# chapter twenty-three

# Optical instruments

WHEN a telescope or a microscope is used to view an object, the appearance of the final image is determined by the cone of rays entering the eye. A discussion of optical instruments and their behaviour must therefore be preceded by a consideration of the image formed by the eye, and we must now recapitulate some of the points about the eye mentioned in previous pages.

Firstly, the image formed by the eye lens L must appear on the retina R at the back of the eye if the object is to be clearly seen, Fig. 23.1. Secondly, the normal eye can focus an object at infinity (the "far point" of the normal eye), in which case the eye is said to be "unaccommodated". Thirdly, the eye can see an object in greatest detail when it is placed at a certain distance $D$ from the eye, known as the *least distance of distinct vision*, which is about 25 cm, for a normal eye (p. 508). The point at a distance $D$ from the eye is known as its "near point".

### Visual Angle

Consider an object O placed some distance from the eye, and suppose $\theta$ is the angle in radians subtended by it at the eye, Fig. 23.1. Since

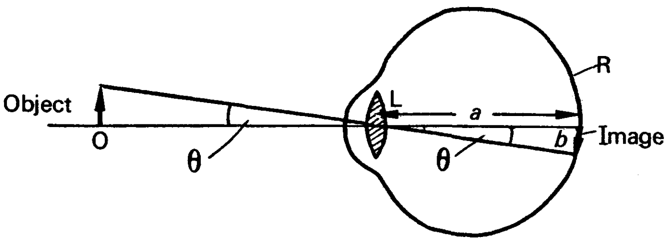



FIG. 23.1. Length of image on retina, and visual angle.

vertically opposite angles are equal, it follows that the length $b$ of the image on the retina is given by $b = a\theta$, where $a$ is the distance from R to L. But $a$ is a constant; hence $b \propto \theta$. We thus arrive at the important conclusion that *the length of the image formed by the eye is proportional to the* **angle** *subtended at the eye by the object.* This angle is known as the *visual angle*; the greater the visual angle, the greater is the apparent size of the object.

Fig. 23.2 (i) illustrates the case of an object moved from A to B, and viewed by the eye in both positions. At B the angle $\beta$ subtended at the eye is greater than the visual angle $\alpha$ subtended at A. Hence the object

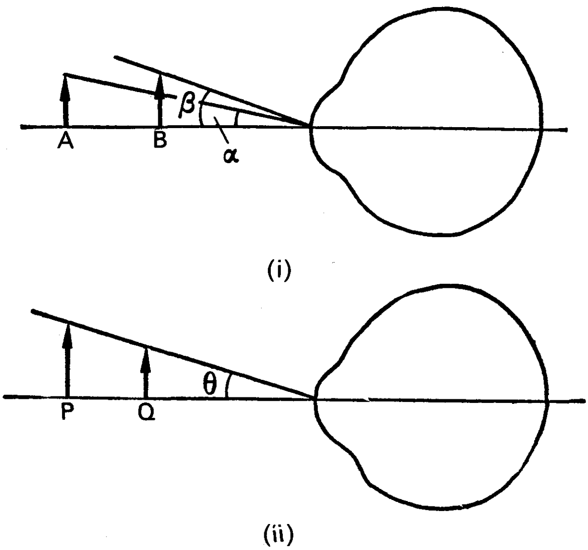



FIG. 23.2. Relation between visual angle and length of image.

appears larger at B than at A, although its physical size is the same. Fig. 23.2 (ii) illustrates the case of two objects, at P, Q respectively, which subtend the same visual angle $\theta$ at the eye. The objects thus appear to be of equal size, although the object at P is physically bigger than that at Q. It should be remembered that an object is not clearly seen if it is brought closer to the eye than the near point.

**Angular Magnification**

Microscopes and telescopes are instruments designed to increase the visual angle, so that the object viewed can be made to appear much larger with their aid. Before they are used the object may subtend a certain angle $\alpha$ at the eye; when they are used the final images may subtend an increased angle $\alpha'$ at the eye. The *angular magnification, M,* of the instrument is defined as the ratio

$$\mathbf{M} = \frac{\alpha'}{\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and this is also popularly known as the *magnifying power* of the instrument. It should be carefully noted that we are concerned with visual angles in the theory of optical instruments, and not with the physical sizes of the object and the image obtained.

**Microscopes**

At the beginning of the seventeenth century single lenses were developed as powerful magnifying glasses, and many important discoveries in human and animal biology were made with their aid. Shortly afterwards two or more convex lenses were combined to form powerful microscopes, and with their aid HOOKE, in 1648, discovered the existence of "cells" in animal and vegetable tissue.

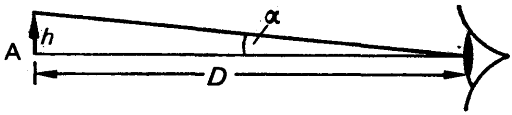
A microscope is an instrument used for viewing *near* objects. When it is in normal use, therefore, the image formed by the microscope is usually at the least distance of distinct vision, $D$, from the eye, i.e., at the near point of the eye. With the unaided eye (i.e., without the instrument), the object is seen clearest when it is placed at the near point. Consequently the angular magnification of a microscope in *normal* use is given by

$$M = \frac{\alpha'}{\alpha},$$

where $\alpha'$ is the angle subtended at the eye by the image at the near point, and $\alpha$ is the angle subtended at the unaided eye by the object at the near point.

## Simple Microscope or Magnifying Glass

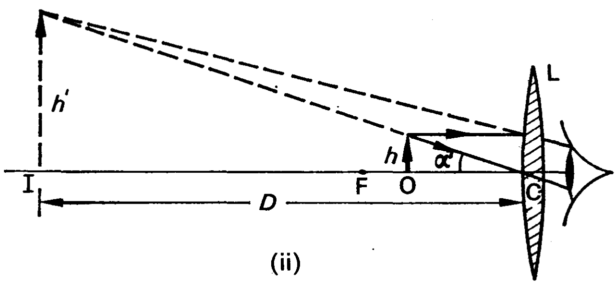
Suppose that an object of length $h$ is viewed at the near point, A, by the unaided eye, Fig. 23.3 (i). The visual angle, $\alpha$, is then $h/D$ in radian




(i)

FIG. 23.3 (i). Visual angle with unaided eye.

measure. Now suppose that a convex lens L is used as a magnifying glass to view the same object. An erect, magnified image is obtained when the object O is nearer to L than its focal length (p. 486), and the observer moves the lens until the image at I is situated at his near point. If the observer's eye is close to the lens at C, the distance IC is then equal to $D$, the least distance of distinct vision, Fig. 23.3 (ii). Thus the new visual angle $\alpha'$ is given by $h'/D$, where $h'$ is the length of the virtual image, and it can be seen that $\alpha'$ is greater than $\alpha$ by comparing Fig. 23.3 (i) with Fig. 23.3 (ii).




(ii)

FIG. 23.3 (ii). Simple microscope, or magnifying glass.

The angular magnification, $M$, of this simple microscope can be evaluated in terms of $D$ and the focal length $f$ of the lens. From the definition of $M$ (p. 526), $M = \alpha'/\alpha$.

But $$\alpha' = \frac{h'}{D}, \alpha = \frac{h}{D}$$

$$\therefore \quad M = \frac{h'}{D} \bigg/ \frac{h}{D} = h'/h \quad . \quad . \quad . \quad \text{(i)}$$

Now $h'/h$ is the "linear magnification" produced by the lens, and is given by $h'/h = v/u$, where $v$ is the image distance CI and $u$ is the object distance CO (see p. 488). Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, with the usual notation, we have

$$1 + \frac{v}{u} = \frac{v}{f},$$

by multiplying throughout by $v$.

$$\therefore \quad \frac{v}{u} = \frac{v}{f} - 1 = \frac{D}{f} - 1$$

since $v = \text{CI} = D$.

$$\therefore \quad \frac{h'}{h} = \frac{D}{f} - 1.$$

$$\therefore \quad M = \frac{D}{f} - 1 \quad . \quad . \quad . \quad . \quad (2)$$

from (i) above.

If the magnifying glass has a focal length of 2 cm, $f = +2$ as it is converging; also, if the least distance of distinct vision is 25 cm, $D = -25$ as the image is virtual, see Fig. 23.3 (ii). Substituting in (2),

$$M = \frac{-25}{+2} - 1 = -13\tfrac{1}{2}.$$

Thus the angular magnification is $13\frac{1}{2}$. The position of the object O is given by substituting $v = -25$ and $f = +2$ in the lens equation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, from which the object distance $u$ is found to be $+1{\cdot}86$ cm.

From the formula for $M$ in (2), it follows that a lens of *short* focal length is required for high angular magnification.

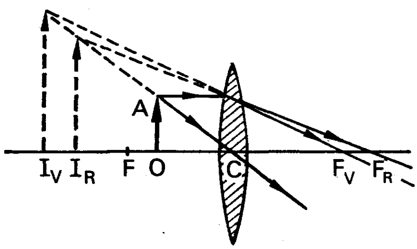



FIG. 23.4. Dispersion with magnifying glass.

When an object OA is viewed through a convex lens acting as a *magnifying glass*, various coloured virtual images, corresponding to $I_R$, $I_V$ for red and violet rays for example, are formed, Fig. 23.4. The top point of each image lies on the line CA. Hence each image subtends the same angle at the eye close to the lens, so that the colours received by the eye will practically overlap. Thus the virtual image seen in a magnifying glass is almost free of chromatic aberration. A little colour is observed at the edges as a result of spherical aberration. A *real* image formed by a lens has chromatic aberration, as explained on p. 513.

### Magnifying Glass with Image at Infinity

We have just considered the normal use of the simple microscope, in which case the image formed is at the near point of the eye and the eye is accommodated (p. 527). When the image is formed at infinity, however, which is not a normal use of the microscope, the eye is undergoing the least strain and is then unaccommodated (p. 508). In this case the object must be placed at the focus, F, of the lens. Fig. 23.5.

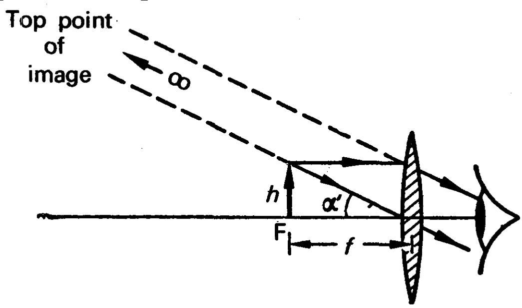



FIG. 23.5. Final image at infinity.

Suppose that the focal length of the lens is $f$. The visual angle $\alpha'$ now subtended at the eye is then $h/f$ if the eye is close to the lens, and hence the angular magnification, $M$, is given by

$$M = \frac{\alpha'}{\alpha} = \frac{h/f}{h/D},$$

as $\alpha = h/D$, Fig. 23.3 (i).

$$\therefore \quad M = \frac{D}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

When $f = +2$ cm and $D = -25$ cm, $M = -12\frac{1}{2}$. The angular magnification was $-13\frac{1}{2}$ when the image was formed at the near point (p. 528). It can easily be verified that the angular magnification varies between $12\frac{1}{2}$ and $13\frac{1}{2}$ when the image is formed between infinity and the near point, and the maximum angular magnification is thus obtained when the image is at the near point.

### Compound Microscope

From the formula $M = D/f - 1$, $M$ is greater the smaller the focal length of the lens. As it is impracticable to decrease $f$ beyond a certain limit, owing to the mechanical difficulties of grinding a lens of short focal length (great curvature), *two* separated lenses are used to obtain a high angular magnification, and constitute a *compound* microscope. The lens nearer to the object is called the *objective*; the lens through which the final image is viewed is called the *eye-piece*. The objective and the eye-piece are both converging, and both have small focal lengths for a reason explained later (p. 531).

When the microscope is used, the object O is placed at a slightly *greater* distance from the objective than its focal length. In Fig. 23.6, $F_o$ is the focus of this lens. An inverted real image is then formed at $I_1$ in the microscope tube, and the eye-piece is adjusted so that a large virtual image is formed by it at $I_2$. Thus $I_1$ is *nearer* to the eye-piece

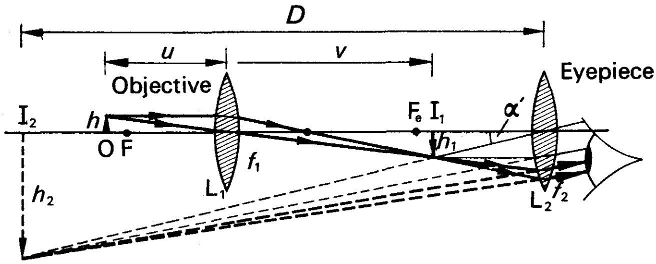



FIG. 23.6. Compound microscope in *normal* use.

than the focus $F_e$ of this lens. It can now be seen that the eye-piece functions as a simple magnifying glass, used for viewing the image formed at $I_1$ by the objective.

## Angular Magnification with Microscope in Normal Use

When the microscope is in normal use the image at $I_2$ is formed at the least distance of distinct vision, $D$, from the eye (p. 527). Suppose that the eye is close to the eye-piece, as shown in Fig. 23.6. The visual angle $\alpha'$ subtended by the image at $I_2$ is then given by $\alpha' = h_2/D$, where $h_2$ is the height of the image. With the unaided eye, the object subtends a visual angle given by $\alpha = h/D$, where $h$ is the height of the object, see Fig. 23.3 (i).

$$\therefore \quad \text{angular magnification, } M = \frac{\alpha'}{\alpha}$$

$$= \frac{h_2/D}{h/D} = \frac{h_2}{h}.$$

Now $\frac{h_2}{h}$ can be written as $\frac{h_2}{h_1} \times \frac{h_1}{h}$, where $h_1$ is the length of the intermediate image formed at $I_1$.

$$\therefore \quad M = \frac{h_2}{h_1} \cdot \frac{h_1}{h} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

The ratio $h_2/h_1$ is the linear magnification of the "object" at $I_1$ produced by the *eye-piece*, and we have shown on p. 528 that the linear magnification is also given by $\frac{v}{f_2} - 1$, where $v$ is the image distance from the lens and $f_2$ is the focal length. Since $v = D$ numerically in this case (the image at $I_2$ is at a distance $-D$ from the eye-piece), it follows that

$$\frac{h_2}{h_1} = \frac{D}{f_2} - 1 \qquad \text{(ii)}$$

Also, the ratio $h_1/h$ is the linear magnification of the object at O produced by the *objective* lens. Thus if the distance of the image $I_1$ from this lens is denoted by $v$, we have

$$\frac{h_1}{h} = \frac{v}{f_1} - 1 \qquad \text{(iii)}$$

$$\therefore \quad M = \frac{h_2}{h_1} \cdot \frac{h_1}{h} = \left(\frac{D}{f_2} - 1\right)\left(\frac{v}{f_1} - 1\right) \qquad (4)$$

It can be seen that if $f_1$ and $f_2$ are small, $M$ is large. Thus the angular magnification is high if the focal lengths of the objective and the eye-piece are small.

## Microscope with Image at Infinity

The compound microscope can also be used with the final image formed at infinity, which is not the normal use of the instrument. In this case the eye is unaccommodated, or "at rest". The image of the object in the objective must now be formed at the focus, $F_e$, of the eye-piece, as shown in Fig. 23.4, and the visual angle $\alpha'$ subtended at the eye by the final image at infinity is then given by $\alpha' = h_1/f_2$, where $h_1$ is the length of the image at $I_1$ and $f_2$ is the focal length of the eye-piece.

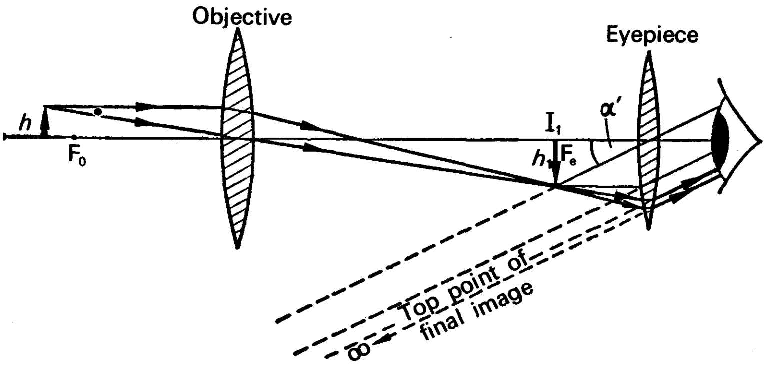



FIG. 23.7. Microscope with image at infinity.

The angular magnification, $M$, is given by $M = \alpha'/\alpha$, where $\alpha = h/D$ (p. 527).

$$\therefore \quad M = \frac{\alpha'}{\alpha} = \frac{h_1/f_2}{h/D} = \frac{h_1}{h} \cdot \frac{D}{f_2}$$

But, from above, $\frac{h_1}{h} = \frac{v}{f_1} - 1$.

$$\therefore \quad M = \left(\frac{v}{f_1} - 1\right)\frac{D}{f_2} \quad . \quad . \quad . \quad (5)$$

Comparing equations (4) and (5), it can be seen, since $D$ is a negative (virtual) quantity, that the angular magnification is greater when the final image is formed at the near point. Further, it will be noted that the eye-piece is nearer to the image at $I_1$ in the latter case.

## The Best Position of the Eye. The Eye-Ring

When an object is viewed by an optical instrument, only those rays from the object which are bounded by the perimeter of the objective lens enter the instrument. The lens thus acts as a *stop* to the light from the object. Similarly, the only rays from the image causing the sensation of vision are those which enter the pupil of the eye. The pupil thus acts as a natural stop to the light from the image; and with a given objective, the best position of the eye is one where it collects as much light as possible from that passing through the objective.

Fig. 23.8 illustrates three of the rays from a point X on an object at O placed in front of a compound microscope. Two of the rays are refracted at the boundary of the objective $L_1$ to pass through Y on the real image at $I_1$, while the ray $OC_1$ through the middle $C_1$ of the objective passes straight through to Y. The cone of light is then incident on the eye-piece lens $L_2$, where it is refracted and forms the point T on the final image, corresponding to X on the object. Now the central ray of the beams of light incident on $L_1$ from every point on the object passes through $C_1$, the centre of the objective lens. The central ray of the emergent beams from the eye-piece $L_2$ thus passes through the image of $C_1$ in $L_2$. By similar reasoning, we arrive at the conclusion that *all*

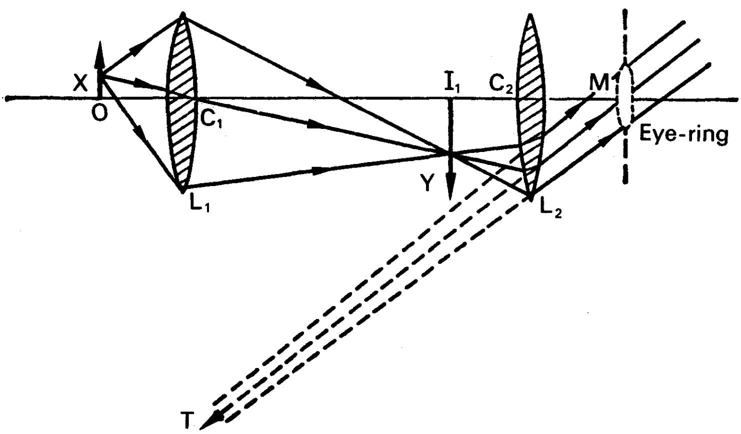



FIG. 23.8. Position of eye-ring.

*the emergent rays pass through the image M of the objective in the eye-piece.* This image is known as the *eye-ring*, and the best position of the eye is thus at M.

Suppose the objective is 16 cm from $L_2$, which has a focal length of 2 cm. The image distance, $v$, in $L_2$ is given by $\frac{1}{v} + \frac{1}{(+16)} = \frac{1}{(+2)}$, from which $v = 2{\cdot}3$ cm. Thus M is a short distance from the eye-piece, and in practice the eye should be farther from the eye-piece than in Fig. 23.8. This is arranged in commercial microscopes by having a circular opening fixed at the eye-ring distance from the eye-piece, so that the observer's eye has automatically the best position when it is placed close to the opening.

## Angular Magnification of Telescopes

*Telescopes* are instruments used for viewing distant objects, and they are used extensively at sea and at astronomical observatories. The first telescope is reputed to have been made about 1608, and in 1609 Galileo made a telescope through which he observed the satellites of Jupiter and the rings of Saturn. The telescope thus paved the way for great astronomical discoveries, particularly in the hands of KEPLER. Newton also designed telescopes, and was the first person to suggest the use of curved mirrors for telescopes free from chromatic aberration (see p. 513).

If $\alpha$ is the angle subtended at the unaided eye by a distant object, and $\alpha'$ is the angle subtended at the eye by its image when a telescope is used, the angular magnification $M$ of the instrument is given by

$$M = \frac{\alpha'}{\alpha}.$$

It should be carefully noted that $\alpha$ is *not* the angle subtended at the unaided eye by the object at the near point, as was the case with the microscope, because the telescope is used for viewing distant objects.

## Astronomical Telescope in Normal Adjustment

An astronomical telescope made from lenses consists of an objective of long focal length and an eye-piece of short focal length, for a reason given on p. 534. Both lenses are converging. *The telescope is in normal adjustment when the final image is formed at infinity*, and the eye is then unaccommodated when viewing the image. The unaided eye is also unaccommodated when the distant object is viewed, as the latter may be considered to be at infinity.

Fig. 23.9 illustrates the formation of the final image when the telescope is used normally. The image I of the distant object is formed at the focus, $F_o$, of the objective since the rays incident on the latter are parallel; and since the final image is formed at infinity, the focus $F_e$ of the eye-piece must also be at $F_0$. Fig. 23.9 shows three of the many rays from the *top* point of the object, marked $a$, and three of the many rays from the foot of the object, marked $b$. These rays pass respectively through the top and foot of the image I, as shown.

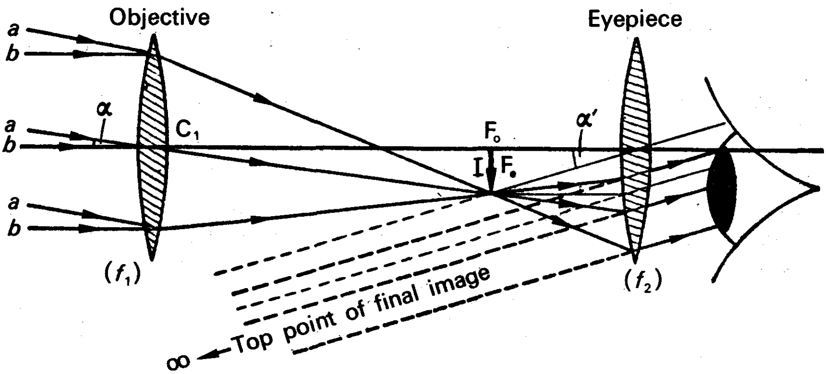



FIG. 23.9. Telescope in *normal* use.

We can now obtain an expression for the angular magnification, $M$, of the telescope; in so doing we shall assume that the eye is close to the eye-piece. Since the length between the objective and the eye-piece is very small compared with the distance of the object from either lens, we can take the angle $\alpha$ subtended at the unaided eye by the object as that subtended at the objective lens, Fig. 23.9. The angle $\alpha'$ subtended at the eye when the telescope is used is given by $\alpha' = h/f_2$, where $h$ is the length of the image I and $f_2$ is the focal length of the eye-piece.

But $$\alpha = h/f_1,$$

where $f_1$ is the focal length of the objective, since I is at a distance $f_1$ from $C_1$.

$$\therefore \quad M = \frac{\alpha'}{\alpha} = \frac{h/f_2}{h/f_1}$$

$$\therefore \quad \mathbf{M} = \frac{\mathbf{f_1}}{\mathbf{f_2}} \quad . \quad . \quad . \quad . \quad (6)$$

Thus the angular magnification is equal to the ratio of the focal length of the objective ($f_1$) to that of the eye-piece ($f_2$). For high angular magnification, it follows from (6) that the objective should have a long focal length and the eye-piece a short focal length.

It will be noted that the distance between the lenses is equal to the sum ($f_1 + f_2$) of their focal lengths. This provides a simple method of setting up two convex lenses to form an astronomical telescope when their focal lengths are known.

## The Eye-ring, and Relation to Angular Magnification

As we explained in the case of the microscope, the rays which pass through the telescope from the distant object are those bounded by the objective lens. Fig. 23.10 illustrates three rays from a point on the distant object which pass through the objective, forming an image at Y. The eye-ring, M, the best position for the eye, is the circular image of the objective in the eye-piece $L_2$, and we can calculate its position as

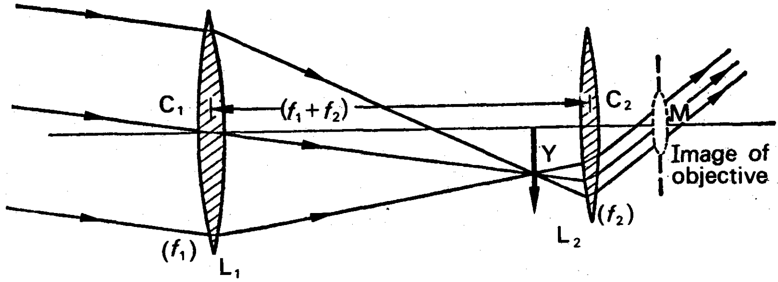



FIG. 23.10. Eye-ring relation to magnification.

$C_1C_2 = f_1 + f_2$, from previous. As the focal length of $L_2 = f_2$, the distance $C_2M$, or $v$, is given by $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$,

$$\text{i.e.,} \quad \frac{1}{v} + \frac{1}{+(f_1+f_2)} = \frac{1}{+(f_2)}$$

from which
$$v = \frac{f_2}{f_1}(f_1 + f_2).$$

Now the objective diameter: eye-ring diameter $= C_1C_2 : C_2M$

$$= u : v = (f_1 + f_2) : \frac{f_2}{f_1}(f_1 + f_2)$$
$$= f_1/f_2.$$

But the angular magnification of the telescope $= f_1/f_2$ (p. 534). Thus the angular magnification, $M$, is also given by

$$M = \frac{\text{diameter of objective}}{\text{diameter of eye-ring}}, \quad . \quad . \quad (7)$$

the telescope being in normal adjustment.

## Telescope with Final Image Near Point

When a telescope is used, the final image can be formed at the near point of the eye instead of at infinity. The eye is then "accommodated", and although the image is still clearly seen, the telescope is *not* in normal adjustment (p. 533). Fig. 23.11 illustrates the formation of the final image.

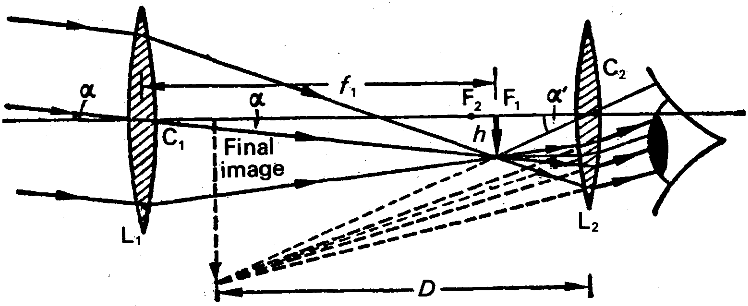



FIG. 23.11. Final image at near point.

The objective forms an image of the distant object at its focus $F_1$, and the eye-piece is moved so that the image is nearer to it than its focus $F_2$, thus acting as a magnifying glass.

The angle $\alpha$ subtended at the unaided eye is practically that subtended at the objective $L_1$. Thus $\alpha = h/f_1$, where $h$ is the length of the image in the objective and $f_1$ its focal length. The angle $\alpha'$ subtended at the eye by the final image $= h/u$, if the eye is close to the eye-piece, where $u = F_1C_2 =$ the distance of the image at $F_1$ from the eye-piece.

Thus angular magnification, $M = \dfrac{\alpha'}{\alpha} = \dfrac{h/u}{h/f_1}$

$$\therefore \quad M = \frac{f_1}{u} \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

As the final image is formed at a numerical distance $D$ from the eye-piece $L_2$, we have $v = -D$ when $f = +f_2$. Thus, from $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$,

$$\frac{1}{-D} + \frac{1}{u} = \frac{1}{+f_2},$$

from which
$$u = \frac{f_2 D}{f_2 + D}.$$

Substituting in (i) for $u$,

$$\therefore \quad M = \frac{f_1}{f_2}\left(\frac{f_2 + D}{D}\right)$$

$$\therefore \quad M = \frac{f_1}{f_2}\left(1 + \frac{f_2}{D}\right) \quad . \quad . \quad . \quad \text{(8)}$$

The angular magnification when the telescope is in normal adjustment (i.e., final image at infinity) $= \dfrac{f_1}{f_2}$ (p. 534). Hence, from (8), the angular magnification is increased in the ratio $\left(1 + \dfrac{f_2}{D}\right) : 1$ when the final image is formed at the near point.

## Terrestrial Telescope

From Fig. 23.11, it can be seen that the top point of the distant object is above the axis of the lens, but the top point of the final image is below the axis. Thus the image in an astronomical telescope is *inverted.* This instrument is suitable for astronomy because it makes little difference if a star, for example, is inverted, but it is useless for viewing objects on the earth or sea, in which case an erect image is required.

A *terrestrial telescope* provides an erect image. In addition to the objective and eye-piece of the astronomical telescope, it has a converging lens L of focal length $f$ between them, Fig. 23.12. L is placed at a distance $2f$ in front of the inverted real image $I_1$ formed by the objective, in which case, as shown on p. 488, the image I in L of $I_1$ (i) is inverted, real, and the same size as $I_1$, (ii) is also at a distance $2f$ from L.

Thus the image I is now the same way up as the distant object. If I is

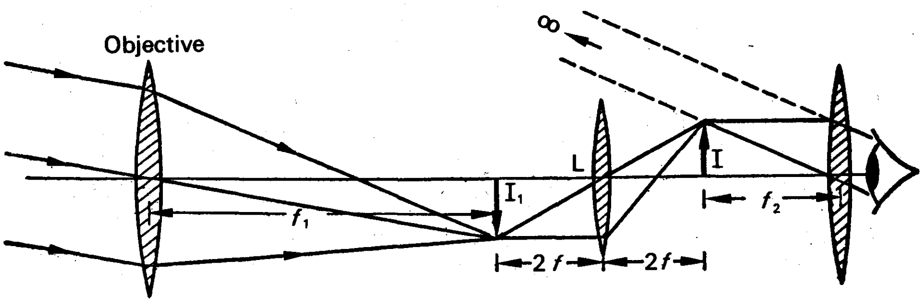



FIG. 23.12. Terrestrial telescope.

at the focus of the eye-piece, the final image is formed at infinity and is also erect.

The lens L is often known as the "erecting" lens of the telescope, as its only function is that of inverting the image $I_1$. Since the image I produced by L is the same size as $I_1$, the presence of L does not affect the magnitude of the angular magnification of the telescope, which is thus $f_1/f_2$ (p. 538). The erecting lens, however, reduces the intensity of the light emerging through the eye-piece, as light is reflected at the lens surfaces. Yet another disadvantage is the increased length of the telescope when L is used; the distance from the objective to the eye-piece is now $(f_1 + f_2 + 4f)$, Fig. 23.12, compared with $(f_1 + f_2)$ in the astronomical telescope.

## Galileo's Telescope

About 1610, with characteristic genius, Galileo designed a telescope which provides an erect image of an object with the aid of only two lenses. The *Galilean telescope* consists of an objective which is a converging lens of long focal length, and an eye-piece which is a *diverging* lens of short focal length. The distance between the lenses is equal to the *difference* in the magnitudes of their focal lengths, i.e., $C_1C_2 =$

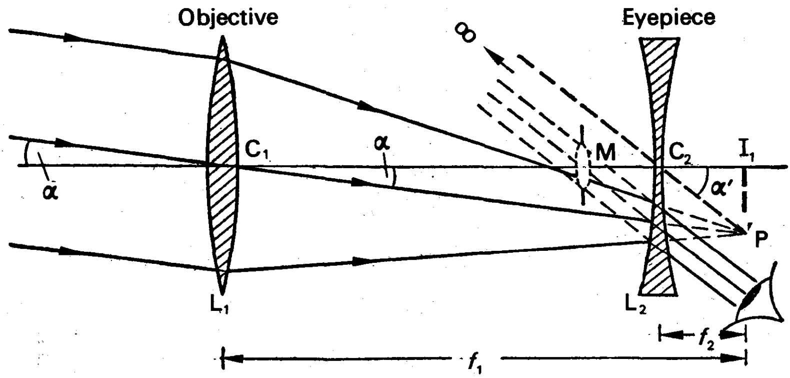



FIG. 23.13. Galilean telescope.

$f_1 - f_2$, where $f_1, f_2$ are the focal lengths of the objective and eye-piece respectively, Fig. 23.13. The image of the distant object in the objective $L_1$ would be formed at $I_1$, where $C_1I_1 = f_1$, in the absence of the diverging lens $L_2$; but since $L_2$ is at a distance $f_2$ from $I_1$, the rays falling on the eye-piece are refracted through this lens so that they emerge parallel. It will now be noted from Fig. 23.13 that an observer sees the top point of the final image above the axis of the lenses, and hence *the image is the same way up as the distant object.*

In Fig. 23.13, the rays converging to P emerge parallel after passing through the eye-piece $L_2$. The top point of the image formed at infinity is thus a virtual image in $L_2$ of the virtual object P. But a ray $C_2P$ through the middle of $L_2$ passes straight through the lens, and this will also be a ray which passes through the top point of the image at infinity. Thus the three parallel rays shown emerging from the eye-piece in Fig. 23.13 are parallel to the line $PC_2$. Hence if the eye is placed close to the diverging lens, the angle $\alpha'$ subtended at it by the image at infinity is angle $I_1C_2P$.

The angle $\alpha$ subtended at the eye by the distant object is practically equal to the angle subtended at the objective, Fig. 23.13. Now $\alpha = h/C_1I_1 = h/f_1$, where $f_1$ is the objective focal length and $h$ is the length $I_1P$; and $\alpha' = h/C_2I_1 = h/f_2$.

$$\therefore \quad \text{angular magnification, } M = \frac{\alpha'}{\alpha} = \frac{h/f_2}{h/f_1}$$

$$\therefore \quad M = \frac{f_1}{f_2} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Thus for high angular magnification, an objective of long focal length ($f_1$) and an eye-piece of short focal length ($f_2$) are required, as in the case of the astronomical telescope (see p. 534).

**Advantage and Disadvantage of Galilean Telescope. Opera Glasses**

The distance $C_1C_2$ between the objective and the eye-piece in the Galilean telescope is $(f_1 - f_2)$; the distance between the same lenses in the terrestrial telescope is $(f_1 + f_2 + 4f)$, p. 537. Thus the Galilean telescope is a much shorter instrument than the terrestrial telescope, and is therefore used for *opera glasses.*

As already explained (p. 532), the eye-ring is the image of the objective in the eye-piece. But the eye-piece is a diverging lens. Thus the eye-ring is virtual, and corresponds to M, which is between $L_1$ and $L_2$ (Fig. 23.13). Since it is impossible to place the eye at M, the best position of the eye in the circumstances is as close as possible to the eye-piece $L_2$, and consequently the field of view of the Galilean telescope is very limited compared with that of the astronomical or terrestrial telescope. This is a disadvantage of the Galilean telescope.

**Final Image at Near Point**

The final image in a Galilean telescope can also be viewed at the near point of the eye, when the telescope is not in normal adjustment. Fig. 23.14 illustrates the formation of the erect image in this case. The distance $C_2I_1$ is now more than the focal length $f_2$ of the eye-piece; and since $C_2I_2 = D$, the least distance of distinct vision, we have $v = -D$ (the image in $L_2$ is virtual) and $f_2$ is negative. Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we obtain

$$\frac{1}{-D} + \frac{1}{u} = \frac{1}{-f_2},$$

assuming $f_2$ is the *numerical* value of the diverging lens focal length, from which $u = \frac{-f_2D}{D - f_2}$.

With the usual notation, the angular magnification, $M = \frac{\alpha'}{\alpha}$.

But $\alpha' = h/u, \alpha = h/f_1$. Thus $M = f_1/u$.

But
$$u = \frac{-f_2D}{D - f_2}$$

$$\therefore \quad M = \frac{f_1}{f_2}\left(\frac{f_2}{D} - 1\right)$$

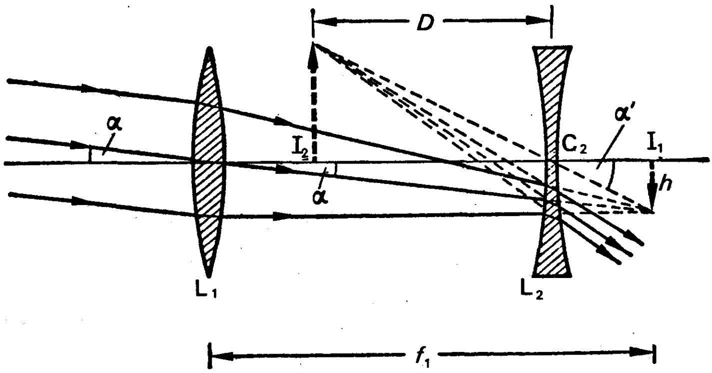



FIG. 23.14. Final image at near point.

**Measurement of magnifying power of telescope and microscope**

*Method* 1. *The magnifying power of a telescope* can be measured by placing a well-illuminated large scale S at one end of the laboratory, and viewing it through the telescope at the other end. If the telescope consists of converging lenses O, E acting as objective and eye-piece respectively, the distance

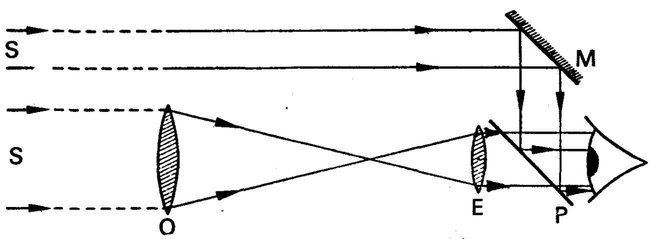



FIG. 23.15. Measurement of magnifying power.

between O, E is the sum of their focal lengths when the instrument is in normal use (p. 534), Fig. 23.15. By means of a plane mirror M and a plane piece of glass P, the divisions on the scale S can be superimposed on its image seen through the telescope, and the ratio of the divisions in equal lengths of the field of view can thus be determined. Since the ratio of the angles subtended at the eye by the image and by the scale is equal to the ratio, $a$, of the divisions, the magnifying power is equal to $a$. There must be no parallax between the scale and the final image.

*Method* 2. The magnifying power of a telescope is the ratio of the diameters of the objective and eye-ring (p. 535). The eye-ring is the image of the objective in the eye-piece, and is obtained by pointing the telescope to the sky and holding a ground glass screen near the eye-piece. A circle, which is the image of the objective in the eye-piece, is observed on the screen, and its diameter, $d$, is measured. The magnifying power is then given by the ratio $d_0/d$, where $d_0$ is the diameter of the objective.

This method is particularly useful when the telescope is fixed in a tube, as it is in practice. When a telescope is set up as in Fig. 23.10, a piece of cardboard with a circular hole can be placed round the objective lens to define its diameter, and the eye-ring found by placing a screen near the eye-piece.

*The magnifying power of a microscope* can be found by placing two similar scales in front of it, one being 25 cm from the eye. The other scale is placed near the objective, and the eye-piece is moved until the image of this scale coincides with the first scale by the method of no parallax, both eyes being used. The magnifying power is then given by the ratio of the number of divisions occupying the same length.

## Prism Binoculars

Prism binoculars are widely used as field glasses, and consist of short astronomical telescopes containing two right-angled isosceles prisms between the objective and eye-piece, Fig. 23.16. These lenses are both converging, and they would produce an inverted image of the distant object if they were alone used. The purpose of the two prisms is to invert the image and obtain a final *erect* image.

One prism A, is placed with its refracting edge vertical, while the other, B, is placed with its refracting edge horizontal. As shown in

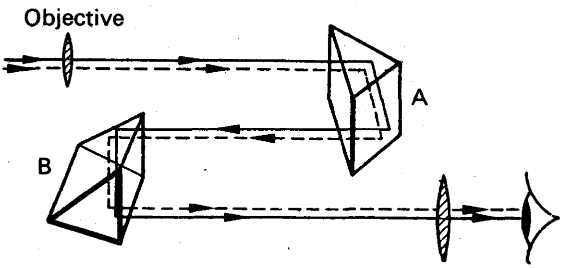



FIG. 23.16. Prism binoculars.

Fig. 23.11, the image formed by the objective alone is inverted. Prism A, however, turns it round in a horizontal direction, and prism B inverts it in a vertical direction, both prisms acting as reflectors of light (see p. 449). The image produced after reflection at B is now the same way up, and the same way round, as the original object. Since the eye-piece is a convex lens acting as a magnifying glass, it produces a final image the same way up as the image in front of it, and hence the final image is the same way up as the distant object.

Fig. 23.16 illustrates the path of two rays through the optical system. Since the optical path of a ray is about 3 times the distance $d$ between the objective and the eye-piece, the system is equivalent optically to an astronomical telescope of length $3d$. The focal lengths of the objective and eye-piece in the prism binoculars can thus provide the same angular magnification as an astronomical telescope 3 times as long. The compactness of the prism binocular is one of its advantages; another advantage is the wide field of view obtained, as it is an astronomical telescope (p. 541).

## Projection Lantern

The projection lantern is used for showing slides on a screen, and the *essential features* of the apparatus are illustrated in Fig. 23.17. S is a slide whose image is formed on the screen A by adjusting the position of an achromatic objective lens L.

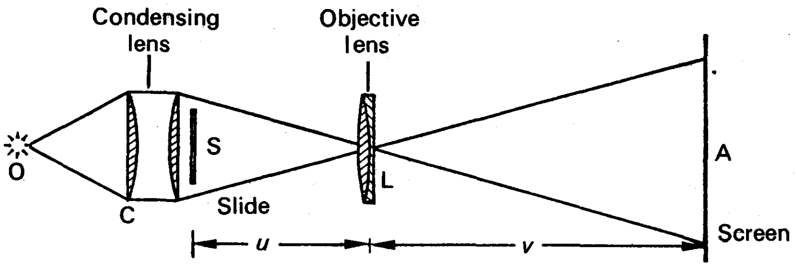



FIG. 23.17. Projection lantern.

The illumination of the slide must be as high as possible, otherwise the image of it on the screen is difficult to see clearly. For this purpose a very bright point source of light, O, is placed near a *condensing lens* C, and the slide S is placed immediately in front of C. The condensing lens consists of a plano-convex lens arrangement, which concentrates the light energy from O in the direction of S, and it has a short focal length. The lens L and the source O are arranged to be conjugate foci for the lens C (i.e., the image of O is formed at L), in which case (i) all the light passes through L, and (ii) an image of O is not formed on the screen. Fig. 23.17 illustrates the path of the beam of light from O which forms the image of S on the screen.

The linear magnification, $m$, of the slide is given by $m = \frac{v}{u}$, where $v$, $u$ are the respective screen and slide distances from L. Now $\frac{v}{u} = \frac{v}{f} - 1$ (see p. 528). Thus the required high magnification is obtained by using an objective whose focal length is small compared with $v$.

## Pinhole Camera

The *pinhole camera* consists essentially of a closed box with a pinhole in front and a photographic plate at the back on which the image is formed. The principle was first discovered by PORTA about 1600, who found that clear images were formed on a screen at the back of the box when objects were placed in front of the pinhole.

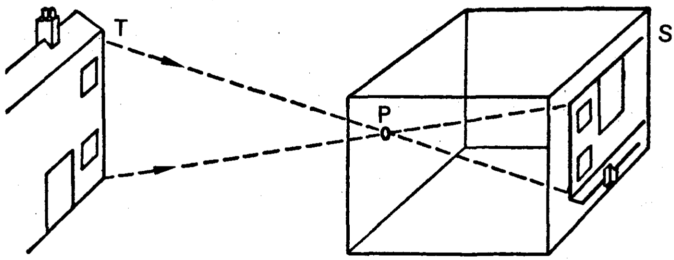



FIG. 23.18. Action of pinhole camera (*not to scale*).

The simple camera utilises the principle that rays of light normally travel in straight lines. As the pinhole P is small, a very narrow cone of rays pass through P from a point T on a house, for example, in front of the box; thus a well-defined image of T is obtained on the photographic plate S, Fig. 23.18. Similarly, other points on the building give rise to clear images on the screen. If the pinhole is enlarged a blurred image is obtained, as the rays from different points on the building then tend to overlap.

The pinhole camera is used by surveyors to photograph the outline of buildings, as the image obtained is free from the distortion produced by the lens in a normal camera.

## Photographic Camera; *f*-number

The photographic camera consists essentially of a *lens system* L, a *light-sensitive film* F at the back, and a *focusing arrangement*, Fig. 23.19.

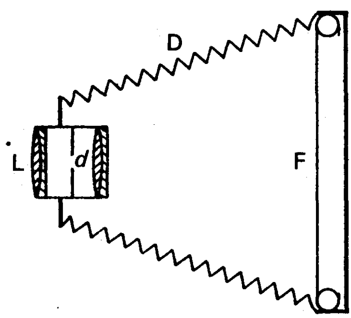



FIG. 23.19. Photographic camera.

The latter is usually a concertina-shaped canvas bag D, which adjusts the distance of the lens L from F. The lens in the camera is an achromatic doublet (p. 515), and the use of two lenses diminishes spherical aberration (p. 518). An *aperture* or *stop* of diameter $d$ is provided, so that the light is incident centrally on the lens, thus diminishing distortion.

The amount of luminous flux falling on the image in a camera is proportional to the area of the lens aperture, or to $d^2$, where $d$ is the diameter of the aperture. The area of the image formed is proportional to $f^2$, where $f$ is the focal length of the lens, since the length of the image formed is proportional to the focal length, as illustrated by Fig. 23.20. It therefore follows that the

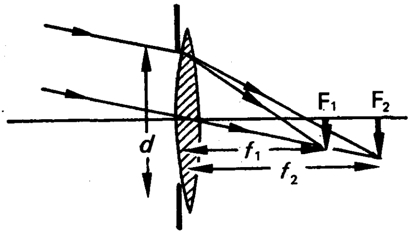



FIG. 23.20. Brightness of image.

luminous flux per unit area of the image, or *brightness B*, of the image, is proportional to $d^2/f^2$. The time of exposure, $t$, for activating the chemicals on the given negative is inversely proportional to $B$. Hence

$$t \propto \frac{f^2}{d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

The *relative aperture* of a lens is defined as the ratio $d/f$, where $d$ is the diameter of the aperture and $f$ is the focal length of the lens. The aperture is usually expressed by its *f-number*. If the aperture is $f$-4, this means that the diameter $d$ of the aperture is $f/4$, where $f$ is the focal length of the lens. An aperture of $f$-8 means a diameter $d$ equal to $f/8$, which is a smaller aperture than $f/4$.

Since the time $t$ of exposure is proportional to $f^2/d^2$, from (i), it follows that the exposure required with an aperture $f$-8 ($d = f/8$) is 16 times that required with an aperture $f$-2 ($d = f/2$). The $f$-numbers on a camera are 2, 2·8, 3·5, 4, 4·8, for example. On squaring the values of $f/d$ for each number, we obtain the relative exposure times, which are 4, 8, 12, 16, 20, or 1, 2, 3, 4, 5.

## Depth of Field

An object will not be seen by the eye until its image on the retina covers at least the area of a single cone, which transmits along the optic nerve light energy just sufficient to produce the sensation of vision. As a basis of calculation in photography, a circle of finite diameter about 0·25 mm viewed 250 mm away will just be seen by the eye, as a fairly sharp point, and this is known as the *circle of least confusion*. It corresponds to an angle of about 1/1000th radian subtended by an object at the eye.

On account of the lack of resolution of the eye, a camera can take clear pictures of objects at different distances. Consider a point object O in front of a camera lens A which produces a point image I on a film, Fig. 23.21. If XY represents the diameter of the circle of least confusion round I, the eye will see all points in the circle as reasonably sharp points. Now rays from the lens aperture to the edge of XY meet at $I_1$ beyond I, and also at $I_2$ in front of I. The point images $I_1$, $I_2$ correspond

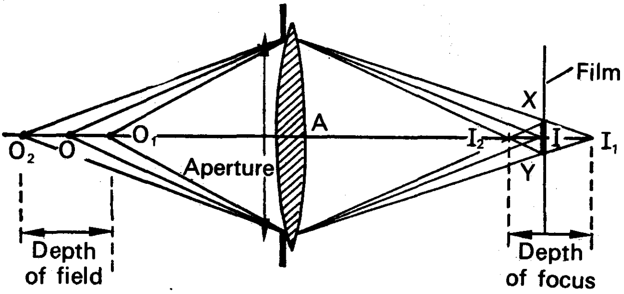



FIG. 23.21. Depth of field.

to point objects $O_1$, $O_2$ on either side of O, as shown. Consequently the images of all objects between $O_1$, $O_2$ are seen clearly on the film.

The distance $O_1O_2$ is therefore known as the *depth of field.* The distance $I_1I_2$ is known as the *depth of focus.* The depth of field depends on the lens aperture. If the aperture is made smaller, and the diameter XY of the circle of least confusion is unaltered, it can be seen from Fig. 23.21 that the depth of field increases. If the aperture is made larger, the depth of field decreases.

## The Mount Palomar Telescope

The construction of the largest telescope in the world is one of the most fascinating stories of scientific skill and invention. The major feature of the telescope is a parabolic *mirror*, 5 metres across, which is made of pyrex, a low expansion glass. The glass itself took more than six years to grind and polish, having been begun in 1936, and the front of the mirror is coated with aluminium, instead of being covered with silver, as it lasts much longer. The huge size of the mirror enables enough light from very distant stars and planets to be collected and brought to a focus for them to be photographed. Special cameras are incorporated in the instrument to photograph the universe. This method has the advantage that plates can be exposed for hours, if necessary, to the object to be studied, enabling records to be made. It is used to obtain useful information about the building-up and breaking-down of the elements in space (thus assisting in atomic energy research), to investigate astronomical theories of the universe, and to photograph Mars.

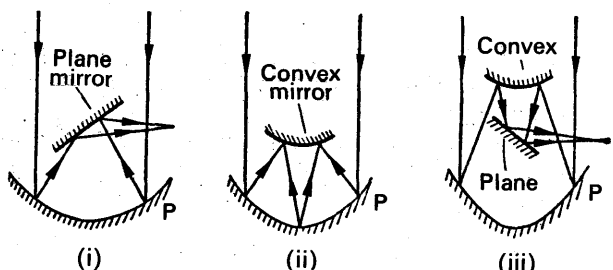



FIG. 23.22. (i). Newton reflector. (ii). Cassegrain reflector. (iii). Coudé reflector.

Besides the main parabolic mirror P, seven other mirrors are used in the 5 metre telescope. Some are plane, Fig. 23.22 (i), while others are convex, Fig. 23.22 (ii), and they are used to bring the light to a more convenient focus, where the image can be photographed, or magnified several hundred times by an eye-piece for observation. The various methods of focusing the image were suggested respectively by *Newton*, *Cassegrain*, and *Coudé*, the last being a combination of the former two methods.

## EXAMPLES

**1.** What do you understand by the magnifying power of an astronomical telescope? Illustrate your answer with a ray diagram depicting the use of the instrument to view stars in the heavens. If such a telescope has an object glass of focal length 50 cm and an eye lens of focal length 5 cm, what is its magnifying power? If it is assumed that the eye is placed very close to the eye lens and that the pupil of the eye has a diameter of 3 mm, what will be the diameter of the object glass if all the light passing through the object glass is to emerge as a beam which fills the pupil of the eye? Assume that the telescope is pointed directly at a particular star. (*W.*)

First part. See text.

Second part. Assuming the telescope is in *normal* adjustment, the final image is formed at infinity. The magnifying power of the telescope is then $\frac{50\text{ cm}}{5\text{ cm}}$, or 10. See p. 529.

If all the light emerging from the eye-piece fills the pupil of the eye, the pupil is at the eye-ring. See p. 534. The eye-ring is the image of the objective in the eye-piece. Since the distance, $u$, from the objective to the eye-piece $= 50 + 5 = 55$ cm, the eye-ring distance, $v$, is given by

$$\frac{1}{v} + \frac{1}{(+55)} = \frac{1}{(+5)}$$

from which $$v = 5\cdot5 \text{ cm.}$$

This is the position of the pupil of the eye. The magnification of the objective is given by

$$\frac{\text{eye-ring diameter}}{\text{objective diameter}} = \frac{v}{u} = \frac{5\cdot5}{55} = \frac{1}{10}$$

$$\therefore \quad \frac{3\text{ mm}}{\text{objective diameter}} = \frac{1}{10}$$

$$\therefore \quad \text{objective diameter} = 3\text{ cm.}$$

**2.** What do you understand by (*a*) the apparent size of an object, and (*b*) the magnifying power of a microscope? A model of a compound microscope is made up of two converging lenses of 3 and 9 cm focal length at a fixed separation of 24 cm. Where must the object be placed so that the final image may be at infinity? What will be the magnifying power if the microscope as thus arranged is used by a person whose nearest distance of distinct vision is 25 cm? State what is the best position for the observer's eye and explain why. (*L.*)

First part. (*a*) The apparent size of an object is proportional to the visual angle. See p. 525. (*b*) The magnifying power of a microscope is defined on p. 527.

Second part. (i) Suppose the objective A is 3 cm focal length, and the eye-piece B is 9 cm focal length, Fig. 23.23. If the final image is at infinity, the image $I_1$ in the objective must be 9 cm from B, the focal length of the eye-piece. See p. 531. Thus the image distance $LI_1$, from the objective A = 24 − 9 = 15 cm. The object distance OL is thus given by

$$\frac{1}{(+15)}+\frac{1}{u}=\frac{1}{(+3)},$$

from which $u = OL = 3\frac{3}{4}$ cm.

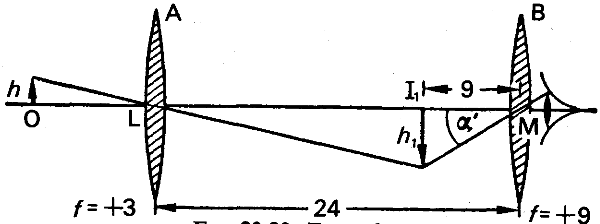



FIG. 23.23. Example.

(ii) The angle $\alpha'$ subtended at the observer's eye is given by $\alpha' = h_1/9$, where $h_1$ is the height of the image at $I_1$, Fig. 23.23. Without the lenses, the object subtends an angle $\alpha$ at the eye given by $\alpha = h/25$, where $h$ is the height of the object, since the least distance of distinct vision is 25 cm.

$$\therefore \quad \text{magnifying power } M = \frac{\alpha'}{\alpha} = \frac{h_1/9}{h/25} = \frac{25}{9} \times \frac{h_1}{h}$$

But

$$\frac{h_1}{h} = \frac{LI_1}{LO} = \frac{15}{3\frac{3}{4}} = 4$$

$$\therefore \quad M = \frac{25}{9} \times 4 = 11\frac{1}{9}$$

The best position of the eye is at the eye-ring, which is the image of the objective A in the eye-piece B (p. 534).

3. A Galilean telescope has an object-glass of 12 cm focal length and an eye lens of 5 cm focal length. It is focused on a distant object so that the final image seen by the eye appears to be situated at a distance of 30 cm from the eye lens. Determine the angular magnification obtained and draw a ray diagram. What are the advantages of prism binoculars as compared with field glasses of the Galilean type? (*N.*)

Suppose $I_2$ is the final image, distant 30 cm from the eye lens $L_2$. Fig. 23.24. The corresponding object is $I_1$. Since $I_2$ is a virtual image in $L_2$, $v = I_2L_2 = -30, f = -5, u = L_2I_2$. From the lens equation for $L_2$, we have

$$\frac{1}{(-30)}+\frac{1}{u}=\frac{1}{(-5)}$$

from which $u = -6$ cm.

Thus $I_1$ is a virtual object for $L_2$.

The angular magnification, $M$, is given by $M = \alpha'/\alpha$. Now $\alpha' = h_1/L_2I_1$, and $\alpha = h_1/L_1I_1$.

$$\therefore \quad M = \frac{h_1/L_2I_1}{h_1/L_1I_1} = \frac{L_1I_1}{L_2I_1}$$

But $L_2I_1 = 6$ cm, from above, and $L_1I_1$ = focal length of $L_1 = 12$ cm, since the object is distant.

$$\therefore \quad M = \frac{12}{6} = 2$$

The advantages of the prism binoculars are given on p. 541.

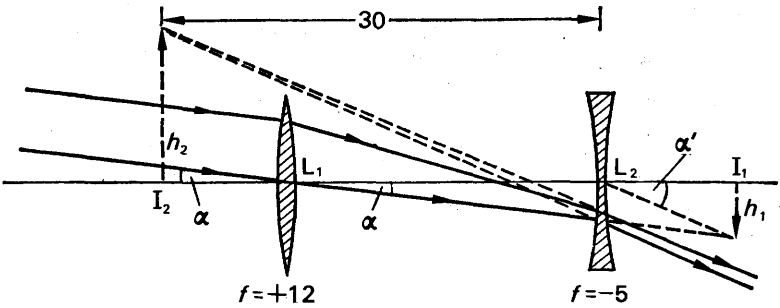



FIG. 23.24. Example.

**4.** Describe the optical system of a projection lantern. A lantern is required for the projection of slides 7·5 cm square on to a screen 2·1 m square. The distance between the front of the lantern and the screen is to be 20 m. What focal length of projection lens would you consider most suitable?

First part. See text.

Second part. Suppose O is the slide, L is the projection lens, and S is the screen, Fig. 23.25. The linear magnification, $m$, due to the lens is given by

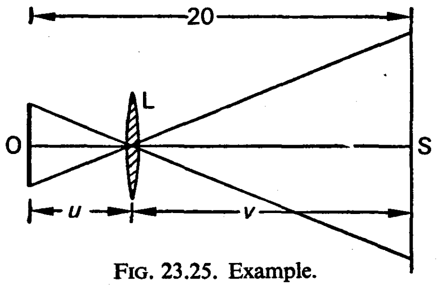



FIG. 23.25. Example.

$$m = \frac{210\text{ cm}}{7{\cdot}5\text{ cm}} = 28$$

$$\therefore \quad LS : LO = 28 : 1$$

$$\therefore \quad LS = v = \frac{28}{29} \times 20\text{ m}$$

and $$LO = u = \frac{1}{29} \times 20\text{ m.}$$

Applying the lens equation,

$$\therefore \quad \frac{1}{560/29} + \frac{1}{20/29} = \frac{1}{f}$$

from which $$f = \frac{560}{841}\text{ m} = 67\text{ cm (to nearest cm)}$$

## EXERCISES 23

1. An object is viewed with a normal eye at (i) the least distance of distinct vision, (ii) 40 cm, (iii) 100 cm from the eye. Find the ratio of the visual angles in the three cases, and raw diagrams in illustration.

2. Where should the final image be formed when (*a*) a telescope, (*b*) a microscope is in *normal* use? Define the angular magnification (magnifying power) of a telescope and a microscope.

3. Explain the essential features of the astronomical telescope. Define and deduce an expression for the magnifying power of this instrument.

A telescope is made of an object glass of focal length 20 cm and an eyepiece of 5 cm, both converging lenses. Find the magnifying power in accordance with your definition in the following cases: (*a*) when the eye is focused to receive parallel rays, and (*b*) when the eye sees the image situated at the nearest distance of distinct vision which may be taken as 25 cm. (*L.*)

4. Explain the action of a microscope consisting of two thin lenses and show on a diagram the paths of three rays from a non-axial object point to the eye. Distinguish clearly between rays and construction lines.

A microscope having as eyepiece a thin lens of focal length 5·0 cm is set up by an observer whose least distance of distinct vision is 25·0 cm. An observer with defective eyesight has to withdraw the eyepiece by 0·50 cm in order that the image may be at his least distance of distinct vision. Find the nature of the defect and specify the nature and the focal length of the spectacle lens he needs to make his least distance of distinct vision 25·0 cm. (*L.*)

5. Describe, with the help of diagrams, how (*a*) a single biconvex lens can be used as a magnifying glass, (*b*) two biconvex lenses can be arranged to form a microscope. State (i) one advantage, (ii) one disadvantage, of setting the microscope so that the final image is at infinity rather than at the near point of the eye.

A centimetre scale is set up 5 cm in front of a biconvex lens whose focal length is 4 cm. A second biconvex lens is placed behind the first, on the same axis, at such a distance that the final image formed by the system coincides with the scale itself and that 1 mm in the image covers 2·4 cm in the scale. Calculate the position and focal length of the second lens. (*O. & C.*)

6. Compare and contrast the optical properties of (*a*) an astronomical telescope, (*b*) a Galilean telescope, (*c*) a reflecting telescope. Draw ray diagrams for (*a*) and (*b*) showing the path through each instrument of a non-axial pencil of rays from a distant object.

Describe one method by which the image in an astronomical telescope may be made erect. (*L.*)

7. What is the *eye-ring* of a telescope?

For an astronomical telescope in normal adjustment deduce expressions for the size and position of the eye-ring in terms of the diameter of the object glass and the focal lengths of the object glass and eye-lens.

Discuss the importance of (i) the magnitude of the diameter of the object glass, (ii) the structure of the object glass, (iii) the position of the eye. (*L.*)

8. Show, by means of a ray diagram, how an image of a distant extended object is formed by an astronomical refracting telescope in normal adjustment (i.e. with the final image at infinity).

A telescope objective has focal length 96 cm and diameter 12 cm. Calculate the focal length and minimum diameter of a simple eyepiece lens for use with the telescope, if the magnifying power required is $\times$ 24, and all the light transmitted by the objective from a distant point on the telescope axis is to fall on the eyepiece. Derive any formulae you use.

If the eyepiece is an equiconvex lens made from glass of refractive index 1·6, calculate the radius of curvature of its faces and the minimum thickness of the lens at its centre. (*O. & C.*)

**9.** Draw the path of two rays, from a point on an object, passing through the optical system of a compound microscope to the final image as seen by the eye.

If the final image formed coincides with the object, and is at the least distance of distinct vision (25 cm) when the object is 4 cm from the objective, calculate the focal lengths of the objective and eye lenses, assuming that the magnifying power of the microscope is 14. (*L.*)

**10.** Define *magnifying power* (*angular magnification*) of an optical instrument.

An astronomical telescope consists of two thin converging lenses which are 25·00 cm apart when the telescope is in normal adjustment. The distance between the lenses is reduced to 24·50 cm and a virtual image of an infinitely distant object is then formed 28·00 cm from the eye lens. Calculate the values of the focal lengths of the two lenses and the magnifying power of the instrument with this adjustment, supposing the eye to be placed at the eye lens.

For this same adjustment show on a labelled diagram (not to scale) the relative positions of the principal foci of the two lenses and the construction lines showing the relation of the final image to the intermediate image. On the separate diagram show the paths through the instrument of two rays from a non-axial point. One of the rays should pass through the centre of the objective and the other through its periphery. (*L.*)

**11.** Distinguish between the *magnification* produced by, and the *magnifying power* of, an optical system.

Draw a ray diagram showing the action of a simple astronomical telescope (assume two lenses only) in forming separate images of two stars which are close together and near the axis, the final images being at infinity.

If the objective has a diameter of 30 cm and a focal length of 3 metres, and the focal length of the eyepiece is 1·2 cm, calculate (*a*) the magnifying power of the telescope, (*b*) the diameter of the image of the objective formed by the eyepiece. (*O. & C.*)

**12.** Briefly describe an optical instrument which includes a reflecting prism. What is the function of the prism and what is the principle governing its action?

What are the advantages in using a prism rather than a silvered mirror in the apparatus you describe?

Parallel rays of light fall normally on the face $AC$ of a total reflection prism $ABC$ of refractive index 1·5 which has angle $A$ exactly 45°, angle $B$ approximately 90° and angle $C$ approximately 45°. After total internal reflections in the prism two beams of parallel light emerge from the hypotenuse face, the angle between them being 6°. Calculate the value of angle $B$. You may assume that for small angles $\sin i/\sin r$ equals $i/r$.

How would you discover whether the angle is more or less than 90°? (*O. & C.*)

13. Give a detailed description of the optical system of the compound microscope, explaining the problems which arise in the design of an object lens for a microscope.

A compound microscope has lenses of focal length 1 cm and 3 cm. An object is placed 1·2 cm from the object lens; if a virtual image is formed 25 cm from the eye, calculate the separation of the lenses and the magnification of the instrument. (*O. & C.*)

14. A projection lantern contains a condensing lens and a projection lens. Show clearly in a ray diagram the function of these lenses.

A lantern has a projection lens of focal length 25 cm and is required to be able to function when the distance from lantern to screen may vary from 6 m to 12 m. What range of movement for the lens must be provided in the focusing arrangement? What is the approximate value of the ratio of the magnifications at the two extreme distances? (*W.*)

15. A converging lens of focal length 20 cm and a diverging lens of focal length 10 cm are arranged for use as an opera glass. Draw a ray diagram to scale showing how the final image at infinity is produced, describing briefly how you do this, and derive the magnifying power.

When an object is placed 60 cm in front of the converging lens and the lenses are separated by a distance $x$, a real image is formed 30 cm beyond the diverging lens. Calculate $x$. (*C.*)

16. An astronomical telescope consisting of an objective focal length 60 cm and an eyepiece of focal length 3 cm is focused on the moon so that the final image is formed at the minimum distance of distinct vision (25 cm) from the eyepiece. Assuming that the diameter of the moon subtends an angle of $\frac{1}{2}°$ at the objective, calculate (*a*) the angular magnification, (*b*) the actual size of the image seen.

How, with the same lenses, could an image of the moon, 10 cm in diameter, be formed on a photographic plate? (*C.*)

17. Explain, with the aid of a ray diagram, how a simple astronomical telescope employing two converging lenses may form an apparently enlarged image of a distant extended object. State with reasons where the eye should be placed to observe the image.

A telescope constructed from two converging lenses, one of focal length 250 cm, the other of focal length 2 cm, is used to observe a planet which subtends an angle of $5 \times 10^{-5}$ radian. Explain how these lenses would be placed for normal adjustment and calculate the angle subtended at the eye of the observer by the final image.

How would you expect the performance of this telescope for observing a star to compare with one using a concave mirror as objective instead of a lens, assuming that the mirror had the same diameter and focal length as the lens. (*O. & C.*)