

chapter twenty-four

Velocity of light Photometry

VELOCITY OF LIGHT

FOR many centuries the velocity of light was thought to be infinitely large; from about the end of the seventeenth century, however, evidence began to be obtained which showed that the speed of light, though enormous, was a finite quantity. Galileo, in 1600, attempted to measure the velocity of light by covering and uncovering a lantern at night, and timing how long the light took to reach an observer a few miles away. Owing to the enormous speed of light, however, the time was too small to measure, and the experiment was a failure. The first successful attempt to measure the velocity of light was made by RÖMER, a Danish astronomer, in 1676.

Römer's Astronomical Method

Römer was engaged in recording the eclipses of one of Jupiter's satellites or moons, which has a period of 1.77 days round Jupiter. The period of the satellite is thus very small compared with the period of the earth round the sun (one year), and the eclipses of the satellite occur very frequently while the earth moves only a very small distance in its orbit. Thus the eclipses may be regarded as *signals sent out from Jupiter* at comparatively short intervals, and observed on the earth; almost like a bright lamp covered at regular intervals at night and viewed by a distant observer.

The earth makes a complete revolution round the sun, S , in one year. Jupiter makes a complete revolution round the sun in about $11\frac{3}{4}$ years, and we shall assume for simplicity that the orbits of the earth and Jupiter are both circular, Fig. 24.1. At some time, Jupiter (J_1) and the earth (E_1) are on the same side of the sun, S , and in line with each other, and the earth and Jupiter are then said to be in *conjunction**. Suppose that an eclipse, or "signal", is now observed on the earth E_1 . If $E_1J_1 = x$, and c is the velocity of light, the time taken for the "signal" to reach E_1 is x/c ; and if the actual time when the eclipse occurred was a (which is not known), the time T_1 of the eclipse *recorded on the earth* is given by

$$T_1 = a + \frac{x}{c} \quad \dots \dots \dots (i)$$

The earth and Jupiter now move round their respective orbits, and

* Astronomers use the terms 'conjunction' and 'opposition' in relation to the positions of the *sun* and Jupiter. With this usage, the latter are in 'opposition' here.

at some time, about $6\frac{1}{2}$ months later, the earth (E_2) and Jupiter (J_2) are on opposite sides of the sun S and in line with each other, Fig. 24.1. The earth and Jupiter are then said to be in *opposition*. During the $6\frac{1}{2}$ months

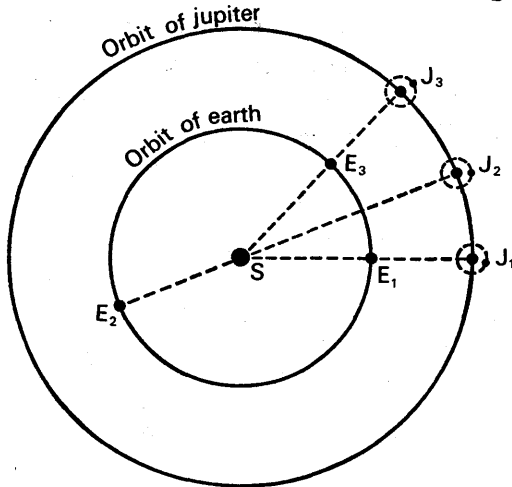


FIG. 24.1. Römer's method.

suppose that m eclipses have occurred at regular intervals T , i.e., T is the actual time between successive eclipses: the time for the interval between the 1st and m th eclipses is then $(m - 1)T$. In the position J_2, E_2 , however, the light travels a distance J_2E_2 , or $(x + d)$, from Jupiter to the earth, where d is the diameter of the earth's orbit round the sun. The time taken to travel this distance = $(x + d)/c$. Thus the time T' recorded on the earth at E_2 when the m th eclipse occurs is given by

$$T' = a + (m - 1)T + \frac{x + d}{c} \quad \dots \quad (ii)$$

But, from (i), $T_1 = a + \frac{x}{c}$

Subtracting, $T' - T_1 = I = (m - 1)T + \frac{d}{c}$, $\dots \dots \dots$ (iii)

where I is the interval recorded on the earth for the time of m eclipses, from the position of conjunction of Jupiter and the earth to their position of opposition. By similar reasoning to the above, the interval I_1 recorded on the earth for m eclipses from the position of opposition (J_2, E_2) to the next position of conjunction of (J_3, E_3) is given by

$$I_1 = (m - 1)T - \frac{d}{c} \quad \dots \dots \dots \quad (iv)$$

The reason why I_1 is less than I is that the earth is moving towards Jupiter from E_2 to E_3 , and away from Jupiter from E_1 to E_2 .

Römer observed that the m th eclipse between the position J_1, E_1 to the position J_2, E_2 occurred later than he expected by about $16\frac{1}{2}$ minutes; and he correctly deduced that the additional time was due to the time

taken by light to travel across the earth's orbit. In (iii), $(m - 1) T$ was the time expected for $(m - 1)$ eclipses, and d/c was the extra time ($16\frac{1}{2}$ minutes) recorded on the earth. Since $d = 300\,000\,000$ km approximately, the velocity of light, $c = 300\,000\,000 / (16\frac{1}{2} \times 60) = 300\,000$ km per second approximately.

Römer also recorded that the time I for m eclipses from the position E_1, J_1 to the position E_2, J_2 was about 33 minutes more than the time I_1 , for m eclipses between position E_2, J_2 to the position E_3, J_3 . But, subtracting (iv) from (iii),

$$I - I_1 = \frac{2d}{c} \quad \dots \dots \dots (1)$$

$$\therefore \frac{2d}{c} = 33 \text{ mins} = 33 \times 60 \text{ secs}$$

$$\therefore c = 2d / (33 \times 60) = 2 \times 300\,000\,000 / (33 \times 60)$$

$$\therefore c = 300\,000 \text{ km per second (approx.)}$$

Maximum and minimum observed periods. When the earth E_1 is moving directly away from Jupiter at J_a , the apparent period T' of the satellite is a maximum. Suppose the earth moves from E_1 to E_2 in the time T' , Fig. 24.2.

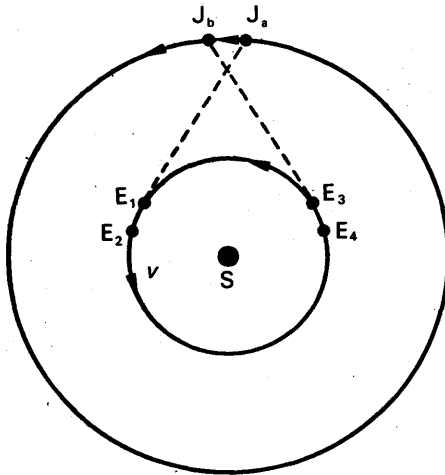


FIG. 24.2. Maximum and minimum periods.

Then if T is the actual period, v is the velocity of the earth, and c is the velocity of light, it follows that

$$T' = T + \frac{E_1E_2}{c} = T + \frac{vT'}{c}$$

$$\therefore T' \left(1 - \frac{v}{c} \right) = T \quad \dots \dots \dots (i)$$

When the earth is moving directly towards Jupiter at J_b , the apparent period T'' is a minimum. Suppose the earth moves from E_3 to E_4 in this time. Then

$$T'' = T - \frac{E_3 E_4}{c} = T - \frac{vT''}{c}$$

$$\therefore T'' \left(1 + \frac{v}{c}\right) = T \quad \dots \quad (ii)$$

From (i) and (ii), it follows that

$$T' \left(1 - \frac{v}{c}\right) = T'' \left(1 + \frac{v}{c}\right)$$

$$\therefore \frac{T'}{T''} = \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} = \text{ratio of maximum and minimum observed periods}$$

Bradley's Aberration Method

Römer's conclusions about the velocity of light were ignored by the scientists of his time. In 1729, however, the astronomer BRADLEY observed that the angular elevation of a "fixed" star varied slightly according to the position of the earth in its orbit round the sun. For some time he was puzzled by the observation. But while he was being rowed across a stream one day, he noticed that the boat drifted slightly downstream; and he saw immediately that the difference between the actual and observed angular elevation of the star was due to a combination of the velocity of the earth in its orbit (analogous to the velocity of the stream) with that of the velocity of light (analogous to the velocity of the boat). Thus if the earth were stationary, a telescope T would have to point in the true direction AS of a star S to observe it; but since the earth is moving in its orbit round the sun with a velocity v , T would have to be directed along MN to observe the star, where MN makes a small angle α with the direction SA , Fig. 24.3 (i).

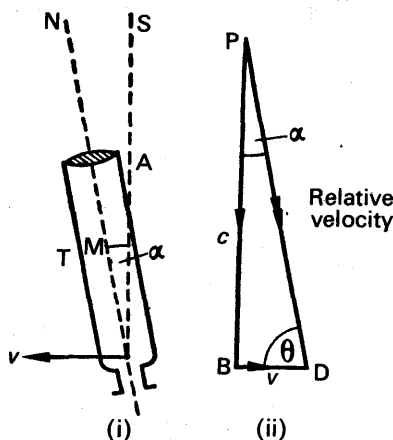


FIG. 24.3. Bradley's aberration method.

The direction of MN is that of the *relative velocity* between the earth and the light from S, which is found by subtracting the velocity v from the velocity c of the light. This is easily done by drawing the triangle of velocities PBD, in which PB, parallel to SA, represents c in magnitude and direction, while BD represents a velocity equal and opposite to v , Fig. 24.3 (ii). The resultant of PB and BD is then PD, which is parallel to NM, and PD represents the relative velocity.

The angle α between the true and apparent directions of the star is known as the *aberration*. From Fig. 24.3 (ii), it follows that

$$\frac{v}{c} = \frac{\sin \alpha}{\sin \theta},$$

where θ is the apparent altitude of the star.

$$\therefore c = \frac{v \sin \theta}{\sin \alpha} \quad (2)$$

Since α is very small, $\sin \alpha$ is equal to α in radians. By using known values of v , θ , and α , Bradley calculated c , the velocity of light, and obtained a value close to Römer's value. The aberration α is given by half the difference between the maximum and minimum values of the apparent altitude, θ , of the star.

Fizeau's Rotating Wheel Method. A Terrestrial Method

In 1849 FIZEAU succeeded in measuring the velocity of light with apparatus on the earth, for the first time. His method, unlike Römer's and Bradley's method, is thus known as a *terrestrial method*.

Fizeau's apparatus is illustrated in Fig. 24.4. A bright source at O emits light which is converged to a point H by means of the lens and the plane sheet of glass F, and is then incident on a lens B. H is at the focus of B, and the light thus emerges parallel after refraction through the latter and travels several miles to another lens C. This lens brings the light to a focus at M, where a silvered plane mirror is positioned, and the light is now returned back along its original path to the glass plate F. An image of O can thus be observed through F by a lens E.

The rim of a toothed wheel, W, which can rotate about a horizontal axis Q, is placed at H, and is the important feature of Fizeau's method.

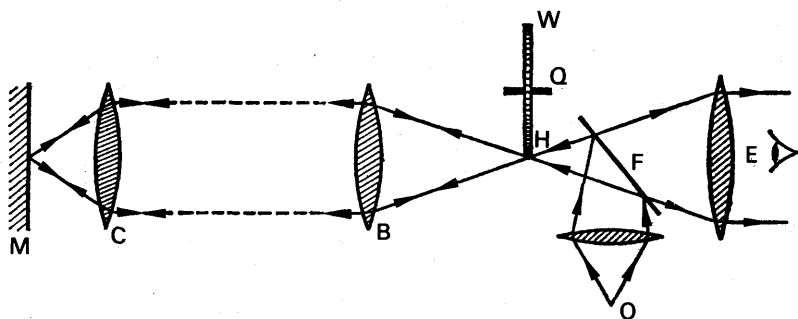


FIG. 24.4. Fizeau's rotating wheel method.

The teeth and the gaps of W have the same width, Fig. 24.5. As W is rotated, an image is observed through E as long as the light passes through the wheel towards E. When the speed of rotation exceeds about 10 cycles per second, the succession of images on the retina causes an image of O to be seen continuously. As the speed of W is further increased, however, a condition is reached when the returning light passing through a gap of W and reflected from M arrives back at the wheel to find that the tooth *next* to the gap has taken the place of the gap. Assuming the wheel is now driven at a constant speed, it can be seen that light continues to pass through a gap in the wheel towards M but always arrives back at W to find its path barred by the neighbouring tooth. The field of view through E is thus dark. If the speed of W is now doubled a bright field of view is again observed, as the light passing through a gap arrives back from M to find the next *gap* in its place, instead of a tooth as before.

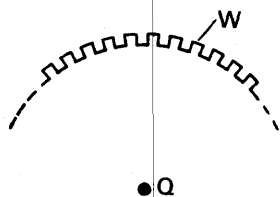


FIG. 24.5.
Rotating wheel.

Fizeau used a wheel with 720 teeth, and first obtained a dark field of view through E when the rate of revolution was 12.6 revs per second. The distance from H to M was 8633 metres. Thus the time taken for the light to travel from H to M and back = $2 \times 8633/c$ seconds, where c is the velocity of light in metres per second. But this is the time taken by a tooth to move to a position corresponding to a neighbouring gap. Since there are 2×720 teeth and gaps together, the time = $1/(2 \times 720)$ of the time taken to make one revolution, or $1/(2 \times 720 \times 12.6)$ seconds, as 12.6 revs are made in one second.

$$\begin{aligned} \therefore \frac{2 \times 8633}{c} &= \frac{1}{2 \times 720 \times 12.6} \\ \therefore c &= 2 \times 8633 \times 2 \times 720 \times 12.6 \\ &= 3.1 \times 10^8 \text{ metres per sec.} \end{aligned}$$

The *disadvantage* of Fizeau's method is mainly that the field of view can never be made perfectly dark, owing to the light diffusely reflected at the teeth towards E. To overcome this disadvantage the teeth of the wheel were bevelled, but a new and more accurate method of determining the velocity of light was devised by FOUCAULT in 1862.

Foucault's Rotating Mirror Method

In Foucault's method a plane mirror M_1 is rotated at a high constant angular velocity about a vertical axis at A, Fig. 24.6. A lens L is placed so that light from a bright source at O_1 is reflected at M_1 and comes to a focus at a point P on a concave mirror C. The centre of curvature of C is at A, and consequently the light is reflected back from C along its original path, giving rise to an image coincident with O_1 . In order to observe the image, a plate of glass G is placed at 45° to the axis of the lens, from which the light is reflected to form an image at B_1 .

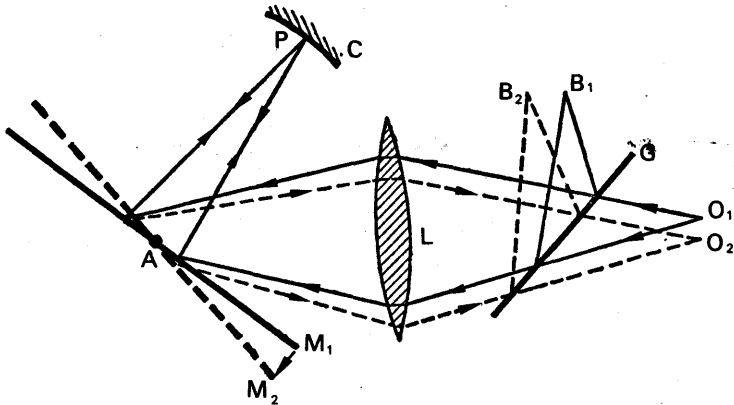


FIG. 24.6. Foucault's rotating mirror method.

Suppose the plane mirror M_1 begins to rotate. The light reflected by it is then incident on C for a fraction of a revolution, and if the speed of rotation is 2 revs per sec, an intermittent image is seen. As the speed of M_1 is increased to about 10 revs per sec the image is seen continuously as a result of the rapid impressions on the retina. As the speed is increased further, the light reflected from the mirror flashes across from M_1 to C, and returns to M_1 to find it displaced by a very small angle θ to a new position M_2 . An image is now observed at B_2 , and by measuring the displacement, B_1B_2 , of the image Foucault was able to calculate the velocity of light.

Theory of Foucault's Method

Consider the point P on the curved mirror from which the light is always reflected back to the plane mirror, Fig. 24.7. When the plane mirror is at M_1 , the image of P in it is at I_1 , where $AI_1 = AP = a$, the radius of curvature of C (see p. 396). The rays incident on the lens L from the plane mirror appear to come from I_1 . When the mirror is at M_2 the image of P in it is at I_2 , where $AI_2 = AP = a$, and the rays incident on L from the mirror now appear to come from I_2 . Now the

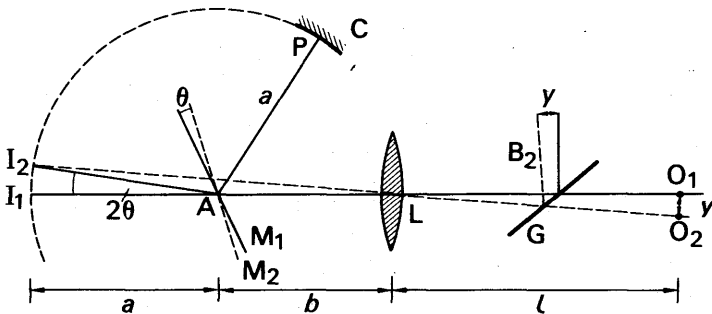


FIG. 24.7. Explanation of Foucault's method.

mirror has rotated through an angle θ from M_1 to M_2 , and the direction PA of the light incident on it is constant. The angle between the reflected rays is thus 2θ (see p. 393), and hence $I_2A_1I_1 = 2\theta$.

$$\therefore I_2I_1 = a \times 2\theta = 2a\theta \quad \dots \quad (i)$$

The images O_1, O_2 formed by the lens, L, are the images of I_1, I_2 in it, as the light incident on L from the mirror appear to come from I_1, I_2 . Hence $I_2I_1 : O_1O_2 = I_1L : LO_1$.

$$\therefore \frac{I_2I_1}{y} = \frac{(a+b)}{l}$$

where $y = O_1O_2$, $AL = b$, and $LO_1 = l$.

$$\therefore I_2I_1 = \frac{(a+b)y}{l} \quad \dots \quad (ii)$$

From (i) and (ii), it follows that

$$2a\theta = \frac{(a+b)y}{l}$$

$$\therefore \theta = \frac{(a+b)y}{2al} \quad \dots \quad (iii)$$

The angle θ can also be expressed in terms of the velocity of light, c , and the number of revolutions per second, m , of the plane mirror. The angular velocity of the mirror is $2\pi m$ radians per second, and hence the time taken to rotate through an angle θ radians is $\theta/2\pi m$ secs. But this is the time taken by the light to travel from the mirror to C and back, which is $2a/c$ secs.

$$\therefore \frac{\theta}{2\pi m} = \frac{2a}{c}$$

$$\therefore \theta = \frac{4\pi ma}{c} \quad \dots \quad (iv)$$

From (iii) and (iv), we have

$$\frac{(a+b)y}{2al} = \frac{4\pi ma}{c}$$

$$\therefore c = \frac{8\pi ma^2l}{(a+b)y} \quad \dots \quad (3)$$

As m, a, l, b are known, and the displacement $y = O_1O_2 = B_1B_2$ and can be measured, the velocity of light c can be measured.

The disadvantage of Foucault's method is mainly that the image obtained is not very bright, making observation difficult. Michelson (p. 559) increased the brightness of the image by placing a large lens between the plane mirror and C, so that light was incident on C for a greater fraction of the mirror's revolution. Since the distance a was increased at the same time, Fig. 24.7, the displacement of the image was also increased.

The velocity of light in water was observed by Foucault, who placed a pipe of water between the plane mirror and C. He found that, with

the number of revolutions per second of the mirror the same as when air was used, the displacement y of the image B was *greater*. Since the velocity of light $= 8\pi ma^2l/(a + b) y$, from (3), it follows that the velocity of light in water is *less* than in air. Newton's "corpuscular theory" of light predicted that light should travel faster in water than in air (p. 680), whereas the "wave theory" of light predicted that light should travel slower in water than in air. The direct observation of the velocity of light in water by Foucault's method showed that the corpuscular theory of Newton could not be true.

Michelson's Method for the Velocity of Light

The velocity of light, c , is a quantity which appears in many fundamental formulæ in advanced Physics, especially in connection with the theories concerning particles in atoms and calculations on atomic (nuclear) energy. EINSTEIN has shown, for example, that the energy W released from an atom is given by $W = mc^2$ joules, where m is the decrease in mass of the atom in kilogrammes and c the velocity of light in metres per second. A knowledge of the magnitude of c is thus important. A. A. MICHELSON, an American physicist, spent many years of his life in measuring the velocity of light, and the method he devised is regarded as one of the most accurate.

The essential features of Michelson's apparatus are shown in Fig. 24.8. X is an equiangular octagonal steel prism which can be rotated at constant speed about a vertical axis through its centre. The faces of the prism are highly polished, and the light passing through a slit from a very bright source O is reflected at the surface A towards a plane mirror B. From B the light is reflected to a plane mirror L, which is placed so that the image of O formed by this plane mirror is at the focus of a large concave mirror HD. The light then travels as a parallel

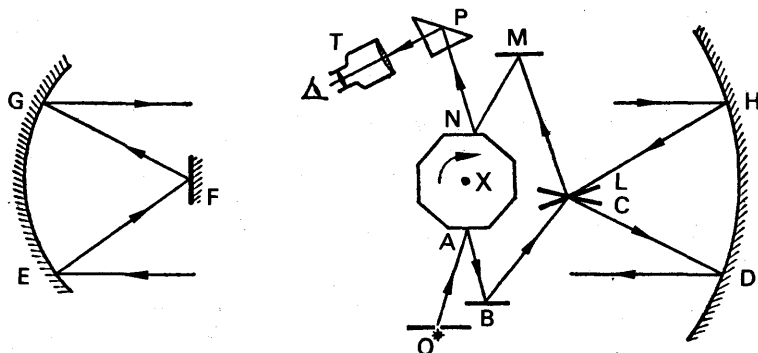


FIG. 24.8. Michelson's rotating prism method.

beam to another concave mirror GE a long distance away, and it is reflected to a plane mirror F at the focus of GE. The light is then reflected by the mirror, travels back to H, and is there reflected to a plane mirror C placed just below L and inclined to it as shown. From C the light is reflected to a plane mirror M, and is then incident on the face N of the octagonal prism opposite to A.

The final image thus obtained is viewed through T with the aid of a totally reflecting prism P.

The image is seen by light reflected from the top surface of the octagonal prism X. When the latter is rotated the image disappears at first, as the light reflected from A when the prism is just in the position shown in Fig. 24.8 arrives at the opposite face to find this surface in some position inclined to that shown. When the speed of rotation is increased and suitably adjusted, however, the image reappears and is seen in the same position as when the prism X is at rest. *The light reflected from A now arrives at the opposite surface in the time taken for the prism to rotate through 45° , or $\frac{1}{8}$ th of a revolution*, as in this case the surface on the left of N, for example, will occupy the latter's position when the light arrives at the upper surface of X.

Suppose d is the total distance in metres travelled by the light in its journey from A to the opposite face; the time taken is then d/c , where c is the velocity of light. But this is the time taken by X to make $\frac{1}{8}$ th of a revolution, which is $1/8m$ secs if the number of revolutions per second is m .

$$\begin{aligned}\therefore \frac{1}{8m} &= \frac{d}{c} \\ \therefore c &= 8md \text{ metres per sec.} \quad \dots \quad (4)\end{aligned}$$

Thus c can be calculated from a knowledge of m and d .

Michelson performed the experiment in 1926, and again in 1931, when the light path was enclosed in an evacuated tube 1.6 km long. Multiple reflections were obtained to increase the effective path of the light. A prism with 32 faces was also used, and Michelson's result for the velocity of light *in vacuo* was 299 774 kilometres per second. Michelson died in 1931 while he was engaged in another measurement of the velocity of light.

EXAMPLES

1. Describe carefully Fizeau's method of determining the speed of propagation of light by means of a toothed wheel. Given that the distance of the mirror is 8000 m, that the revolving disc has 720 teeth, and that the first eclipse occurs when the angular velocity of the disc is $13\frac{3}{4}$ revolutions per second, calculate the speed of propagation of light. (*W*.)

First part. See text.

Second part. Suppose c is the speed of light in metres per second.

$$\therefore \text{time to travel to mirror and back} = \frac{2 \times 8000}{c} \text{ s.}$$

$$\text{But time for one tooth to occupy the next gap's position} = \frac{1}{13\frac{3}{4}} \times \frac{1}{2 \times 720} \text{ s.}$$

$$\therefore \frac{2 \times 8000}{c} = \frac{1}{13\frac{3}{4} \times 2 \times 720}$$

$$\begin{aligned}\therefore c &= 2 \times 8000 \times 13\frac{3}{4} \times 2 \times 720 \\ &= 3.2 \times 10^8 \text{ m s}^{-1}.\end{aligned}$$

2. A beam of light is reflected by a rotating mirror on to a fixed mirror, which sends it back to the rotating mirror from which it is again reflected, and then makes an angle of 18° with its original direction. The distance between the two mirrors is 10^4 m, and the rotating mirror is making 375 revolutions per sec. Calculate the velocity of light. (L)

Suppose OA is the original direction of the light, incident at A on the mirror in the position M_1 , B is the fixed mirror, and AC is the direction of the light reflected from the rotating mirror when it reaches the position M_2 , Fig. 24.9.

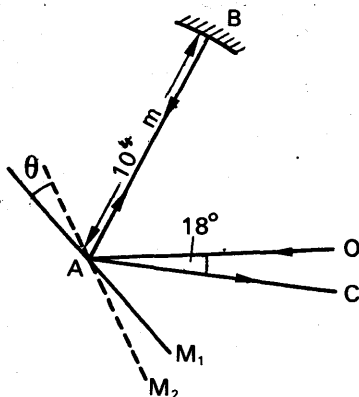


FIG. 24.9. Example.

The angle θ between M_1 , M_2 is $\frac{1}{2} \times 18^\circ$, since the angle of rotation of a mirror is half the angle of deviation of the reflected ray when the incident ray (BA in this case) is kept constant. Thus $\theta = 9^\circ$.

$$\text{Time taken by mirror to rotate } 360^\circ = \frac{1}{375} \text{ s.}$$

$$\therefore \text{ time taken to rotate } 9^\circ = \frac{9}{360} \times \frac{1}{375} \text{ s.}$$

But this is also the time taken by the light to travel from A to B and back, which is given by $2 \times 10^4/c$, where c is the velocity of light in m s^{-1} .

$$\therefore \frac{2 \times 10^4}{c} = \frac{9}{360} \times \frac{1}{375}$$

$$\therefore c = \frac{2 \times 10^4 \times 360 \times 375}{1} = 3 \times 10^8 \text{ m s}^{-1}.$$

Photometry

Standard Candle. The Candela

Light is a form of energy which stimulates the sensation of vision. The sun emits a continuous stream of energy, consisting of ultra-violet, visible, and infra-red radiations (p. 344), all of which enter the eye; but only the energy in the visible radiations, which is called *luminous energy*, stimulates the sensation of vision. In Photometry we are concerned only with the luminous energy emitted by a source of light.

Years ago the luminous energy per second from a candle of specified

wax material and wick was used as a standard of luminous energy. This was called the *British Standard Candle*. The luminous energy per second from any other source of light was reckoned in terms of the standard candle, and its value was given at 10 candle-power (10 c.p.) for example. As the standard candle was difficult to reproduce exactly, the standard was altered. It was defined as one-tenth of the intensity of the flame of the *Vernon Harcourt pentane lamp*, which burns a mixture of air and pentane vapour under specified conditions. Later it was agreed to use as a standard the *international standard candle*, which is defined in terms of the luminous energy per second from a particular electric lamp filament maintained under specified conditions, but the precision of this standard was found to be unsatisfactory. In 1948, a unit known as the *candela*, symbol "cd", was adopted. This is defined as the luminous intensity of $1/600\,000$ metre² ($1/60$ cm²) of the surface of a black body at the temperature of freezing platinum under 101 325 newtons per metre² pressure. A standard is maintained at the National Physical Laboratory.

Illumination and its Units

If a lamp S of 1 candela is placed 1 metre away from a small area A and directly in front of it, the *illumination* of the surface of A is said to be 1 *metre-candle* or *lux*, Fig. 24.10. If the same lamp is placed 1 centimetre away from A, instead of 1 m, the illumination of the surface is said to be 1 *cm-candle* (or 1 *phot*). The SI unit of illumination is the lux (see also p. 564). The "foot-candle" has been used as a unit; the distance of 1 metre in Fig. 24.10 is replaced by 1 foot. $1 \text{ lux} = 10^{-4} \text{ phot} = 9.3 \times 10^{-2} \text{ foot-candle}$. It is recommended that offices should have an intensity of illumination of about 90 lux, and that the intensity of illumination for sewing dark materials in workrooms should be about 200 lux.

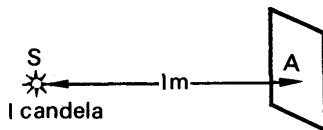


FIG. 24.10. Metre-candle (lux).

Luminous Flux, F

In practice a source of light emits a continuous stream of energy, and the name *luminous flux* has been given to the *luminous energy emitted per second*. The unit of luminous flux is the **lumen**, lm. Since a lumen is a certain amount of "energy per second", or "power", there must be a relation between the lumen and the watt, the mechanical unit of power; and experiment shows that 621 lumens of a green light of wavelength 5.540×10^{-10} m is equivalent to 1 watt.

Solid Angle

A lamp radiates luminous flux in all directions round it. If we think of a particular small lamp and a certain direction from it, for example that of the corner of a table, we can see that the flux is radiated towards the corner in a cone whose apex is the lamp. A thorough study of

photometry must therefore include a discussion of the measurement of an angle in three dimensions, such as that of a cone, which is known as a *solid angle*.

An angle in two dimensions, i.e., in a plane, is given in radians by the ratio s/r , where s is the length of the arc cut off by the bounding lines of the angle on a circle of radius r . In an analogous manner, the solid angle, ω , of a cone is defined by the relation

$$\omega = \frac{S}{r^2} \quad \dots \quad (5)$$

where S is the area of the surface of a sphere of radius r cut off by the bounding (generating) lines of the cone, Fig. 24.11 (i). Since S and r^2 both have the dimensions of (length)², the solid angle ω is a ratio.

When $S = 1 \text{ m}^2$, and $r = 1 \text{ m}$, then $\omega = 1$ from equation (5). Thus *unit solid angle* is subtended at the centre of a sphere of radius

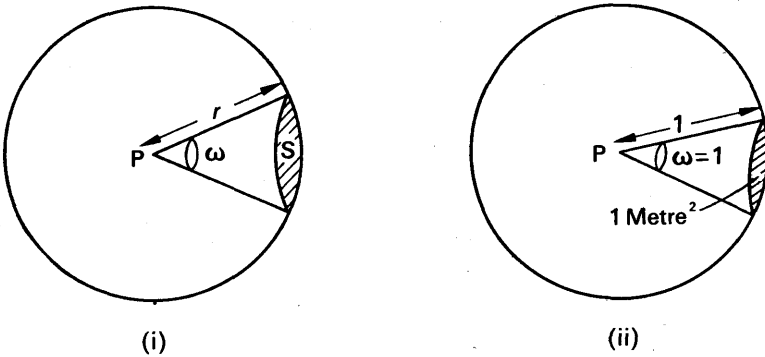


FIG. 24.11. (i). Solid angle at P. (ii). Unit solid angle at P.

1 m by a cap of surface area 1 m², Fig. 24.11 (ii). It is called "1 steradian", sr. The solid angle all round a point is given from (5) by

$$\frac{\text{total surface area of sphere}}{r^2}$$

i.e., by $\frac{4\pi r^2}{r^2}$, or 4π .

Thus the solid angle all round a point is 4π sr. The solid angle all round a point on one side of a plane is thus 2π sr.

Luminous Intensity of Source, I

Experiment shows that the luminous flux from a source of light varies in different directions; to be accurate, we must therefore consider the luminous flux emitted in a particular direction. Suppose that we consider a small lamp P, and describe a cone PCB of small solid angle ω about a particular direction PD as axis, Fig. 24.12. The *luminous intensity, I, of the source in this direction* is then defined by the relation

$$I = \frac{F}{\omega} \quad \dots \quad (6)$$

where F is the luminous flux contained in the small cone. Thus the luminous intensity of the source is the *luminous flux per unit solid angle* in the particular direction. It can now be seen that "luminous intensity" is a measure of the "luminous flux density" in the direction concerned.

The unit of luminous intensity of a source is the *candela*, defined on p. 562, and the luminous intensity was formerly known as the *candle-power* of the source. When the luminous flux, F , in the cone in Fig. 24.12 is 1 lumen (the unit of luminous flux), and the solid angle, ω , of the cone is 1 unit, it follows from equation (6) that $I = 1$ candela. Thus *the lumen can be defined as the luminous flux radiated within unit solid angle by a uniform source of one candela*. A small source of I candela radiates $4\pi I$ lumens all round it.

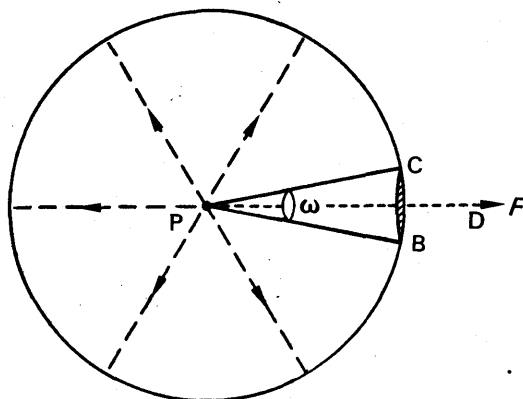


FIG. 24.12. Luminous intensity of source.

Illumination of Surface

On p. 562 we encountered various units of *illumination of a surface*; these were the metre-candle or the lux (lx), which is the SI unit, the cm-candle and the foot-candle. The illumination is defined generally as the *luminous flux per unit area* falling on the part of the surface under consideration. Thus if F is the luminous flux incident on a small area A , the intensity of illumination, E , is given by

$$E = \frac{F}{A} \quad \dots \quad (7)$$

When $F = 1$ lumen and $A = 1 \text{ m}^2$, then $E = 1 \text{ lm m}^{-2}$. Thus 1 m-candle of illumination is equivalent to an illumination of 1 lumen per m^2 of the surface. Similarly, 1 cm-candle (p. 562) is equivalent to an illumination of 1 lumen per cm^2 of the surface. $1 \text{ lux} = 1 \text{ lm m}^{-2}$.

Suppose a lamp of 50 candela illuminates an area of 2 m^2 at a distance 20 m away. The flux F emitted all round the lamp $= I\omega = I \times 4\pi = 50 \times 4\pi = 200\pi$ lumens. The flux per unit area at a distance of 20 m away is given by

$$\frac{200\pi}{4\pi r^2} = \frac{200\pi}{4\pi \times 20^2} \text{ lumens per m}^2$$

∴ flux falling on an area of 2 m²
 = 2 × $\frac{200\pi}{4\pi \times 20^2}$ = 0.25 lumens.

The reader should take pains to distinguish carefully between the meaning of "luminous intensity" and "illumination" and their units. The former refers to the *source* of light and is measured in candelas (formerly candle-power); the latter refers to the *surface* illuminated and is measured in lux or metre-candles. Further, "luminous intensity" is defined in terms of unit solid angle, which concerns three dimensions whereas "illumination" is defined in terms of unit area, which concerns two dimensions.

Relation between Luminous Intensity (*I*) and Illumination (*E*)

Consider a *point* source of light of uniform intensity and a small part X of a surface which it illuminates. If the source of light is doubled, the illumination of X is doubled because the luminous flux incident on it is twice as much. Thus

$$E \propto I \quad \dots \quad (i)$$

where *E* is the illumination due to a point source of luminous intensity *I* at a given place.

The illumination of the surface also depends on the distance of X from the source. Suppose two spheres of radii *r*₁, *r*₂ are drawn round a point source of intensity *I*, such as S in Fig. 24.13 (i). The same amount

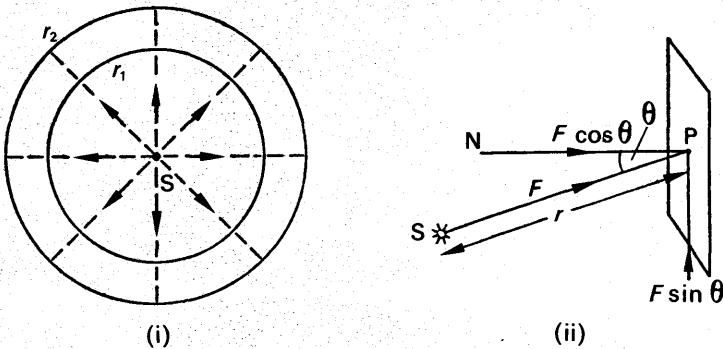


FIG. 24.13. (i). Inverse square law. (ii). Lambert's cosine rule.

of luminous flux *F* spreads over the surface area ($4\pi r^2$) of both spheres, and hence $E_1 : E_2 = F/4\pi r_1^2$, where E_1, E_2 are the values of the illumination at the surface of the smaller and larger spheres respectively. Thus $E_1 : E_2 = r_2^2 : r_1^2$. It can hence be seen that the illumination due to a given point source varies inversely as the square of the distance from the source, i.e.,

$$E \propto \frac{1}{r^2} \quad \dots \quad (ii)$$

In the eighteenth century LAMBERT showed that the illumination round a particular point P on a surface is proportional to $\cos \theta$, where θ is the angle between the normal PN at P to the surface and the line SP joining the source S to P, Fig. 24.13 (ii). A rigid proof of Lambert's law is given shortly, but we can easily see qualitatively why the cosine rule is true. The luminous flux F illuminating P is in the direction SP, and thus has a component $F \cos \theta$ along NP and a component $F \sin \theta$ parallel to the surface, Fig. 24.13. The latter does not illuminate the surface. Hence the effective part of the flux F is $F \cos \theta$.

From equations (i) and (ii) and the cosine law, it follows that the illumination E round P due to the source S of luminous intensity I is given by

$$E = \frac{I \cos \theta}{r^2} \quad \dots \dots \dots (8)$$

where $SP = r$. This is a fundamental equation in Photometry, and it is proved rigidly on p. 567. In applying it in practice one has to take into account that (i) a "point source" is difficult to realise, (ii) the area round the point considered on the surface should be very small so that the flux incident all over it can be considered the same, (iii) the intensity I of a source varies in different directions, (iv) the actual value of illumination round a point on a table, for example, is not only due to the electric lamp above it but also to the luminous flux diffusely reflected towards the point from neighbouring objects such as walls.

Example: Suppose that we are required to calculate the intensity I of a lamp S fixed 4 m above a horizontal table, if the value of the illumination at a point P on the table 3 m to one side of the vertical through the lamp is 6 m-candles, Fig. 24.14.

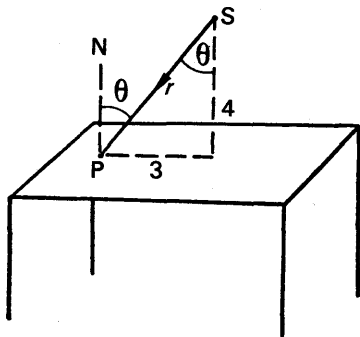


FIG. 24.14. Example.

The illumination, E , at P is given by

$$E = \frac{I \cos \theta}{r^2} \quad \dots \dots \dots (i)$$

where I is the luminous intensity of S, θ is the angle between SP and the normal PN to the table at P, and $r = SP$.

But

$$r^2 = 4^2 + 3^2 = 25, \text{ i.e., } r = 5,$$

$$\cos \theta = \frac{4}{r} = \frac{4}{5},$$

and

$$E = 6 \text{ m-candles.}$$

Substituting in (i),

$$\therefore 6 = \frac{I \times \frac{4}{5}}{25}$$

$$\therefore I = 187.5 \text{ candelas}$$

Proof of $E = I \cos \theta/r^2$. Consider a source S of intensity I illuminating a very small area A round a point P on a surface, Fig. 24.15. If ω is the solid angle at S in the cone obtained by joining S to the boundary of the area, the illumination at P is given by

$$E = \frac{F}{A} = \frac{I\omega}{A}$$

as $I = \frac{F}{\omega}$ (p. 563). Now, by definition,

$$\omega = \frac{A_1}{r^2},$$

where A_1 is the area cut off on a sphere of centre S and radius r ($= SP$) by the generating lines of the cone.

$$\therefore E = \frac{I\omega}{A} = \frac{I A_1}{r^2 A},$$

But

$$A_1 = A \cos \theta,$$

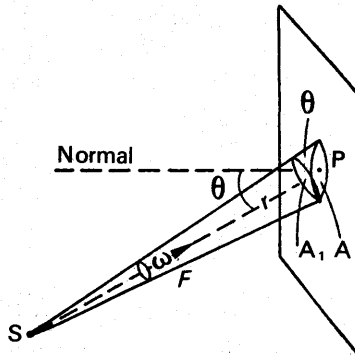


FIG. 24.15. Proof of $E = I \cos \theta/r^2$.

since A_1 is the projection of A on the sphere of centre S, and θ is the angle between the two areas as well as the angle made by SP with the normal to the surface, Fig. 24.15.

$$\therefore E = \frac{I A \cos \theta}{r^2 A} = \frac{I \cos \theta}{r^2}$$

Luminance of a Surface: Reflection and Transmission Factors

The *luminance* of a surface in a given direction is defined as the luminous flux per unit area *coming from* the surface in the particular direction. The luminance of white paint on the wall of a room is considerably higher than the luminance of a brown-painted panel in the middle of the wall; the luminance of a steel nib is much greater than that of a dark ebonite penholder.

The “luminance” of a particular surface should be carefully distinguished from the “illumination” of the surface, which is the luminous flux per sq m *incident on* the surface. Thus the illumination of white chalk on a particular blackboard is practically the same as that of the neighbouring points on the board itself, whereas the luminance of the chalk is much greater than that of the board. The difference in luminance

ance is due to the difference in the *reflection factor*, r , of the chalk and board, which is defined by the relation

$$r = \frac{B}{E} \dots \dots \dots (9)$$

where B is the luminance of the surface and E is the illumination of the surface. Thus the luminance, B , is given by

$$B = rE \dots \dots \dots (10)$$

Besides reflection, the luminance of a surface may be due to the transmission of luminous flux through it. The luminance of a pearl lamp, for example, is due to the transmission of luminous flux through its surface. The *transmission factor*, t , of a substance is a ratio which is defined by

$$t = \frac{F}{E} \dots \dots \dots (11)$$

where F is the luminous flux per m^2 transmitted through the substance and E is the luminous flux per m^2 incident on the substance. Thus

$$F = tE \dots \dots \dots (12)$$

The Lummer-Brodhun Photometer

A *photometer* is an instrument which can be used for comparing the luminous intensities of sources of light. One of the most accurate forms of photometer was designed by LUMMER and BRODHUN, and the essential features of the instrument are illustrated in Fig. 24.16 (i).

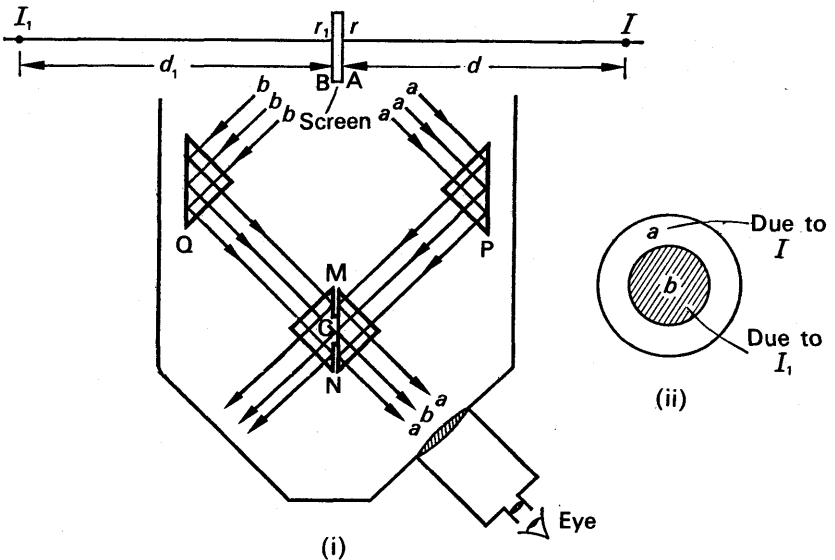


FIG. 24.16. Lummer-Brodhun photometer.

Lamps of luminous intensities I_1 and I_2 respectively are placed on opposite sides of a white opaque screen, and some of the diffusely-reflected light from the opposite surfaces A, B is incident on two identical totally reflecting prisms P, Q, Fig. 24.16 (i). The light reflected from the prisms then passes towards the "Lummer-Brodhun cube", which is the main feature of the photometer. This consists of two right-angled isosceles prisms in optical contact at their central portion C, but with the edges of one cut away so that an air-film exists at M, N all round C between the prisms. The rays leaving the prism P are thus transmitted through the central portion C of the "cube", but totally reflected at the edges. Similarly, the rays reflected from the prism Q towards the "cube" are transmitted through the central portion but totally reflected at the edges. An observer of the "cube" thus sees a central circular patch b of light due initially to the light from the source of intensity I_1 , and an outer portion α due initially to the light from the source of intensity I_2 Fig. 24.16 (ii).

Comparison of Luminous Intensities

In general, the brightness of the central and outer portions of the field of view in a Lummer-Brodhun photometer is different, so that one appears darker than the other. By moving one of the sources, however, a position is obtained when both portions appear equally bright, in which case they cannot be distinguished from each other and the field of view is uniformly bright. A "photometric balance" is then said to exist.

Suppose that the distances of the sources I_1 and I from the screen are d_1, d respectively, Fig. 24.16. The intensity of illumination, E , due to the source I is generally given by $E = I \cos \theta / d^2$ (p. 566). But $\theta = 0$ in this case, as the line joining the source to the screen is normal to the screen. Hence, since $\cos 0^\circ = 1$, $E = I / d^2$. Similarly, the intensity of illumination, E_1 , at the screen due to the source I_1 is given by $E_1 = I_1 / d_1^2$. Now the luminance of the surface B = $r_1 E_1$, where r_1 is the reflection factor of the surface (p. 568); and the luminance of the surface A = rE , where r is the reflection factor of this surface. Hence, for a photometric balance,

$$r_1 E_1 = rE$$

$$\therefore \frac{r_1 I_1}{d_1^2} = \frac{rI}{d^2} \quad \dots \dots \dots (i)$$

If the reflection factors r_1, r of A, B are equal, equation (i) becomes

$$\frac{I_1}{d_1^2} = \frac{I}{d^2}$$

$$\therefore \frac{I_1}{I} = \frac{d_1^2}{d^2}$$

The ratio of the intensities are hence proportional to the squares of the corresponding distances of the sources from the screen.

The reflection factors r_1, r are not likely to be exactly equal, however, in which case another or *auxiliary lamp* is required to compare the

candle-powers I_1, I . The auxiliary lamp, I_2 , is placed on the right side, say, of the screen at a distance d_2 , and one of the other lamps is placed on the other side. A photometric balance is then obtained,

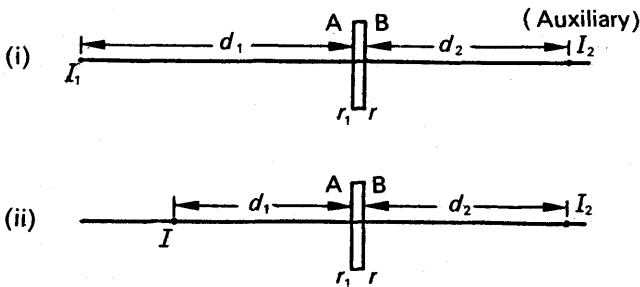


FIG. 24.17. Comparison of luminous intensities.

Fig. 24.17 (i). In this case, if I_1 is the intensity of the lamp and d_1 is its distance from the screen,

$$r_1 \frac{I_1}{d_1^2} = r \frac{I_2}{d_2^2} \dots \dots \dots \quad (ii)$$

The remaining lamp I is then used instead of the lamp I_1 , and a photometric balance is again obtained by moving this lamp, keeping the position of the lamp I_2 unaltered, Fig. 24.17 (ii). Suppose the distance of the lamp I from the screen is d . Then

$$r_1 \frac{I}{d^2} = r \frac{I_2}{d_2^2} \dots \dots \dots \quad (iii)$$

From (ii) and (iii), it follows that

$$\begin{aligned} r_1 \frac{I_1}{d_1^2} &= r_1 \frac{I}{d^2} \\ \therefore \frac{I_1}{d_1^2} &= \frac{I}{d^2} \\ \therefore \frac{I_1}{I} &= \frac{d_1^2}{d^2} \dots \dots \dots \quad (13) \end{aligned}$$

The intensities I_1, I are hence proportional to the squares of the corresponding lamp distances from the screen. It should be noted from (13) that the auxiliary lamp's intensity is not required in the comparison of I_1 and I , nor is its constant distance d_2 from the screen required.

Measurement of Illumination

It was pointed out at the beginning of the chapter that the maintenance of standards of illumination plays an important part in safeguarding our health. It is recommended that desks in class-rooms and

offices should have an illumination of 60–110 lux, and workshops an illumination of 120–180 lux; for sewing dark materials an intensity of 180–300 lux is recommended, while 1200 lux is suggested for the operating table in a hospital.

Photovoltaic cell. There are two types of meters for measuring the illumination of a particular surface. A modern type is the photovoltaic cell, which may consist of a cuprous oxide and copper plate, made by

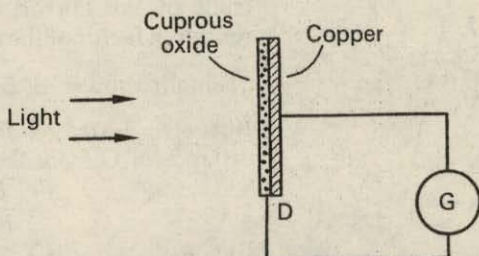


FIG. 24.18. Photovoltaic cell.

oxidising one side of a copper disc D, Fig. 24.18. When the oxide surface is illuminated, electrons are emitted from the surface whose number is proportional to the incident luminous energy, and a current flows in the microammeter or sensitive moving-coil galvanometer G which is proportional to the illumination. The galvanometer is previously calibrated by placing a standard lamp at known distances from the disc D, and its scale reads lux directly. Fig. 24.19 illustrates an "AVO Light-meter", which operates on this principle; it is simply laid on the surface whose illumination is required and the reading is then taken.

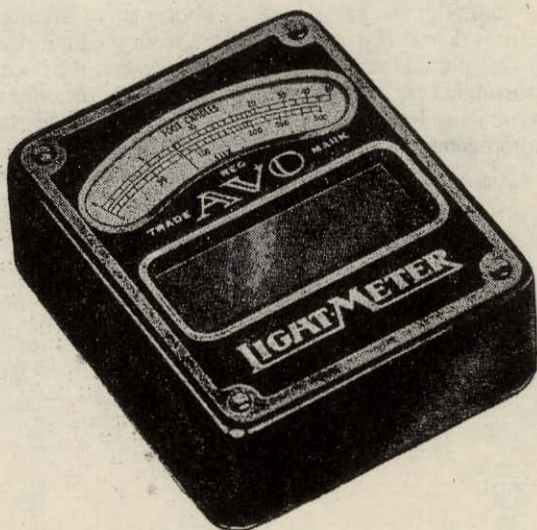


FIG. 24.19. AVO light-meter.

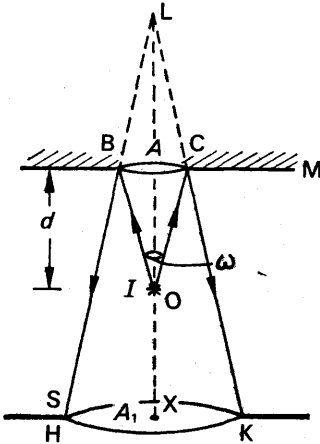


FIG. 24.20. Plane mirror illumination

$$\therefore E = \frac{I}{LX^2} \quad \dots \quad (i)$$

The image of O in the plane mirror is at L, which is at a distance LX from the screen S. It follows from (i) that, owing to light reflected from the mirror, the illumination of S is the same as that obtained by a lamp of I candela at the position of the image of O.

EXAMPLE

Define *lumen* and *lux*, and show how they are related. Describe and explain how you would make an accurate comparison of the illuminating powers of two lamps of the same type. A photometric balance is obtained between two lamps A and B when B is 100 cm from the photometer. When a block of glass G is placed between A and the photometer, balance is restored by moving B through 5 cm. Where must B be placed in order to maintain the balance when two more blocks, identical with G, are similarly placed between A and the photometer. (L.)

First part. See text.

Second part. Suppose I_1, I_2 are the intensities of A, B respectively, and d is the distance of A from the photometer P. Then, originally,

$$\frac{I_1}{d^2} = \frac{I_2}{100^2} \quad \dots \quad (i)$$

When G is placed in position, the balance is restored by moving B 105 cm. from P, Fig. 24.21 (i). If t is the transmission factor of G, the effective intensity of A is tI_1 , and hence

$$\frac{t I_1}{d^2} = \frac{I_2}{105^2} \quad \dots \quad (ii)$$

Dividing (ii) by (i),

$$\therefore t = \frac{100^2}{105^2} \quad \dots \quad (iii)$$

Illumination due to plane mirror.
 Consider a small lamp of I candela at O in front of a plane mirror M, Fig. 24.20. The flux F from O in a small cone OBC of solid angle ω is $I\omega$, and falls on an area A of the mirror. This flux is reflected to illuminate an area A_1 , or HK, on a screen S in front of the mirror. Assuming the reflection factor of the mirror is unity,

$$\text{illumination of S, } E = \frac{F}{A_1} = \frac{I\omega}{A_1}$$

But $\omega = \text{area } A/d^2$, where d is distance of O from the mirror.

$$\therefore E = \frac{IA}{d^2 A_1}$$

But $A/A_1 = d^2/LX^2$, from similar triangles LBC, LHK.

When two more blocks are placed beside G, the effective intensity of $A = t \times t \times t I_1 = t^3 I_1$, Fig. 24.21 (ii). Thus if the distance of B from P is now x .

$$\frac{t^3 I_1}{d^2} = \frac{I_2}{x^2} \dots \dots \dots (iv)$$

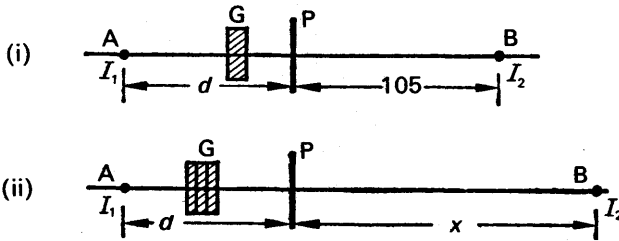


FIG. 24.21. Plane mirror illumination

Dividing (iv) by (i)

$$\therefore t^3 = \frac{100^2}{x^2}$$

From (iii)

$$\therefore \left(\frac{100^2}{105^2}\right)^3 = \frac{100^2}{x^2}$$

$$\therefore x^2 = \frac{100^2 \times 105^6}{100^6} = \frac{105^6}{100^4}$$

$$\therefore x = \frac{105^3}{100^2} = 116 \text{ cm.}$$

EXERCISES 24

Velocity of Light

1. In Fizeau's rotating wheel experiment the number of teeth was 720 and the distance between the wheel and reflector was 8633 metres. Calculate the number of revolutions per second of the wheel when extinction first occurs, assuming the velocity of light is 3.13×10^8 metres per second. What are the disadvantages of Fizeau's method?

2. Draw a diagram of Foucault's method of measuring the velocity of light. How has the velocity of light in water been shown to be less than in air? The radius of curvature of the curved mirror is 20 metres and the plane mirror is rotated at 20 revs per second. Calculate the angle in degrees between a ray incident on the plane mirror and then reflected from it after the light has travelled to the curved mirror and back to the plane mirror (velocity of light = 3×10^8 m s⁻¹).

3. Draw a diagram showing the arrangement of the apparatus and the path of the rays of light in Fizeau's toothed wheel method for measuring the velocity of light. What are the chief difficulties met with in carrying out the experiment?

If the wheel has 150 teeth and 150 spaces of equal width and its distance from the mirror be 12 kilometres, at what speed, in revolutions per minute, will the first eclipse occur? (*N.*)

4. Explain how the velocity of light was first determined, and describe one more recent method of measuring it.

A beam of light after reflection at a plane mirror, rotating 2000 times per minute, passes to a distant reflector. It returns to the rotating mirror from which it is reflected to make an angle of 1° with its original direction. Assuming that the velocity of light is $300\,000\text{ km s}^{-1}$, calculate the distance between the mirrors. (*L.*)

5. Describe and explain in detail one accurate terrestrial method for measuring the speed of light in air.

How does the speed of light depend on (a) the wavelength of the light, (b) the medium through which it travels? (*O. & C.*)

6. Explain why the velocity of light is difficult to measure by a direct terrestrial method; illustrate your answer with an estimate of the orders of magnitude of the quantities involved in the assessment.

Describe, with the aid of a labelled diagram, Michelson's method for the determination of the velocity of light.

What are the advantages of this method over the earlier one developed by Foucault? (*N.*)

7. Describe any *one* terrestrial method by which the velocity of light has been determined. Why is a knowledge of the value of this velocity so important?

A beam of light can be interrupted $(1.0 \pm 0.0002) \times 10^7$ times each second on passing through a certain crystal device. Such a beam is reflected from a distant mirror and returned through the same crystal. It is found that for certain positions of the mirror very little light emerges from the crystal a second time. Explain this. One such position of the mirror is known to be between 18 and 26 metres from the crystal. Calculate a more accurate value for this distance, assuming the velocity of light to be $(3.0 \pm 0.002) \times 10^{10}\text{ cm s}^{-1}$. Estimate the error in this calculated value. (*C.*)

8. Describe a method of measuring the speed of light. Explain precisely what observations are made and how the speed is calculated from the experimental data.

A horizontal beam of light is reflected by a vertical plane mirror *A*, travels a distance of 250 metres, is then reflected back along the same path and is finally reflected again by the mirror *A*. When *A* is rotated with constant angular velocity about a vertical axis in its plane, the emergent beam is deviated through an angle of 18 minutes. Calculate the number of revolutions per second made by the mirror.

If an atom may be considered to radiate light of wavelength 5000 \AA for a time of 10^{-10} second, how many cycles does the emitted wave train contain? (*O. & C.*)

9. Give one reason why it is important to know accurately the velocity of light.

A glass prism whose cross-section is a regular polygon of n sides has its faces silvered and is mounted so that it can rotate about a vertical axis through the centre of the cross-section, with the silvered faces vertical. An intense narrow horizontal beam of light from a small source is reflected from one face of the prism to a small distant mirror and back to the same face, where it is reflected and used to form an image of the source which can be observed. Describe the behaviour of this image if the prism is set into rotation and slowly accelerated to a high speed. If the distance between the mirrors is D and the velocity of light is c , find an expression for the values of the angular velocity of the prism at which the image will be formed in its original position.

If $c = 3 \times 10^8\text{ m s}^{-1}$, $D = 30\text{ km}$ and the maximum safe speed for the mirror is 700 rev per second, what would be a suitable value for n ? (*O. & C.*)

10. Describe a terrestrial method by which the velocity of light has been measured. How could the method be modified to show that the velocity of light in water is less than that in air? Briefly discuss the theoretical importance of this fact. (C.)

11. Describe a terrestrial method of determining the velocity of light in air, explaining (without detailed calculation) how the result is obtained.

Why do we conclude that in free space red and blue light travel with the same speed, but that in glass red light travels faster than blue? (L.)

Photometry

12. A 30 candelas (cd) lamp X is 40 cm in front of a photometer screen. What is the illumination directly in front of X on the screen? Calculate the cd of a lamp Y which provides the same illumination when placed 60 cm from the screen.

13. A lamp of 800 cd is suspended 16 m above a road. Find the illumination on the road (i) at a point A directly below the lamp, (ii) at a point B 12 m from A.

14. Define the terms *luminous intensity* and *illumination*. Describe an accurate method of comparing the luminous intensities of two sources of light.

A lamp is fixed 4 m above a horizontal table. At a point on the table 3 m to one side of the vertical through the lamp, a light-meter is placed flat on the table. It registers 4 m candles. Calculate the intensity of the lamp. (C.)

15. What is meant by *luminous intensity* and *illumination*? How are they related to each other?

A small source of 32 cd giving out light equally in all directions is situated at the centre of a sphere of 8 m diameter, the inner surface of which is painted black. What is the illumination of the surface?

If the inner surface is repainted with a matt white paint which causes it to reflect diffusely 80 per cent of all lighting falling on it, what will the illumination be? (L.)

16. Describe an accurate form of photometer for comparing the luminous intensities of lamps.

A lamp is 100 cm from one side of a photometer and produces the same illumination as a second lamp placed at 120 cm on the opposite side. When a lightly smoked glass plate is placed before the weaker lamp, the brighter one has to be moved 50 cm to restore the equality of illumination. Find what fraction of the incident light is transmitted by the plate. (L.)

17. How would you compare the luminous intensities of two small lamps?

A small 100 cd lamp is placed 10 m above the centre of a horizontal rectangular table measuring 6 m by 4 m. What are the maximum and minimum values of the illumination on the table due to direct light?

How would your results be changed by the presence of a large horizontal mirror, placed 2 m above the lamp, so as to reflect light down on to the table, assuming that only 80 per cent of the light incident on the mirror is reflected? (W.)

18. Describe one form of a photometer, and explain how you would measure the light loss which results from enclosing a light source by a glass globe. Two small 16 cd lamps are placed on the same side of a screen at

distances of 2 and 5 m from it. Calculate the distance at which a single 32 cd lamp must be placed in order to give the same intensity of illumination on the screen. (N.)

19. Define *lumen*, *metre-candle*. Describe the construction and use of a Lummer-Brodhun photometer.

Twenty per cent of the light emitted by a source of 500 cd is evenly distributed over a circular area 5 m in diameter. What is the illumination at points within this area? (L.)

20. Distinguish between *luminous intensity* and *illumination*. How may the two sources be accurately compared?

A surface receives light normally from a source at a distance of 3 m. If the source is moved closer until the distance is only 2 m, through what angle must the surface be turned to reduce the illumination to its original value? (O. & C.)

21. Define *illumination of a screen*, *luminous intensity of a source of light*. Indicate units in which each of these quantities may be measured.

Describe a reliable photometer, and explain how you would use it to compare the reflecting powers of plaster of Paris and ground glass. (C.)

22. Describe an accurate form of photometer for comparing the luminous intensities of two sources of light.

Two electric lamps, A and B, are found to give equal illuminations on the two sides of a photometer when their distances from the photometer are in the ratio 4 : 5. A sheet of glass is then placed in front of B, and it is found that equality of illumination is obtained when the distances of A and B are in the ratio 16 : 19. Find the percentage of light transmitted by the glass. (C.)