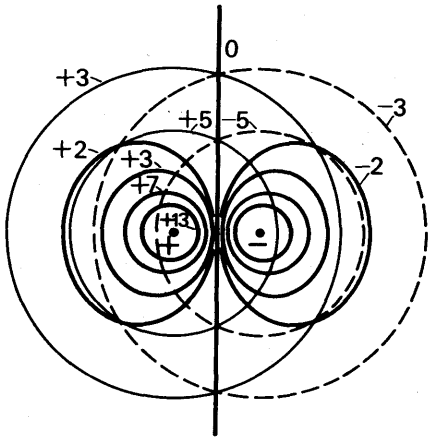


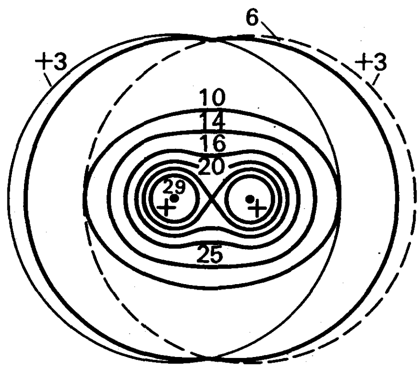
FIG. 30.32. Finding resultant field of two point-charges.

When we set out to consider the electric field due to more charges than one, then we see the advantages of the idea of potential over the idea of field-strength. If we wish to find the field-strength E at the point P in Fig. 30.32, due to the two charges Q_1 and Q_2, we have first to find the force exerted by each on a unit charge at P, and then to compound these forces by the parallelogram method. See Fig. 30.32. On the other hand, if we wish to find the potential at P, we merely calculate the potential due to each charge, and *add the potentials algebraically.*

Quantities which can be added algebraically are called 'scalars'; they may have signs—positive or negative, like a bank balance— but



(i) Opposite charges



(ii) Like charges

FIG. 30.33. Equipotentials in the field of two point charges.

they have no direction: they do not point north, east, south, or west. Quantities which have direction, like forces, are called 'vectors'; they have to be added either by resolution into components, or by the parallelogram method. Either way is slow and clumsy, compared with the addition of scalars. For example, we can draw the equipotentials round a point-charge with compasses; if we draw two sets of them, as in Fig. 30.33 (i) or (ii), then by simple addition we can rapidly sketch the equipotentials around the two charges together.

And when we have plotted the equipotentials, they turn out to be more useful than lines of force. A line of force diagram appeals to the imagination, and helps us to see what would happen to a charge in the field. But it tells us little about the strength of the field—at the best, if it is more carefully drawn than most, we can only say that the field is strongest where the lines are closest. But equipotentials can be labelled with the values of potential they represent; and from their spacing we can find the actual value of the potential gradient, and hence the field-strength. The only difficulty in interpreting equipotential diagrams lies in visualizing the direction of the force on a charge; this is always at right angles to the curves.

Field inside Hollow Conductor. Potential Difference and Gold-leaf Electroscope

If a hollow conductor contains no charged bodies, then, whatever charge there may be on its outside, there is none on its inside. Inside it, therefore, there is no electric field; the space within the conductor is an equipotential volume. If the conductor has an open end, like a can, then most of the space inside it is equipotential, but near its mouth there is a weak field (Fig. 30.34).

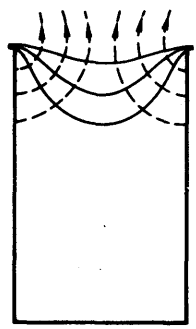


FIG. 30.34. Equipotentials and lines of force near mouth of an open charged can.

The behaviour of the *gold-leaf electroscope* illustrates this point. If we stand the case on an insulator, and connect the cap to it with a wire, then, no matter what charge we give to the cap, the leaves do not diverge (Fig. 30.35). Any charge we give to the cap spreads over the case

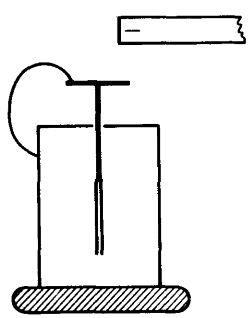


FIG. 30.35. Electroscope cap joined to case.

of the electroscope, but none appears on the leaves, and there is no force acting to diverge them. When, as usual, the cap is insulated and the case earthed, charging the cap sets up a potential difference between it and the case. Charges appear on the leaves, and the field between them and the case makes them diverge (p. 733). If the case is insulated from earth, as well as from the cap, the leaves diverge less; the charge on them and the cap raises the potential of the case and reduces the potential difference between it and the leaves. The field acting on the leaves is thus made weaker, and the force on the leaves less. We can sum up these observations by saying that *the electroscope indicates the potential difference between its leaves and its case.*

Potential of Pear-shaped Conductor

On p. 742 we saw that the surface-density of the charge on a pear-shaped conductor was greatest where the curvature was greatest. The potential of the conductor at various points can be examined by means of the gold-leaf electroscope, the case being earthed. One end of a wire is connected to the cap; some of the wire is then wrapped round an insulating rod, and the free end of the wire is placed on the conductor. As the free end is moved over the conductor, it is observed that the divergence of the leaf remains constant. This result was explained on pp. 756, 757.

Electrostatic Shielding

The fact that there is no electric field inside a close conductor, when it contains no charged bodies, was demonstrated by Faraday in a spectacular manner. He made for himself a large wire cage, supported it on insulators, and sat inside it with his electroscopes. He then had the cage charged by an induction machine—a forerunner of the type we described on p. 737—until painful sparks could be drawn from its outside. Inside the cage Faraday sat in safety and comfort, and there was no deflection to be seen on even his most sensitive electroscope.

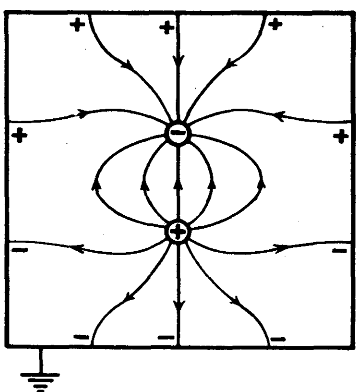


FIG. 30.36. Lines of force round charges.

If we wish to protect any persons or instruments from intense electric fields, therefore, we enclose them in hollow conductors; these are called 'Faraday cages', and are widely used in high-voltage measurements in industry.

We may also wish to prevent charges in one place from setting up an electric field beyond their immediate neighbourhood. To do this we surround the charges with a Faraday cage, and connect the cage to earth (Fig. 30.36). The charge induced on the outside of the cage then runs to earth, and there is no external field. (When a cage is used to shield something *inside* it, it does not have to be earthed.)

Comparison of Static and Current Phenomena

Broadly speaking, we may say that in electrostatic phenomena we meet small quantities of charge, but great differences of potential. On the other hand in the phenomena of current electricity discussed later, the potential differences are small but the amounts of charge transported by the currents are great. Sparks and shocks are common in electrostatics, because they require great potential differences; but they are rarely dangerous, because the total amount of energy available is usually small. On the other hand, shocks and sparks in current electricity are rare, but, when the potential difference is great enough to cause them, they are likely to be dangerous.

These quantitative differences make problems of insulation much more difficult in electrostatic apparatus than in apparatus for use with currents. The high potentials met in electrostatics make leakage currents relatively great, and the small charges therefore tend to disappear rapidly. Any wood, for example, ranks as an insulator for current electricity, but a conductor in electrostatics. In electrostatic experiments we sometimes wish to connect a charged body to earth; all we have then to do is to touch it.

EXAMPLE

Three charges $-1\ \mu C$, $2\ \mu C$ and $3\ \mu C$ are placed respectively at the corners A, B, C of an equilateral triangle of side 2 metres. Calculate (*a*) the potential, (*b*) the electric field, at a point X which is half-way along BC. Fig. 30.37.

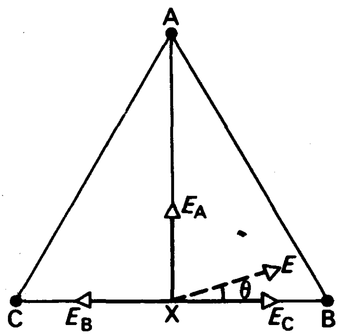


FIG. 30.37. Example.

(*a*) Potential at X due to charge at B is

$$\frac{Q}{4\pi\varepsilon_0 r} = \frac{2\times10^{-6}}{4\pi\varepsilon_0\times1}$$
$$= 18\times10^3\ \text{V}.$$

Similarly potential at X due to the charge at C $= 27\times10^3$ V, and the potential due to A is

$$V_A = \frac{Q}{4\pi\varepsilon_0 r} = \frac{-10^{-6}}{4\pi\varepsilon_0\times\sqrt{3}}$$
$$= -5\times10^3\ \text{V (approx.)}.$$

Since potential is a scalar quantity and can be added algebraically, the net potential at X

$$= (18+27-5)\times10^3\ \text{V}$$
$$= 40\times10^3\ \text{V}.$$

(*b*) The resultant field at X is due to the three electric fields from the three charges.

The field due to B, E_B has magnitude given by

$$E_B = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{2\times10^{-6}}{4\pi\varepsilon_0\times1^2}$$
$$= 18\times10^3\ \text{V m}^{-1}.$$

Similarly $\qquad E_C = 27\times10^3\ \text{V m}^{-1}.$

Since these act along the same straight line the resultant of E_B and $E_C = 9 \times 10^3$ V m^{-1} directed from C to B.

Also,
$$E_A = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{10^{-6}}{4\pi\varepsilon_0 \times (\sqrt{3})^2}$$
$$= 3 \times 10^3 \text{ V m}^{-1}.$$

The resultant field has magnitude, E, given by
$$E^2 = E_A^2 + (E_C - E_B)^2$$
$$= (9+81) \times 10^6$$
$$= 90 \times 10^6$$
$$E = 3\sqrt{10} \times 10^3 \text{ V m}^{-1} = 9{\cdot}5 \times 10^3 \text{ V m}^{-1}.$$

This makes an angle θ with CB, in the direction shown by the dotted line, where
$$\tan\theta = \frac{E_A}{E_C - E_B} = \frac{3 \times 10^3}{9 \times 10^3} = \frac{1}{3}$$
$$\therefore \quad \theta = 18^\circ\, 25'.$$

EXERCISES 30

1. Describe experiments with a gold leaf electroscope:
(*a*) to demonstrate that this instrument indicates the potential difference between its leaves and its case and not necessarily the total charge on its leaves and cap;
(*b*) to compare the quantities of electricity on two conductors of unequal size;
(*c*) to investigate the distribution of electricity on a charged conductor.
In case (*c*) state the results you would expect if the conductor were spherical with a pointed rod attached to it, and describe and explain two practical applications of pointed conductors. (*L.*)

2. Describe, with the aid of a labelled diagram, a Van de Graaff generator, explaining the physical principles of its action.

The high voltage terminal of such a generator consists of a spherical conducting shell of radius 50 cm. Estimate the maximum potential to which it can be raised in air for which electrical breakdown occurs when the electric intensity exceeds 30000 volt cm^{-1}.

State two ways in which this maximum potential could be increased. (*N.*)

3. Define *potential at a point* in an electric field.

Sketch a graph illustrating the variation of potential along a radius from the centre of a charged isolated conducting sphere to infinity.

Assuming the expression for the potential of a charged isolated conducting sphere in air, determine the change in the potential of such a sphere caused by surrounding it with an earthed concentric thin conducting sphere having three times its radius. (*N.*)

4. What is meant by the terms (*a*) *potential* and (*b*) *field strength* in electrostatics? State whether each quantity is a scalar or a vector.

Write down the law which gives the force between two point charges q_1 and q_2 at a distance r apart and use it to derive the electric field strength and the potential due to a point charge q at a distance x from it.

The points A, B and C form an equilateral triangle of side z. Point charges of equal magnitude q are placed at A and B. Find the electric field strength and the potential at C due to these charges when (i) both charges are positive and (ii) the charge at A is positive and the charge at B is negative.

O is the midpoint of AB and POQ is the perpendicular bisector of AB; PO and OQ are very large distances compared with AB. Draw rough graphs to show how the magnitude of the electric field strength and the potential vary along the line POQ in cases (i) and (ii) above. (*O. & C.*)

5. Describe simple electrostatic experiments to illustrate *two* of the following: (*a*) the production of equal and opposite charges by induction; (*b*) the action of points; (*c*) the effect on a charged conductor of the approach of an earthed conductor. (*L.*)

6. Discuss the following, giving examples: *Electrostatic induction, electric discharge from points* and *dielectric strength.*

Describe a modern form of apparatus for obtaining a small current at a very high voltage. (*L.*)

7. An isolated conducting spherical shell of radius 10 cm, in vacuo, carries a positive charge of $1{\cdot}0 \times 10^{-7}$ coulomb. Calculate (*a*) the electric field intensity, (*b*) the potential, at a point on the surface of the conductor. Sketch a graph to show how one of these quantities varies with distance along a radius from the centre to a point well outside the spherical shell. Point out the main features of the graph. (*N.*)

8. Define (*a*) electric intensity, (*b*) difference of potential. How are these quantities related?

A charged oil-drop of radius 0·00013 cm is prevented from falling under gravity by the vertical field between two horizontal plates charged to a difference of potential of 8340 volts. The distance between the plates is 1·6 cm, and the density of oil is 920 kg m^{-3}. Calculate the magnitude of the charge on the drop ($g = 9{\cdot}81$ m s^{-2}). (*O. & C.*)

9. Two plane parallel conducting plates 1·50 cm apart are held horizontal, one above the other, in air. The upper plate is maintained at a positive potential of 1500 volts while the lower plate is earthed. Calculate the number of electrons which must be attached to a small oil drop of mass $4{\cdot}90 \times 10^{-12}$ g, if it remains stationary in the air between the plates. (Assume that the density of air is negligible in comparison with that of oil.)

If the potential of the upper plate is suddenly changed to -1500 volts what is the initial acceleration of the charged drop? Indicate, giving reasons, how the acceleration will change.

10. Show how (i) the surface density, (ii) the intensity of electric field, (iii) the potential, varies over the surface of an elongated conductor charged with electricity. Describe experiments you would perform to support your answer in cases (i) and (iii).

Describe and explain the action of points on a charged conductor; and give two practical applications of the effect. (*L.*)

11. Describe carefully Faraday's ice-pail experiments and discuss the deductions to be drawn from them. How would you investigate experimentally the charge distribution over the surface of a conductor? (*C.*)

12. What is an *electric field*? With reference to such a field define *electric potential.*

Two plane parallel conducting plates are held horizontal, one above the other, in a vacuum. Electrons having a speed of $6{\cdot}0 \times 10^{8}$ cm s^{-1} and moving normally to the plates enter the region between them through a hole in the lower plate which is earthed. What potential must be applied to the other plate so that the electrons just fail to reach it? What is the subsequent motion of these electrons? Assume that the electrons do not interact with one another.

(Ratio of charge to mass of electron is $1 \cdot 8 \times 10^{11}$ coulomb kg^{-1}.) (*N.*)

13. Define the *electric potential V* and the *electric field strength E* at a point in an electrostatic field. How are they related? Write down an expression for the electric field strength at a point close to a charged conducting surface, in terms of the surface density of charge.

Corona discharge into the air from a charged conductor takes place when the potential gradient at its surface exceeds 3×10^6 volt $metre^{-1}$; a potential gradient of this magnitude also breaks down the insulation afforded by a solid dielectric. Calculate the greatest charge that can be placed on a conducting sphere of radius 20 cm supported in the atmosphere on a long insulating pillar; also calculate the corresponding potential of the sphere. Discuss whether this potential could be achieved if the pillar of insulating dielectric was only 50 cm long. (Take ε_0 to be $8 \cdot 85 \times 10^{-12}$ farad $metre^{-1}$.) (*O.*)

14. (i) A needle is mounted vertically, point upwards, on the plate (cap) of a gold leaf electroscope, the blunt end being in metallic contact with the plate. When a negatively charged body is brought close to the needle point, without touching it, and is then withdrawn the gold leaf is left with a permanent deflection. What is the sign of the charge causing this deflection, and how was this charge produced?

(ii) A gold leaf electroscope is so constructed that for a few degrees deflection the gold leaf touches the case and is thereby earthed. Describe and explain the behaviour of the leaf when:

(*a*) a positively charged, insulated body is brought towards the plate (cap) of the electroscope until the leaf touches the case;
(*b*) the positively charged body is then moved slowly closer to the plate;
(*c*) the positively charged body is fixed close to the plate and the air between them is feebly ionized. (*O. & C.*)