WAVES AND SOUND

chapter twenty-five

Oscillations and Waves. Sound Waves

In this chapter we shall study the properties of oscillations and waves in general. Topics in waves which concern particular branches of the subject are discussed elsewhere in this book. We begin with a summary of the results relating to simple harmonic motion already derived (p. 45).

S.H.M.

Simple harmonic motion (S.H.M.) occurs when the force acting on an object or system is directly proportional to its displacement x from a fixed point and is always directed towards this point. If the object executes S.H.M., then the variation of the displacement x with time t can be written as

$$x = a \sin \omega t. \quad \quad (1)$$

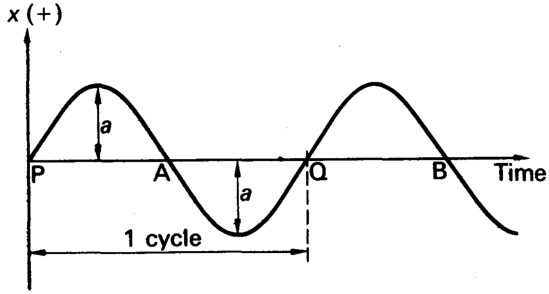

FIG. 25.1 Sine curve.

Here a is the greatest displacement from the mean or equilibrium position; a is the *amplitude*, Fig. 25.1. The constant ω is the 'angular frequency', and $\omega = 2\pi f$ where f is the *frequency* of vibration or number of cycles per second. The period T of the motion, or time to undergo one complete cycle is equal to $1/f$, so that $\omega = 2\pi/T$.

The small oscillation of a pendulum bob or vibrating layer of air is a *mechanical oscillation*, so that x is a displacement from a mean fixed position. Later, *electrical oscillations* are considered. x may then represent the instantaneous charge on the plates of a capacitor when the charge alternates about a mean value of zero. In an *electromagnetic wave*, x may represent the component of the electric or magnetic field vectors at a particular place.

Energy in S.H.M.

On p. 53, it was shown that the sum of the potential and kinetic energies of a body moving with S.H.M. is *constant* and equal to the total energy in the vibration. Further, it was shown that the time averages of the potential energy (P.E.) and kinetic energy (K.E.) are equal; each is half the total energy. In any mechanical oscillation, *there is a continuous interchange or exchange of energy from P.E. to K.E. and back again.*

For vibrations to occur, therefore, an agency is required which can possess and store P.E. and another which can possess and store K.E. This was the case for a mass oscillating on the end of a spring, as we saw on p. 50. The mass stores K.E. and the spring stores P.E.; and interchange occurs continuously from one to the other as the spring is compressed and released alternately. In the oscillations of a simple pendulum, the mass stores K.E. as it swings downwards from the end of an oscillation, and this is changed to P.E. as the height of the bob increases above its mean position.

Note that some agency is needed to accomplish the transfer of energy. In the case of the mass and spring, the force in the spring causes the transfer. In the case of the pendulum, the component of the weight along the arc of the circle causes the change from P.E. to K.E.

Electrical Oscillations

So far we have dealt with mechanical oscillations and energy. The energy in electrical oscillations takes a different form. There are still two types of energy. One is the energy stored in the electric field, and the other that stored in the magnetic field. To obtain electrical oscillations,

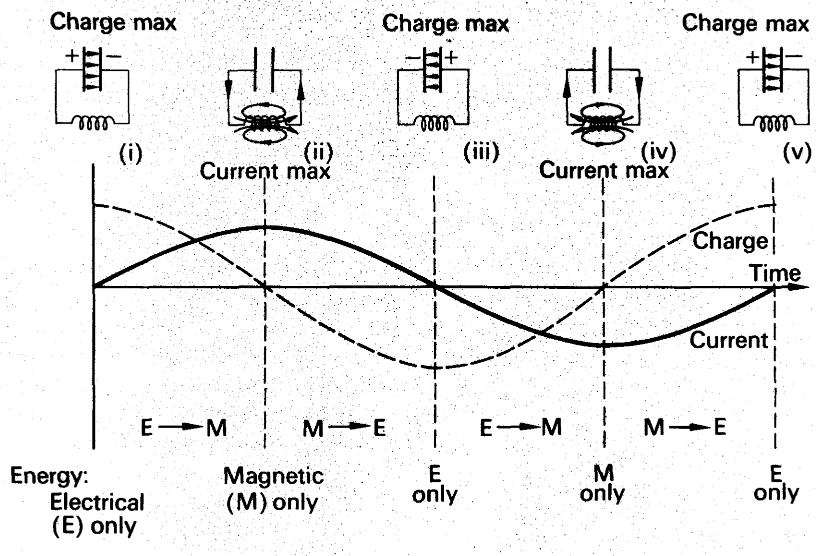

FIG. 25.2 Electrical oscillations – energy exchanges.

an inductor (coil) is used to produce the magnetic field and a capacitor to produce the electric field (see also p. 925).

Suppose the capacitor is charged and there is no current at this moment, Fig. 25.2 (i). A p.d. then exists across the capacitor and an electric field is present between the plates. At this instant all the energy is stored in the electric field, and since the current is zero there is no magnetic energy. Because of the p.d. a current will begin to flow and magnetic energy will begin to be stored in the inductor. Thus there will be a change from electric to magnetic energy. The p.d. is the agency which causes the transfer of energy.

One quarter of a cycle later the capacitor will be fully discharged and the current will be at its greatest, so that the energy is now entirely stored in the magnetic field, Fig. 25.2 (ii). The current continues to flow for a further quarter-cycle until the capacitor is fully charged in the opposite direction, when the energy is again completely stored in the electric field, Fig. 25.2 (iii). The current then reverses and the processes occur in reverse order, Fig. 25.2 (iv), after which the original state is restored and a complete oscillation has taken place, Fig. 25.2 (v). The whole process then repeats, giving continuous oscillations.

Phase of vibrations

Consider an oscillation given by $x_1 = a \sin \omega t$. Suppose a second oscillation has the same amplitude, a, and angular frequency, ω, but reaches the end of its oscillation a fraction, β, of the period T later than the first one. The second oscillation thus *lags behind* the first by a time βT, and so its displacement x_2 is given by

$$x_2 = a \sin \omega(t - \beta T)$$
$$= a \sin (\omega t - \varphi), \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $\varphi = \omega\beta T = 2\pi\beta T/T = 2\pi\beta$. If the second oscillation *leads* the first by a time βT, the displacement is given by

$$x_2 = a \sin (\omega t + \varphi). \quad . \quad . \quad . \quad . \quad . \quad (3)$$

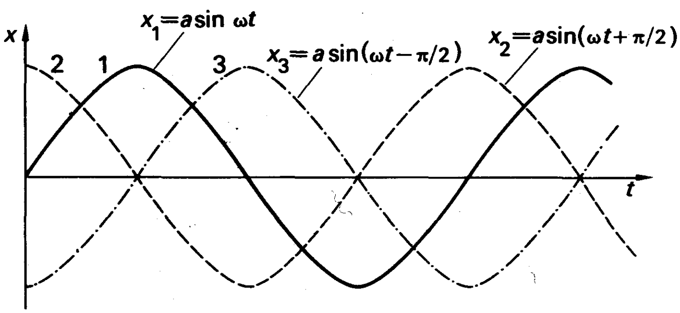

FIG. 25.3 Phase difference.

φ is known as the *phase angle* of the oscillation. It represents the *phase difference* between the oscillations $x_1 = a \sin \omega t$ and $x_2 = a \sin (\omega t - \varphi)$. Graphs of displacement v. time are shown in Fig. 25.3.

Curve 1 represents $x_1 = a \sin \omega t$. Curve 2 represents $x_2 = a \sin (\omega t + \pi/2)$, so that its phase lead is $\pi/2$; this is a lead of one quarter of a period. Curve 3 represents $x_3 = a \sin (\omega t - \pi/2)$ so that its phase lag is $\pi/2$; this is a lag of one quarter of a period on curve 1. If the phase difference is 2π, the oscillations are effectively in phase.

Note that if the phase difference is π, the displacement of one oscillation reaches a positive maximum value at the same instant as the other oscillation reaches a *negative* maximum value. The two oscillations are thus sometimes said to be 'antiphase'.

Damped Vibrations

In practice, the amplitude of vibration in simple harmonic motion does not remain constant but becomes progressively smaller. Such a vibration is said to be *damped*. The diminution of amplitude is due to loss of energy; for example, the amplitude of the bob of a simple pendulum diminishes slowly owing to the viscosity (friction) of the air. This is shown by curve 1 in Fig. 25.4.

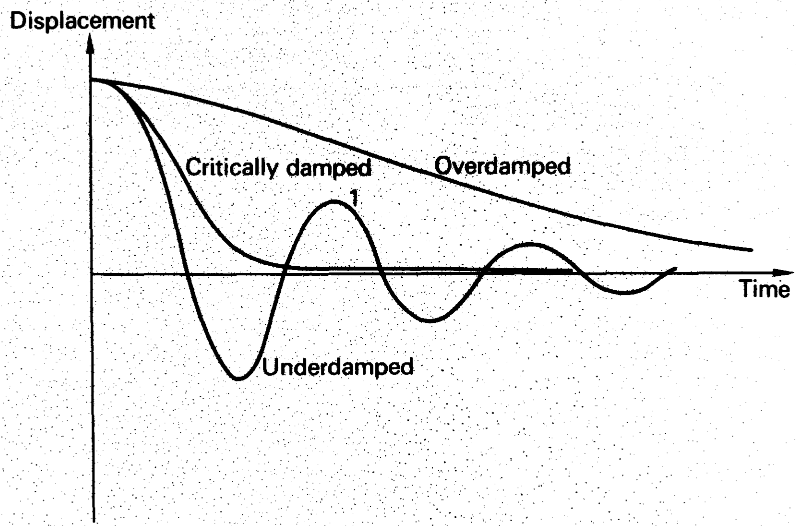

FIG. 25.4 Damped motion.

The general behaviour of mechanical systems subject to various amounts of damping may be conveniently investigated using a coil of a ballistic galvanometer (p. 919). If a resistor is connected to the terminals of a ballistic galvanometer when the coil is swinging, the induced emf due to the motion of the coil in the magnetic field of the galvanometer magnet causes a current to flow through the resistor. This current, by Lenz's Law (p. 896), opposes the motion of the coil and so causes damping. The smaller the value of the resistor, the greater is the degree of damping. The galvanometer coil is set swinging by discharging a capacitor through it. The time period, and the time taken for the amplitude to be reduced to a certain fraction of its original value, are then measured. The experiment can then be repeated using different values of resistor connected to the terminals.

It is found that as the damping is increased the time period increases and the oscillations die away more quickly. As the damping is increased further there is a value of resistance which is just sufficient to prevent the coil from vibrating past its rest position. This degree of damping, called the *critical damping, reduces the motion to rest in the shortest possible time*. If the resistance is lowered further, to increase the damping, no vibrations occur but the coil takes a longer time to settle down to its rest position. Graphs showing the displacement against time for 'underdamped', 'critically damped', and 'overdamped' motion are shown in Fig. 25.4.

When it is required to use a galvanometer as a current-measuring instrument, rather than ballistically to measure charge, it is generally critically damped. The return to zero is then as rapid as possible.

These results, obtained for the vibrations of a damped galvanometer coil, are quite general. All vibrating systems have a certain critical damping, which brings the motion to rest in the shortest possible time.

Forced Oscillations. Resonance

In order to keep a system, which has a degree of damping, in continuous oscillatory motion, some external periodic force must be used. The frequency of this force is called the *forcing frequency*. In order to see how systems respond to a forcing oscillation, we may use an electrical circuit comprising a coil L, capacitor C and resistor R, shown in Fig. 25.5 (see also p. 976).

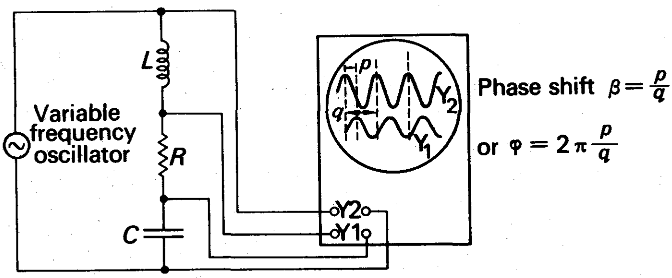

FIG. 25.5 Demonstration of oscillations.

The applied oscillating voltage is displayed on the Y_2 plates of a double-beam oscilloscope (p. 1011). The voltage across the resistor R is displayed on the Y_1 plates. Since the current I through the resistor is given by $I = V/R$, the voltage across R is a measure of the current through the circuit. The frequency of the oscillator is now set to a low value and the amplitude of the Y_1 display is recorded. The frequency is then increased slightly and the amplitude again measured. By taking many such readings, a graph can be drawn of the current through the circuit as the frequency is varied. A typical result is shown in Fig. 25.6 (i).

The phase difference, φ, between the Y_1 and Y_2 displays can be found by measuring the horizontal shift p between the traces, and the length q occupied by one complete waveform. φ is given by $(p/q) \times 2\pi$. A graph of the variation of phase difference between current and applied voltage can then be drawn. Fig. 25.6 (ii) shows a typical curve.

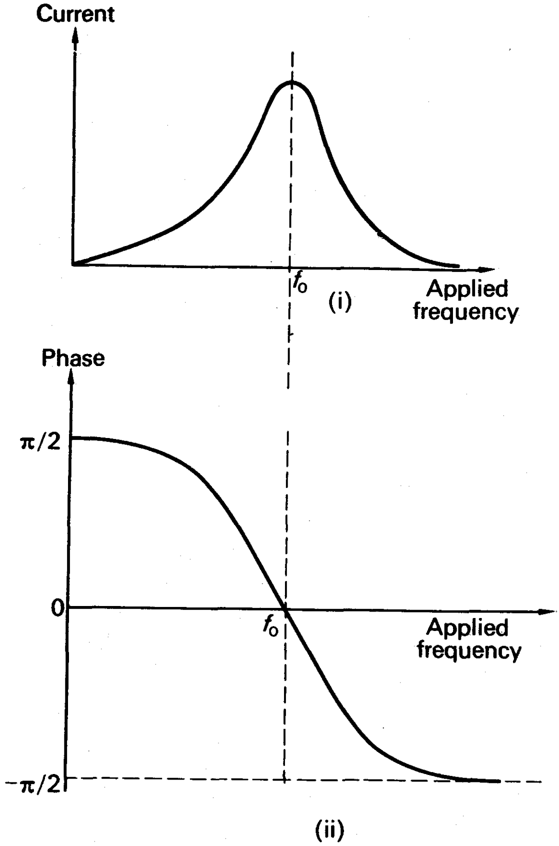

FIG. 25.6 Amplitude and phase in forced vibrations.

The following observations may be made:

1. The current is greatest at a certain frequency f_0. This is the frequency of undamped oscillations of the system, when it is allowed to oscillate on its own. f_0 is called the *natural frequency* of the system. When the forcing frequency is equal to the natural frequency, *resonance* is said to occur. The largest current is then produced.

2. At resonance, the current and voltage are in phase. Well below resonance, the current leads the voltage by $\pi/2$; at very high frequencies the current lags by $\pi/2$. The behaviour of other resonant systems is similar.

3. The forced oscillations always have the same frequency as the forcing oscillations.

Examples of resonance occur in sound and in optics. These are discussed later. It should be noted that considerable energy is absorbed at the resonant frequency from the system supplying the external periodic force.

Waves and Wave-motion

A *wave* allows energy to be transferred from one point to another some distance away without any particles of the medium travelling between the two points. For example, if a small weight is suspended by a string, energy to move the weight may be obtained by repeatedly shaking the other end of the string up and down through a small distance. Waves, which carry energy, then travel along the string from the top to the bottom. Likewise, water waves may spread along the surface from one point A to another point B, where an object floating on the water will be disturbed by the wave. No particles of water at A actually travel to B in the process. The energy in the electromagnetic spectrum, comprising X-rays and light waves, for example, may be considered to be carried by electromagnetic waves from the radiating body to the absorber. Again, sound waves carry energy from the source to the ear by disturbance of the air (p. 585).

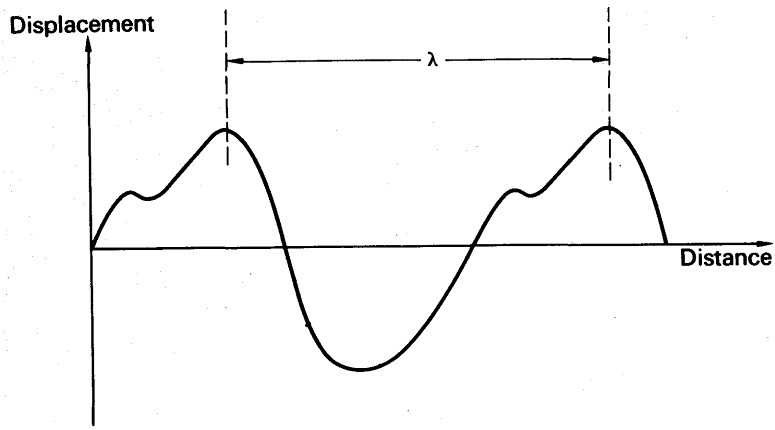

FIG. 25.7 Wave and wavelength.

If the source or origin of the wave oscillates with a frequency f, then each point in the medium concerned oscillates with the same frequency. A snapshot of the wave profile or waveform may appear as in Fig. 25.7 at a particular instant. The source repeats its motion f times per second, so a repeating *waveform* is observed spreading out from it. The distance between corresponding points in successive waveforms, such as two successive crests or two successive troughs, is called the *wavelength*, λ. Each time the source vibrates once, the waveform moves forward a distance λ. Thus in one second, when f vibrations occur, the wave moves forward a distance $f\lambda$. Hence the velocity v of the waves, which is the distance the profile moves in one second, is given by:

$$v = f\lambda.$$

This equation is true for all wave motion, whatever its origin, that is, it applies to sound waves, electromagnetic waves and mechanical waves.

Transverse Waves

A wave which is propagated by vibrations *perpendicular* to the direction of travel of the wave is called a *transverse* wave. Examples of transverse waves are waves on plucked strings and on water. Electromagnetic waves, which include light waves, are transverse waves.

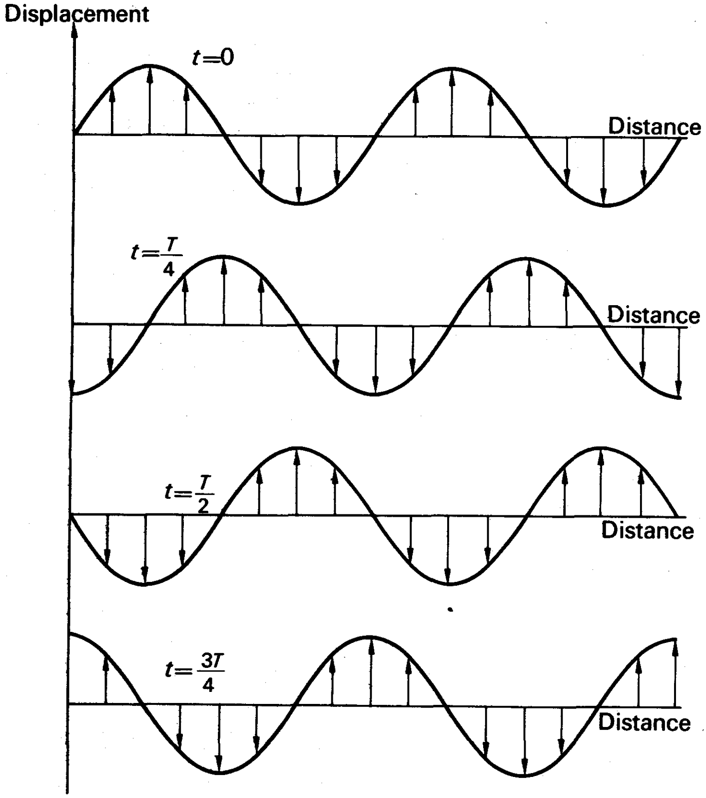

FIG. 25.8 Progressive transverse wave.

The propagation of a transverse wave is illustrated in Fig. 25.8. Each particle vibrates perpendicular to the direction of propagation with the same amplitude and frequency, and the wave is shown successively at $t = 0$, $T/4$, $T/2$, $3T/4$, in Fig. 25.8, where T is the period.

Longitudinal Waves

In contrast to a transverse wave, a *longitudinal* wave is one in which the vibrations occur in the *same* direction as the direction of travel of the wave. Fig. 25.9 illustrates the propagation of a longitudinal wave. The row of dots shows the actual positions of the particles whereas the *graph* shows the *displacement* of the particles from their equilibrium positions. The positions at time $t = 0$, $t = T/4$, $t = T/2$ and $t = 3T/4$

are shown. The diagram for $t = T$ is, of course, the same as $t = 0$. With displacements to R (right) and to L (left), note that:

(i) The displacements of the particles cause regions of high density (*compressions* C) and of low density (*rarefactions* R) to be formed.

(ii) These regions move along with the speed of the wave, as shown by the broken diagonal line.

(iii) Each particle vibrates about its mean position with the same amplitude and frequency.

(iv) The regions of greatest compression are one-quarter wavelength ahead of the greatest displacement in the direction of the wave. This result is important in understanding some processes involving sound waves.

The most common example of a longitudinal wave is a sound wave. This is propagated by alternate compressions and rarefactions of the air.

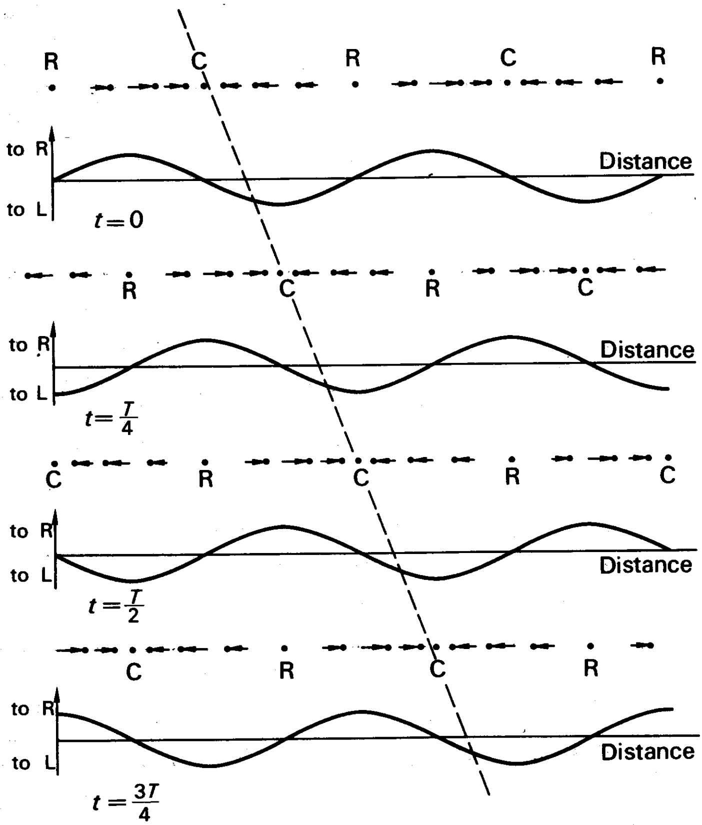

FIG. 25.9 Progressive longitudinal wave.

Progressive Waves

Both the transverse and longitudinal waves described above are *progressive*. This means that the wave profile moves along with the speed of the wave. If a snapshot is taken of a progressive wave, it repeats at equal distances. The repeat distance is the *wavelength* λ. If one point is taken, and the profile is observed as it passes this point, then the profile is seen to repeat at equal intervals of time. The repeat time is the *period*, T.

The vibrations of the particles in a progressive wave are of the same amplitude and frequency. But *the phase of the vibrations changes for different points along the wave.* This can be seen by considering Figs. 25.8 and 25.9. The phase difference may be demonstrated by the following experiment.

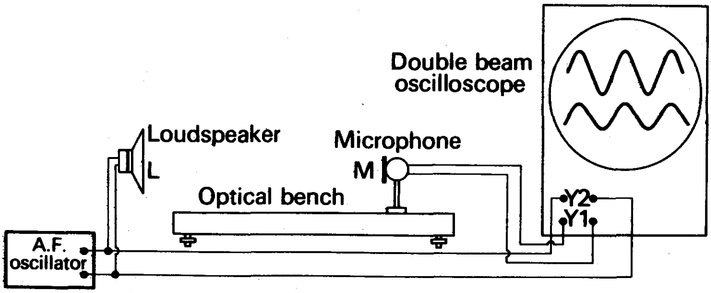

FIG. 25.10 Demonstration of phase in progressive wave.

An audio-frequency (af) oscillator is connected to the loudspeaker L and to the Y_2 plates of a double-beam oscilloscope, Fig. 25.10. A microphone M,

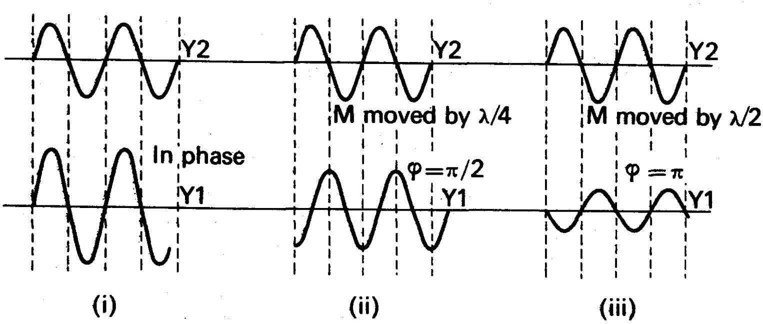

FIG. 25.11 Phase difference and wavelength.

mounted on an optical bench, is connected to the Y_1 plates. When M is moved away from or towards L, the two traces on the screen are as shown in Fig. 25.11 (i) at one position. This occurs when the distance LM is equal to a whole number of wavelengths, so that the signal received by M is in phase with that sent out by L. When M is now moved further away from L through a distance $\lambda/4$, where λ is the wavelength, the appearance on the screen changes to that shown in Fig. 25.11 (ii). The resultant phase change is $\pi/2$, so that the signal now arrives a quarter of a period later. When M is moved a distance $\lambda/2$ from its 'in-phase' position, the signal arrives half a period later, a phase change of π, Fig. 25.11 (iii).

Velocity of Sound in Free Air

The velocity of sound in free air can be found from this experiment. Firstly, a position of the microphone M is obtained when the two signals on the screen are in phase, as in Fig. 25.10. The reading of the position of M on the optical bench is then taken. M is now moved slowly until the phase of the two signals on the screen is seen to change through $\pi/2$ to π and then to be in phase again. The shift of M is then measured. It is equal to λ, the wavelength. From several measurements the average value of λ is found, and the velocity of sound is calculated from $v = f\lambda$, where f is the frequency obtained from the oscillator dial.

Progressive Wave Equation

An equation can be formed to represent generally the displacement y of a vibrating particle in a medium which a *wave* passes. Suppose the wave moves from left to right and that a particle at the origin O then vibrates according to the equation $y = a \sin \omega t$, where t is the time and $\omega = 2\pi f$ (p. 577).

At a particle P at a distance x from O to the right, the phase of the

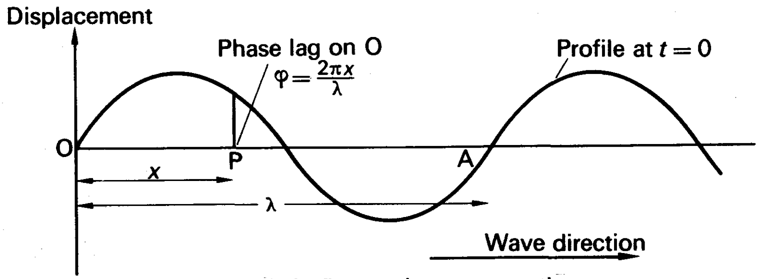

FIG. 25.12 Progressive wave equation.

vibration will be different from that at O, Fig. 25.12. A distance λ from O corresponds to a phase difference of 2π (p. 580). Thus the phase difference φ at P is given by $(x/\lambda) \times 2\pi$ or $2\pi x/\lambda$. Hence the displacement of any particle at a distance x from the origin is given by

$$y = a \sin (\omega t - \varphi)$$

or

$$y = a \sin \left(\omega t - \frac{2\pi x}{\lambda}\right). \quad . \quad . \quad . \quad (4)$$

Since $\omega = 2\pi f = 2\pi v/\lambda$, where v is the velocity of the wave, this equation may be written:

$$y = a \sin \left(\frac{2\pi vt}{\lambda} - \frac{2\pi x}{\lambda}\right)$$

or

$$y = a \sin \frac{2\pi}{\lambda} (vt - x). \quad . \quad . \quad . \quad . \quad (5)$$

Also, since $\omega = 2\pi/T$, equation (4) may be written:

$$y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right). \quad . \quad . \quad . \quad (6)$$

Equations (5) or (6) represent a *plane-progressive wave*. The negative sign in the bracket indicates that, since the wave moves from left to right, the vibrations at points such as P to the right of O will lag on that at O. A wave travelling in the *opposite direction*, from right to left, arrives at P before O. Thus the vibration at P leads that at O. Consequently a wave travelling in the opposite direction is given by

$$y = a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right), \qquad . \quad . \quad . \quad (7)$$

that is, the sign in the bracket is now a plus sign.

As an illustration of calculating the constants of a wave, suppose a wave is represented by

$$y = a \sin\left(2000\pi t - \frac{\pi x}{17}\right),$$

where t is in seconds, y in cm. Then, comparing it with equation (5),

$$y = a \sin \frac{2\pi}{\lambda}(vt - x),$$

we have
$$\frac{2\pi v}{\lambda} = 2000\pi,$$

and
$$\frac{2\pi}{\lambda} = \frac{\pi}{17}.$$

$$\therefore\ \lambda = 2 \times 17 = 34 \text{ cm}$$

and
$$v = 1000\lambda = 1000 \times 34 = 34000 \text{ cm s}^{-1}$$

$$\therefore\ \text{frequency}, f, = \frac{v}{\lambda} = \frac{34000}{34} = 1000 \text{ Hz}$$

$$\therefore\ \text{period}, T, = \frac{1}{f} = \frac{1}{1000} \text{ s}.$$

If two layers of the wave are 180 cm apart, they are separated by 180/34 wavelengths, or by $5\frac{10}{34}\lambda$. Their *phase difference* for a separation λ is 2π; and hence, for a separation $10\lambda/34$, omitting 5λ from consideration, we have:

$$\text{phase difference} = \frac{10}{34} \times 2\pi = \frac{10\pi}{17} \text{ radians.}$$

Principle of Superposition

When two waves travel through a medium, their combined effect at any point can be found by the *Principle of Superposition*. This states that *the resultant displacement at any point is the sum of the separate displacements due to the two waves.*

The principle can be illustrated by means of a long stretched spring ('Slinky'). If wave pulses are produced at each end simultaneously, the two waves pass through the wire. Fig. 25.13(a) shows the stages which occur as the two pulses pass each other. In Fig. 25.13(a) (i), they are some distance apart and are approaching each other, and in Fig. 25.13(a) (ii)

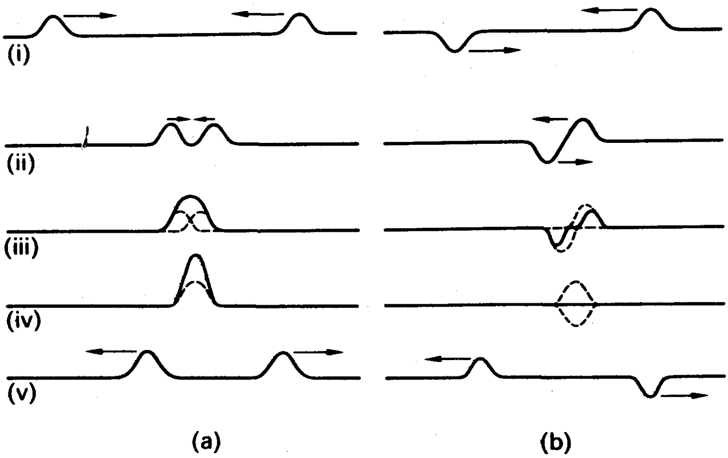

FIG. 25.13 Superposition of waves.

they are about to meet. In Fig. 25.13(a) (iii), the two pulses, each shown by broken lines, are partly overlapping. The resultant is the sum of the two curves. In Fig. 25.13(a) (iv), the two pulses exactly overlap and the greatest resultant is obtained. The last diagram shows the pulses receding from one another. The diagrams in Fig. 25.13(b) show the same sequence of events (i)–(v) but the pulses are equal and opposite. The Principle of Superposition is widely used in discussion of wave phenomena such as interference, as we shall see (p. 688).

Stationary Waves

We have already discussed progressive waves and their properties. Fig. 25.14 shows an apparatus which produces a different kind of wave (see also p. 662). If the weights on the scale-plan are suitably adjusted,

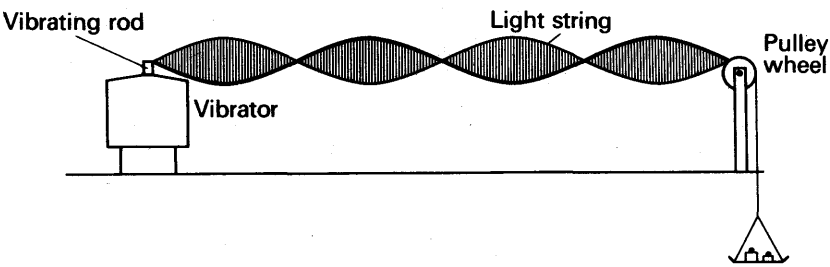

FIG. 25.14 Demonstration of stationary wave.

a number of *stationary vibrating loops* are seen on the string when one end is set vibrating. This time the wave-like profile on the string does *not* move along the medium, which is the string, and the wave is therefore called a *stationary* (or *standing*) *wave*.

The motion of the string when a stationary wave is produced can be seen by using a Xenon stroboscope (strobe). This instrument gives a flashing light whose frequency can be varied. The apparatus is set up in a darkened room and illuminated with the strobe. When the frequency of the strobe is nearly equal to that of the string, the string can be seen moving up and down slowly. Its observed frequency is equal to the difference between the frequency of the strobe and that of the string. Progressive stages in the motion of the string can now be seen and studied, and these are illustrated in Fig. 25.15.

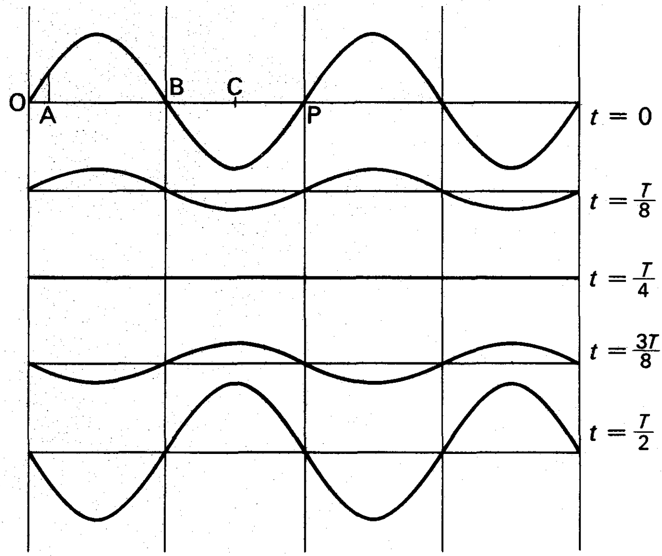

FIG. 25.15 Changes in motion of stationary wave.

The following points should be noted:

1. There are points such as **B** where the displacement is permanently zero. These points are called *nodes* of the stationary wave.

2. At points between successive nodes the vibrations *are in phase*. This property of the stationary wave is in sharp contrast to the progressive wave, where the phase of points near each other are all different (see p. 585). Thus when one point of a stationary wave is at its maximum displacement, *all* points are then at their maximum displacement. When a point (other than a node) has zero displacement, *all* points then have zero displacement.

3. Each point along the wave has a *different amplitude* of vibration from neighbouring points. Again this is different from the case of a progressive wave, where every point vibrates with the same amplitude. Points, e.g. C, which have the greatest amplitude are called *antinodes*.

4. The wavelength is equal to the distance OP, Fig. 25.15. Thus the wavelength λ, is twice the *distance between successive nodes or successive antinodes*. The distance between successive nodes or antinodes is $\lambda/2$; the distance between a node and a neighbouring antinode is $\lambda/4$.

Examples of stationary waves are discussed later in the book. The reader is referred to p. 641 for discussion of stationary waves in sound and to p. 985 for stationary electromagnetic waves.

Stationary Wave Equation

In deriving the wave equation of a progressive wave, we used the fact that the phase changes from point to point (p. 586). In the case of a stationary wave, we may find the equation of motion by considering the *amplitude* of vibration at each point because the amplitude varies while the phase remains constant.

Let ω be the angular frequency of the wave. The vibration of each particle may be represented by the equation

$$y = Y \sin \omega t, \quad . \quad . \quad . \quad . \quad (8)$$

where Y is the amplitude of the vibration at the point considered. Y varies along the wave with the distance x from some origin. If we suppose the origin to be at an antinode, then the origin will have the greatest amplitude, A, say. Now the wave repeats at every distance λ, and it can be seen that the amplitudes at different points vary sinusoidally with their particular distance x. An equation representing the changing amplitude Y along the wave is thus:

$$Y = A \cos \frac{2\pi x}{\lambda} = A \cos kx, \quad . \quad . \quad . \quad (9)$$

where $k = 2\pi/\lambda$. When $x = 0$, $Y = A$; when $x = \lambda$, $Y = A$. When $x = \lambda/2$, $Y = -A$. This equation hence correctly describes the variation in amplitude along the wave, Fig. 25.15. Hence the equation of motion of a stationary wave is, with (8),

$$y = A \cos kx \, . \, \sin \omega t. \quad . \quad . \quad . \quad (10)$$

From (10), $y = 0$ at all times when $\cos kx = 0$. Thus $kx = \pi/2$, $3\pi/2$, $5\pi/2, \ldots$, in this case. This gives values of x corresponding to $\lambda/4$, $3\lambda/4$, $5\lambda/4, \ldots$ These points are *nodes* since the displacement at a node is always zero (p. 590). Thus equation (10) gives the correct distance, $\lambda/2$, between nodes.

A stationary wave can be considered as produced by the superposition of two progressive waves, of the same amplitude and frequency, travelling in opposite directions. This is shown mathematically on p. 643.

Wave Properties, Reflection and Refraction

Any wave motion can be *reflected*. The reflection of light and sound waves, for example, is discussed on pp. 677, 613 respectively.

Waves can also be *refracted*, that is, their direction changes when they enter a new medium. This is due to the change in velocity of the waves on entering a different medium. Refraction of light and sound, for example, is discussed later (pp. 679, 615).

Diffraction

Waves can also be 'diffracted'. *Diffraction* is the name given to the spreading of waves when they pass through apertures or around obstacles.

The general phenomenon of diffraction may be illustrated by using water waves in a ripple tank, with which we assume the reader is familiar. Fig. 25.16 (i) shows the effect of widening the aperture and Fig. 25.16 (ii) the effect of shortening the wavelength and keeping the same width of opening. In certain circumstances in diffraction, reinforcement of the waves, or complete cancellation occurs in particular directions from the aperture, as shown in Fig. 25.16 (i) and (ii). These patterns are called 'diffraction bands' (p. 701).

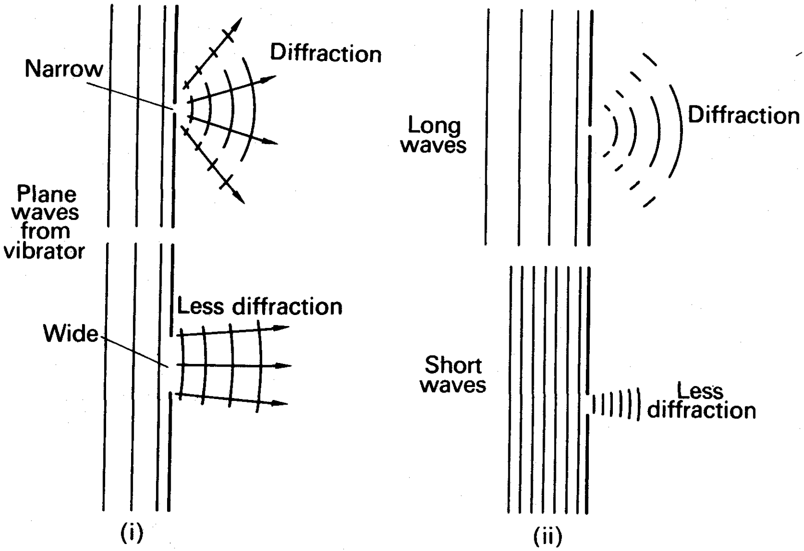

FIG. 25.16 Diffraction of waves.

Generally, the smaller the width of the aperture in relation to the wavelength, the greater is the spreading or diffraction of the waves. This explains why we cannot see round corners. The wavelength of *light waves* is about 6×10^{-5} cm (p. 690). This is so short that no appreciable diffraction is obtained around obstacles of normal size. With very small obstacles or narrow apertures, however, diffraction of light may be appreciable (see p. 707). *Sound waves* have long wavelengths, for example 50 cm, so that diffraction of sound waves occurs easily. For this reason, it is possible to hear round corners. *Electromagnetic waves* can be diffracted, as shown on p. 989.

Interference

When two or more waves of the same frequency overlap, the phenomenon of *interference* occurs. Interference is easily demonstrated in a ripple tank. Two sources, A and B, of the same frequency are used. These produce circular waves which spread out and overlap, and the pattern seen on the water surface is shown in Fig. 25.17.

The interference pattern can be explained from the Principle of Superposition (p. 588). If the oscillations of A and B are in phase, crests from A will arrive at the same time as crests from B at any point on the line RS. Hence by the Principle of Superposition there will be reinforcement or a large wave along RS. Along XY, however, crests from A will arrive before corresponding crests from B. In fact, every point on XY is half a wavelength, λ, nearer to A than to B, so that crests from A arrive at the same time as troughs from B. Thus, by the Principle of Superposition, the resultant is *zero*. At every point along PQ there is a $3\lambda/2$ path difference from A compared to that from B, so that the resultant is also zero along PQ.

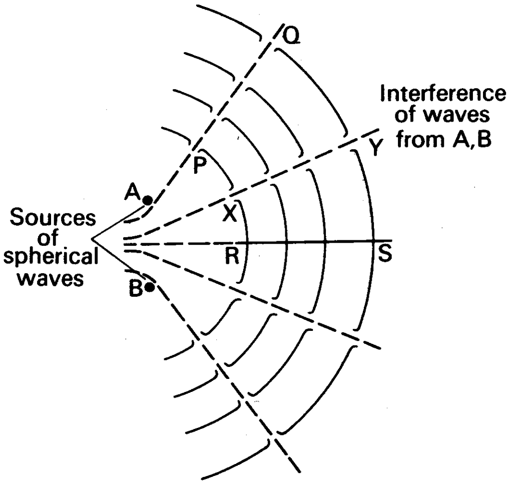

FIG. 25.17 Interference of waves.

Interference of *light waves* is discussed in detail on p. 687. An experiment to demonstrate the interference of *electromagnetic waves* (microwaves) is given on p. 989. The interference of *sound waves* can be

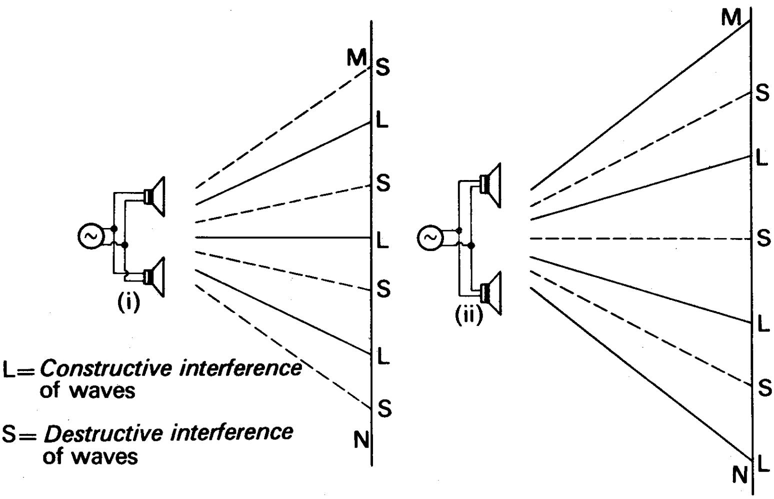

FIG. 25.18 Interference of sound waves.

demonstrated by connecting two loudspeakers in parallel to an audio-frequency oscillator, Fig. 25.18 (i). As the ear or microphone is moved along the line MN, alternate loud (L) and soft (S) sounds are heard according to whether the receiver of sound is on a line of reinforcement (constructive interference) or cancellation (destructive interference) of waves. Fig. 25.18 (i) indicates the positions of loud and soft sounds if the two speakers oscillate in phase. If the connections to *one* of the speakers is reversed, so that they oscillate out of phase, then the pattern is altered as shown in Fig. 25.18 (ii). The reader should try to account for this difference.

Velocity of Waves

We now list, for convenience, the velocity v of waves of various types, which are considered more fully in the appropriate sections of the book:

1. *Transverse wave on string*

$$V = \sqrt{\frac{T}{m}}, \qquad . \quad . \quad . \quad . \quad (11)$$

where T is the tension and m is the mass per unit length.

2. *Sound waves in gas*

$$V = \sqrt{\frac{\gamma p}{\rho}}, \qquad . \quad . \quad . \quad . \quad (12)$$

where p is the pressure, ρ is the density and γ is the ratio of the principal specific heat capacities of the gas.

3. *Longitudinal waves in solid*

$$V = \sqrt{\frac{E}{\rho}}, \qquad . \quad . \quad . \quad . \quad (13)$$

where E is Young's modulus and ρ is the density.

4. *Electromagnetic waves*

$$V = \sqrt{\frac{1}{\mu\varepsilon}}, \qquad . \quad . \quad . \quad . \quad (14)$$

where μ is the permeability and ε is the permittivity of the medium.

SOUND RECEPTION, REPRODUCTION, RECORDING

The Ear

The eye can detect colour changes, which are due to the different frequencies of the light waves; it can also detect variations in brightness, which are due to the different amounts of light energy it receives. In the sphere of sound, the ear is as sensitive as the eye; it can detect notes of different pitch, which are due to the different frequencies of sound waves, and it can also detect loud and soft notes, which are due to different amounts of sound energy falling on the ear per second.

We are not concerned in this book with the complete physiology of the ear. Among other features, it consists of the *outer ear*, A, a canal C

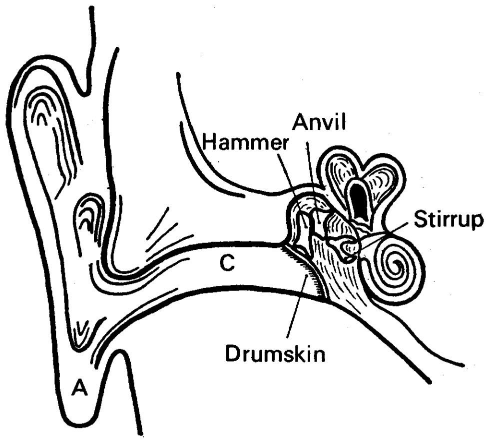

Fig. 25.19. The ear.

leading to the *drumskin*, and bones called the *ossicles*, Fig. 25.19. The ossicles consist of a bone called the 'hammer', fitting into another bone called the 'anvil', which is connected to the third bone called the 'stirrup'. When sound waves occur in the neighbourhood of the ear, they travel down the canal to the drumskin, which is also set into vibration. The part of the hammer in contact with the drumskin then vibrates, and strikes the anvil at the same rate. The motion is thus communicated to the stirrup, and from here it passes by a complicated mechanism to the auditory nerves, which set up the sensation of sound.

In 1843, OHM asserted that the ear perceives a simple harmonic vibration of the air as a simple or pure note. Although the waveforms of notes from instruments are far from being simple harmonic (see p. 610), the ear appears able to analyse a complicated waveform into the sum of a number of simple harmonic waves, which it then detects as separate notes.

Microphones

We now consider the principles of some *microphones*, instruments which convert sound energy to electrical energy. Details of microphones must be obtained from specialist works.

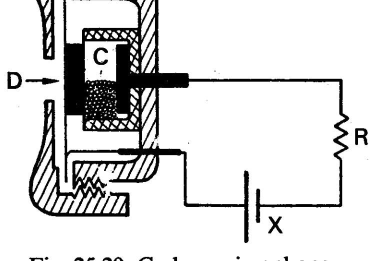

Fig. 25.20. Carbon microphone.

Carbon microphone, Fig. 25.20. This type of microphone is used in the hand set of a telephone. It contains carbon granules, C, whose electrical resistance decreases on compression and increases on release. Thus when sound waves are incident on the diaphragm D, a varying electric current of the same sound or audio frequency is produced along the telephone wires. The carbon microphone is a 'pressure' type since the magnitude of the current depends on the pressure changes in the air.

Ribbon microphone, Fig. 25.21. This is a sensitive microphone, with a uniform response over practically the whole of the audio-frequency (af) range from 40 to 15000 Hz. It has a corrugated aluminium ribbon R clamped between two pole-pieces N, S of a powerful magnet. When sound waves are incident on R, the ribbon vibrates perpendicular to the magnetic field. A varying induced emf of the same frequency is therefore obtained, as shown, and this is passed to an amplifier. The ribbon microphone is a 'velocity' type, since the movement of R depends on the velocity changes in the air particles near it.

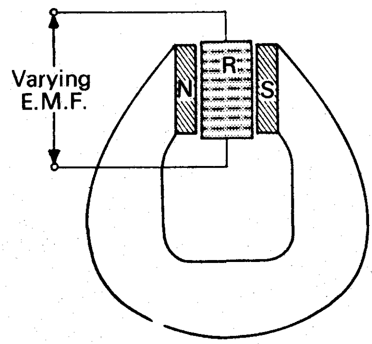

Fig. 25.21. Ribbon microphone.

Moving-coil microphone, Fig. 25.22. This is also a sensitive microphone, widely used. A coil X, made from aluminium tape so that it is very light, is situated in the radial field between the pole-pieces N, S of a magnet M. When sound waves are incident on a diaphragm Y attached to X, the coil vibrates and 'cuts' magnetic flux. The varying induced emf obtained is passed to an amplifier. The moving-coil microphone is a pressure type. To avoid any effect of fluctuations in atmospheric pressure during use, the inside is kept at atmospheric pressure by means of a tube T.

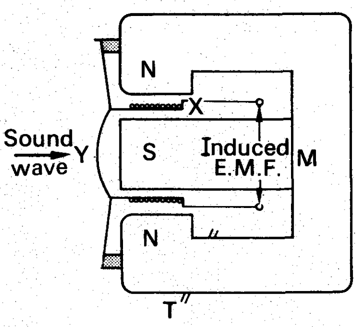

Fig. 25.22. Moving-coil microphone.

The ribbon and moving-coil microphones are examples of *electrodynamic microphones*. They are widely used in the entertainment industry because their frequency response is extremely uniform. Unlike the carbon microphone, which has a relatively poor frequency response, no battery is used, and they are not subject to the background noise and 'hissing' obtained with a carbon microphone.

Loudspeaker. Telephone Earpiece

The *moving-coil loudspeaker* is used to reproduce sound energy from the electrical energy obtained with a microphone. It has a coil C or

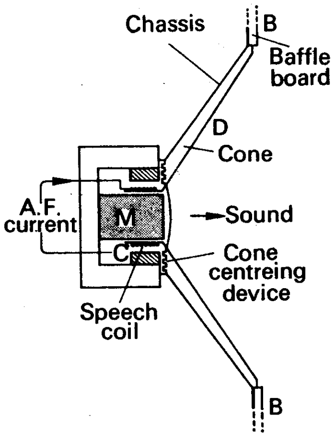

Fig. 25.23. Moving-coil loudspeaker.

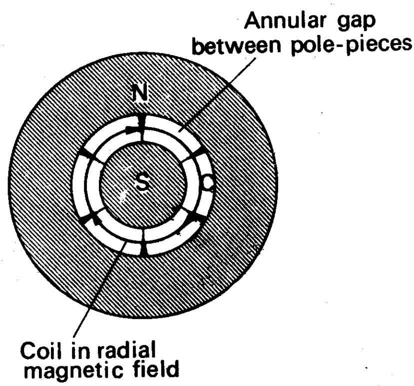

Fig. 25.24. Speech coil in magnetic field.

speech coil, wound on a cylindrical former, which is positioned symmetrically in the radial field of a pot magnet M, Fig. 25.23. A thin cardboard cone D is rigidly attached to the former and loosely connected to a large baffle-board B which surrounds it.

When C carries audio-frequency current, it vibrates at the same frequency in the direction of its axis. This is due to the force on a current-carrying conductor in a magnetic field, Fig. 25.24, whose direction is given by Fleming's left-hand rule. Since the surface area of the cone is large, the large mass of air in contact with it is disturbed and hence a loud sound is produced.

When the cone moves forward, a compression of air occurs in front of the cone and simultaneously a rarefaction behind it. The wave generated behind the cone is then 180° out of phase with that in front. If this wave reaches the front of the cone quickly, it will interfere appreciably with the wave there. Hence the intensity of the wave is diminished. This effect will be more noticeable at low frequency, or long wavelength, as the wave behind then has time to reach the front before the next vibration occurs. Generally, then, the sound would lack low note or bass intensity. The large baffle reduces this effect appreciably. It makes the path from the rear to the front so much longer that interference is negligible.

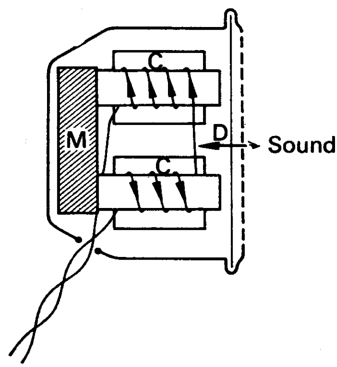

Fig. 25.25. Telephone earpiece.

Telephone earpiece, Fig. 25.25. This has speech coils C wound round soft-iron cores, which have a permanent magnet M between them. The varying speech current produces a corresponding varying attraction on the soft-iron diaphragm D, which thus generates sound waves of the same frequency in the air. The sound is soft because the mass of air in contact with D is small. The permanent magnet is necessary to prevent distortion and to make the movement of D more sensitive to the varying attractive force.

Sound Recording and Reproduction

We now consider the principles of recording sound on tape and on film, and its reproduction. Details are beyond the scope of this book and must be obtained from manuals on the subject.

Tape recorder. Fig. 25.26 (i) illustrates the principle of tape recording in which flexible tape is used. It is coated with a fine uniform layer of a special form of ferric oxide which can be magnetised. The backing is a smooth plastic-base tape. When recording, the tape moves at a constant speed past the narrow gap between the poles of a ring of soft iron which has a coil round it. The coil carries the audio-frequency (af) current due to the sound recorded. On one half of a cycle, that part of the moving tape then in the gap is unmagnetised. On the other half of the same cycle, the next piece of tape in the gap is magnetised in the opposite direction, as shown. The rate at which pairs of such magnets is produced is equal to the frequency of the af current. The strength of the magnets is a measure of the magnetising current and hence of the intensity of the sound recorded.

In 'playback', the magnetised tape is now run at exactly the same speed past the same or another ring, the playback head, Fig. 25.26 (ii). As the small magnets pass the gap between the poles, the flux in the iron changes. An induced emf is thus obtained of the same frequency

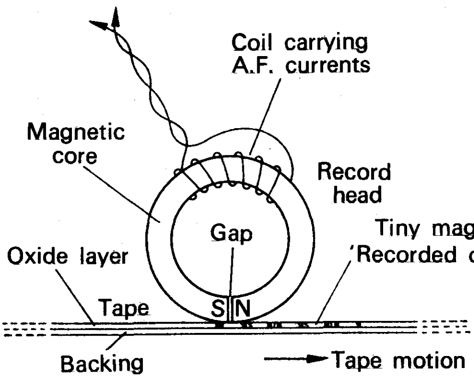

Fig. 25.26 (i). Tape recording.

and strength as that due to the original tape recording. This is amplified and passed to the loudspeaker, which reproduces the sound.

To obtain high-quality sound reproduction from tape, the output from a special high frequency *bias oscillator* is applied to the recording head in addition to the recording signal. This ensures that the magnets formed on the tape have strengths which are proportional to the recording signal. If the bias oscillator was not used, severe distortion due to

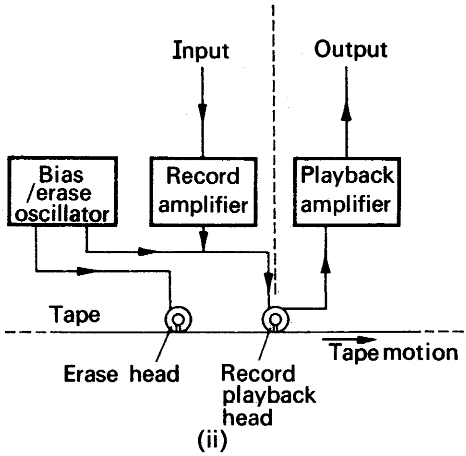

Fig. 25.26 (ii). Sound reproduction.

non-linearity would occur. The bias oscillator can also be used to *erase* the recording on the tape. This is done by applying its output to a coil in a special *erase head.* The erase head is similar to the record head but has a larger gap, so that the tape is in the magnetic field for a longer period. The bias signal takes the magnetic material on the tape through many thousands of hysteresis loops. These become progressively smaller until the magnetism disappears.

Sound Track

One method of recording sound on film, in the form of a *variable area* sound track, is illustrated in Fig. 25.27. A triangular aperture or mask T

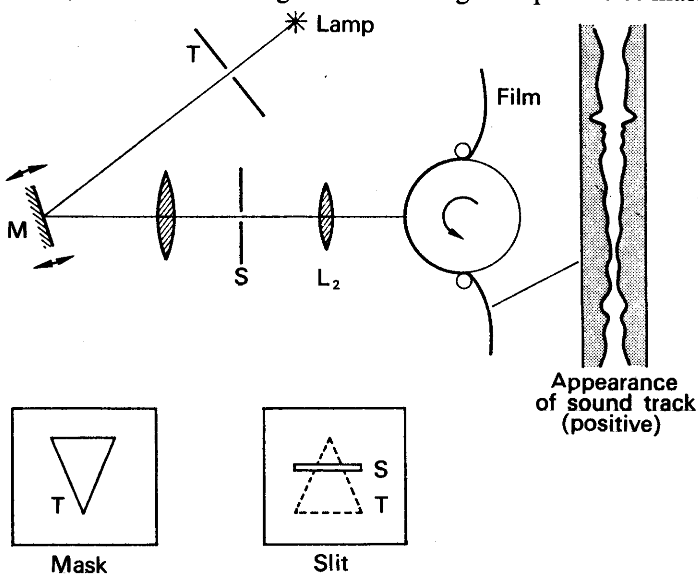

Fig. 25.27. Recording sound on film.

is brightly lit by a high-wattage lamp. After passing through T the light is reflected by a mirror M, and the rays are brought to a focus on to a slit S by a lens L_1. By rotating the mirror slightly, as shown, the image T′ of T, produced by the slit, can be moved up and down. This varies the length of slit illuminated. The light passing through the slit is collected by another lens system L_2 and focused on a strip of moving unexposed film. The mirror M is mounted on the moving system of a galvanometer, so that it moves at the same frequency as the audio-frequency currents passed through the galvanometer coil. The *area* of the film exposed thus varies as the audio-frequency signal. Hence when the film is developed, a permanent sound recording or *sound track* is obtained. A typical length of sound track is shown in Fig. 25.27.

Fig. 25.28 illustrates the principle of reproducing the sound. Light is focused on the sound track and passes through to a *photo-electric cell.* This contains a light-sensitive metal surface such as caesium, which then emits a number of electrons proportional to the light intensity. A current therefore flows in a resistor R. The sound track is coupled to the film, and as it moves, an audio-frequency current flows in R. The pd developed is amplified and passed to the loudspeaker.

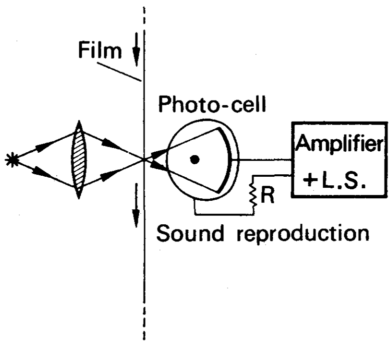

Fig. 25.28. Sound reproduction.

Frequency of Tuning-fork. Falling Plate Method

1. *Comparison method.* A tuning-fork is often used in experiments in sound to provide a note of known frequency. One method of measuring the frequency of a fork P is to compare it with the known frequency of another fork Q by a 'falling plate method'. One end of P is clamped in a vice; a light bristle B is attached to a prong, and rests lightly near the bottom of a smoked glass plate G, Fig. 25.29. The fork Q is similarly placed, and a bristle C, attached to one of its prongs, rests lightly on G. Both forks are sounded by drawing a bow across them, and the thread S suspending G is now burnt. The glass plate, usually in a groove, falls downward past the horizontally vibrating bristles, which then trace out two clear wavy 'tracks' BX, CY on G. Two horizontal lines, MN, are then drawn across BX, CY, and the number of complete waves between MN is counted on each trace. Suppose they are n_1, n_2 respectively, and the frequencies of the corresponding forks are f_1, f_2. Then if t is the time taken by the plate to fall a distance MN,

$$f_1 = \text{number of cycles per second} = \frac{n_1}{t}$$

and $f_2 = n_2/t$. Hence $f_1/f_2 = n_1/n_2$, or

$$f_1 = \frac{n_1}{n_2} \times f_2.$$

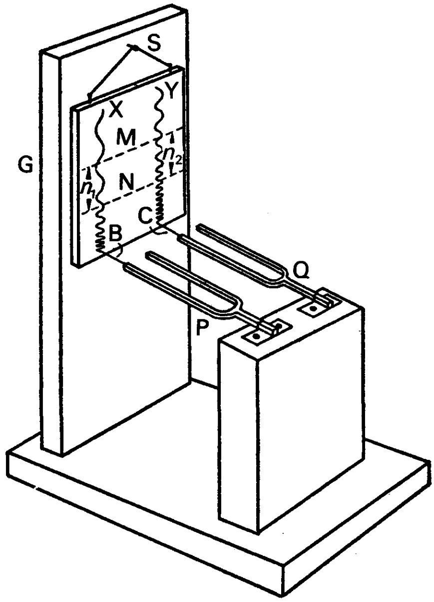

Fig. 25.29. Falling plate method.

Thus, knowing n_1, n_2, f_2, the unknown frequency f_1 can be calculated.

2. *Absolute method.* The unknown frequency f_1 of the tuning-fork P can also be calculated from its own trace. In this case the lengths s_1, s_2 corresponding to an *equal* number of consecutive waves are measured by a travelling microscope. Suppose there are n complete waves between LR, RS, Fig. 25.30. Then f_1 is given by

$$f_1 = n\sqrt{\frac{g}{s_2 - s_1}}, \quad . \quad . \quad . \quad . \quad (15)$$

where $g = 980$ when s_1, s_2 are in centimetres.

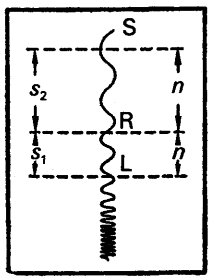

Fig. 25.30. (Not to Scale.)

To prove this formula, for f_1, let t be the time taken by the plate to fall a distance LR. Since the number of waves in LR is the same as in RS, the fork has also vibrated for a time t while the plate falls a distance RS. Thus if t is the time taken by the plate to fall a distance LR, $2t$ is the time it takes to fall a distance LS. Suppose u is the velocity of the plate at the instant when the line L on it reaches the vibrating bristle on the fork.

Then
$$s_1 = ut + \tfrac{1}{2}gt^2 \quad . \quad . \quad . \quad . \quad (i)$$
from the dynamics equation $s = ut + \frac{1}{2}ft^2$, since the acceleration, f, of the plate $= g$, the acceleration due to gravity. As the time taken by the plate to fall a distance LS, or $(s_1 + s_2)$, is $2t$, we have

$$s_1 + s_2 = u \, . \, 2t + \tfrac{1}{2}g(2t)^2. \quad . \quad . \quad . \quad (ii)$$

Multiplying equation (i) by 2, and subtracting from (ii) to eliminate u,

$$s_2 - s_1 = gt^2,$$

$$\therefore\ t = \sqrt{\frac{s_2 - s_1}{g}}$$

$$\therefore\ f_1 = \frac{n}{t} = n\sqrt{\frac{g}{s_2 - s_1}} \qquad . \quad . \quad . \quad . \qquad (16)$$

as given above in equation (15).

The falling-plate method gives only an approximate value of the frequency, as (*a*) it is difficult to determine an *exact* number of waves; (*b*) the attachment of a bristle to the tuning-fork prong lowers its frequency slightly (see p. 622); (*c*) there is friction between the style and the plate.

The Stroboscope

A *stroboscope* is an arrangement which can make a rotating object appear at rest when it is viewed, and thus enables a spinning wheel, for example, to be studied at leisure. The stroboscopic method can be used to determine the frequency of a tuning-fork, which is electrically maintained for the purpose.

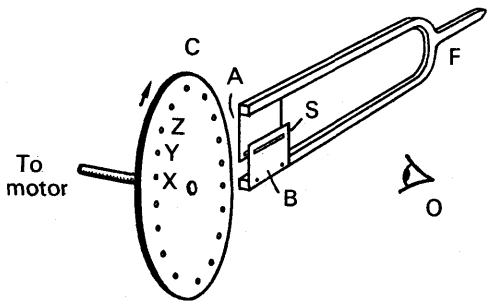

Fig. 25.31. Stroboscope method.

Two light metal plates, A, B, each with a slit S in them, are attached to prongs of the tuning-fork F so that the slits overlap each other when the fork is not sounded, Fig. 25.31. Behind the slit is a vertical circular white card C with black dots spaced at equal distances round the circumference, and the dots on the card can be seen through S. The tuning-fork is set into vibration, and the card is rotated by a motor about a horizontal axis through its centre with increasing speed. At first an observer O, viewing the dots through S, sees them moving round in an opposite direction to that in which the card is rotated. This is because the intermittent glimpses of the card through S occur quicker than the time taken for one dot to reach the place of the dot in front of it, with the result that the dots appear to be moving slowly back. As the speed of the card is increased further, a stage is reached when the dots appear perfectly stationary.

Through the slit S, glimpses of the dots are seen twice in every cycle of vibration of the tuning-fork. When the dots first appear stationary, a particular dot such as X moves to a neighbouring dot position Y at one glimpse of the wheel, then to the next dot position Z at the next glimpse, and so on. At the end of one second, $2f$ glimpses have occurred, where f is the frequency of the fork in Hz. If the wheel has m dots and is now rotating at n rev s^{-1}, there have been $m \times n$ dot successive movements in one second.

$$\therefore\ 2f = mn, \text{ or } f = \frac{mn}{2}. \quad . \quad . \quad . \quad . \quad (17)$$

At twice the speed of revolution, $2n$ rev s^{-1}, the dots are again seen stationary. But this time only half their number is seen, as a particular dot moves through two dot places between successive glimpses. The dots may again be seen stationary at $3n$, $4n$, . . . rev s^{-1} of the wheel if the stress on the wheel at higher speeds is below a dangerous level.

As an illustration, suppose a wheel has 40 equally spaced dots and is viewed stroboscopically by a fork of 300 Hz. If the dots are seen stationary at the lowest angular speed, the number of revs per sec, n, is given by

$$600 = n \times 40, \text{ or } n = 15 \text{ rev s}^{-1}.$$

A neon lamp, providing intermittent flashes of light at a rate which can be varied by an electrical circuit, is used as a stroboscope in industry to adjust critically the speed of rotating wheels or machinery, which then appear stationary. The wear and tear with time of the moving parts of watches have been photographed with the aid of a stroboscope.

Relative movement of dots. It is sometimes impossible to keep the speed of rotation of the wheel in Fig. 25.31 constant, so that the dots appear to move slowly forward or back. Suppose, for example, that 2 dots per second cross the line of view in a backward direction in the case of the fork and wheel just discussed. Instead of 40 × 15 or 600 successive dot movements at 15 revs per second, when the wheel appears stationary, there are now 600 − 2 or 598 successive dot movements. Since there are 40 dots round the wheel,

$$\therefore \text{ new rate of revolution of wheel} = \frac{598}{40} = 14{\cdot}95 \text{ rev s}^{-1}.$$

Suppose that the 40 dots appear stationary again at a wheel rotation of 15 revs per sec when viewed stroboscopically with the fork of frequency 300 Hz, and the fork is now loaded with a small piece of plasticine. The fork frequency is then lowered to a value f'. In this case the time interval between successive glimpses is longer than before, so that the dots appear to move forward. Suppose the movement is 3 dots per 10 seconds across the field of view. The number of successive glimpses of the wheel is $10 \times 2f'$ in 10 seconds. In this time the number of successive dot movements is $10 \times 40 \times 15 - 3$, or 5997.

$$\therefore\ 20f' = 5997$$
$$\therefore\ f' = 299{\cdot}85.$$

Thus the frequency of the fork is lowered by 0·15 Hz.

EXERCISES 25

1. A tuning-fork is considered to produce a 'pure' note. (i) Write down an equation which represents the vibration of the prongs. (ii) Explain how an *exchange of energy* occurs during the motion of the prongs.

2. State, with reasons, whether the following waves are *longitudinal* (L) or *transverse* (T): *sound, waves on plucked string, water waves, light waves*. Draw sketches to illustrate your answer.

3. If the velocity of sound in air is 340 metres per second, calculate (i) the wavelength in cm when the frequency is 256 Hz, (ii) the frequency when the wavelength is 85 cm.

4. A plane-progressive wave is represented by the equation

$$y = 0{\cdot}1 \sin (200\pi t - 20\pi x/17),$$

where y is the displacement in millimetres, t is in seconds and x is the distance from a fixed origin O in metres (m).

Find (i) the frequency of the wave, (ii) its wavelength, (iii) its speed, (iv) the phase difference in radians between a point 0·25 m from O and a point 1·10 m from O, (v) the equation of a wave with double the amplitude and double the frequency but travelling exactly in the opposite direction.

5. Describe how a *sound wave* passes through air, using graphs which illustrate and compare the variation of (i) the *displacement* of the air particles, (ii) the *pressure changes*, while the wave travels.

6. In the *falling-plate* experiment of measuring the frequency of a tuning-fork, the successive lengths occupied by 25 complete waves were 9·85 and 14·1 cm. When the experiment was repeated, the successive lengths occupied by 25 complete waves were 9·0 and 13·0 cm. Calculate the frequency of the fork for each experiment, deriving any formula you employ.

7. Describe an absolute method for determining the frequency of a tuning-fork.

Two forks *A* and *B* vibrate in unison but when two slits are fixed to the prongs of *A*, so that they are in line when the prongs are at rest, 9 beats in 10 sec. are heard when the forks are sounded together. *A* is then made to vibrate in front of a stroboscopic disc, on which are marked 50 equally spaced radial lines. The disc is viewed through the slits and the lines appear at rest when the disc rotates at 25 rev s^{-1}. What is the frequency of *B*?

Describe and explain what would be seen if the speed of rotation of the disc were slightly decreased. (*L*).

8. Describe the nature of the disturbance set up in air by a vibrating tuning-fork and show how the disturbance can be represented by a sine curve. Indicate on the curve the points of (*a*) maximum particle velocity, (*b*) maximum pressure.

What characteristics of the vibration determine the pitch, intensity, and quality respectively of the note? (*N.*)

9. Distinguish between *longitudinal* and *transverse* wave motions, giving examples of each type. Find a relationship between the frequency, wavelength and velocity of propagation of a wave motion.

Describe experiments to investigate quantitatively for sound waves the phenomena of (a) reflection, (b) refraction, (c) interference. (*C.*)

10. State and explain the differences between progressive and stationary waves.

A progressive and a stationary simple harmonic wave each have the same frequency of 250 Hz and the same velocity of 30 m s^{-1}. Calculate (i) the phase difference between two vibrating points on the progressive wave which are 10 cm apart, (ii) the equation of motion of the progressive wave if its amplitude is 0·03 metre, (iii) the distance between nodes in the stationary wave, (iv) the equation of motion of the stationary wave if its amplitude is 0·01 metre.

11. (i) Describe a stroboscope and an experiment to demonstrate one of its uses. Explain the calculation involved. (ii) Explain how sound is recorded and reproduced in a tape recorder. (*L.*)

12. Describe a *ribbon* and a *moving-coil* microphone, and explain with the aid of a diagram how each functions.

Draw a labelled diagram of the principal features of *a moving-coil loudspeaker*. Explain the purpose of the *baffle-board.*

13. Give a brief account of the principle of the stroboscope.

Describe how such a device may be used to determine the frequency of a tuning-fork. (You may, if necessary, suppose that two forms of equal frequency are available.)

Give a short account of any use made outside the laboratory of the stroboscope principle.

When viewed stroboscopically at a frequency of 300 vibrations $second^{-1}$ a circular disc with 40 equally spaced dots appears to have a backwards rotation such that two dots cross the viewing line each second. What is the least rate of rotation of the disc? (*N.*)

14. Explain the terms *damped oscillation, forced oscillation* and *resonance.* Give one example of each.

Describe an experiment to illustrate the behaviour of a simple pendulum (or pendulums) undergoing forced oscillation. Indicate qualitatively the results you would expect to observe.

What factors determine (*a*) the period of free oscillations of a mechanical system, and (*b*) the amplitude of a system undergoing forced oscillation? (*O. & C.*)