chapter twenty-one

Refraction through lenses

A *lens* is a piece of glass bounded by one or two spherical surfaces. When a lens is thicker in the middle than at the edges it is called a *convex* or *converging lens*, Fig. 21.1 (i); when it is thinner in the middle than at the edges it is known as a *concave* or *diverging lens*, Fig. 21.1 (ii). Fig. 21.9, on p. 478, illustrates other types of converging and diverging lenses.

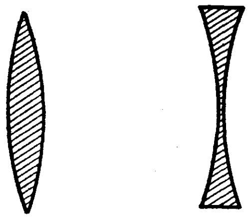

FIG. 21.1.
Converging and diverging lenses.

Lenses were no doubt made soon after the art of glass-making was discovered; and as the sun's rays could be concentrated by these curved pieces of glass they were called "burning glasses". ARISTOPHANES, in 424 B.C., mentions a burning glass. To-day, lenses are used in spectacles, cameras, microscopes, and telescopes, as well as in many other optical instruments, and they afford yet another example of the many ways in which Science is used to benefit our everyday lives.

Since a lens has a curved spherical surface, a thorough study of a lens should be preceded by a discussion of the refraction of light through a curved surface. We shall therefore proceed to consider what happens in this case, and defer a discussion of lenses until later, p. 478.

REFRACTION AT CURVED SPHERICAL SURFACE

Relation Between Object and Image Distances

Consider a curved spherical surface NP, bounding media of refractive indices n_1, n_2 respectively, Fig. 21.2. The medium of refractive index n_1 might be air, for example, and the other of refractive index n_2 might be glass. The centre, C, of the sphere of which NP is part is the centre of curvature of the surface, and hence CP is the radius of curvature, *r*. The line joining C to the mid-point P of the surface is known as its *principal axis*. P is known as the *pole*.

Suppose a point object O is situated on the axis PC in the medium of refractive index n_1. The image of O by refraction at the curved surface can be obtained by taking two rays from O. A ray OP passes straight through along PC into the medium of refractive index n_2, since OP is normal to the surface, while a ray ON *very close* to the axis is refracted at N along NI towards the normal CN, if we assume n_2 is greater than n_1. Thus at the point of intersection, I, of OP and NI is the image O, and we have here the case of a real image.

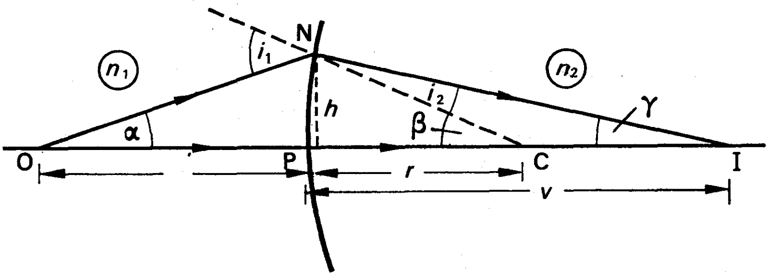

FIG. 21.2. Refraction at curved surface.

Suppose i_1, i_2 are the angles made by ON, NI respectively with the normal, CN, at N, Fig. 21.2. Then, applying "$n \sin i$" is a constant (p. 423),

$$n_1 \sin i_1 = n_2 \sin i_2 \quad . \quad . \quad . \quad . \quad \text{(i)}$$

But if we deal with rays from O very close to the axis OP, i_1 is small; and hence $\sin i_1 = i_1$ in radians. Similarly, $\sin i_2 = i_2$ in radians. From (i), it follows that

$$n_1 i_1 = n_2 i_2 \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

If α, β, γ are the angles with the axis made by ON, CN, IN respectively, we have

$$i_1 = \alpha + \beta, \text{ from the geometry of triangle ONC,}$$

and $$i_2 = \beta - \gamma, \text{ from the geometry of triangle CNI.}$$

Substituting for i_1, i_2 in (ii), we have

$$n_1 (\alpha + \beta) = n_2 (\beta - \gamma)$$

$$\therefore \; n_1 \alpha + n_2 \gamma = (n_2 - n_1) \beta \quad . \quad . \quad . \quad \text{(iii)}$$

If h is the height of N above the axis, and N is so close to P that NP is perpendicular to OP,

$$\alpha = \frac{h}{\text{OP}}, \; \gamma = \frac{h}{\text{PI}}, \; \beta = \frac{h}{\text{PC}},$$

From (iii), using our sign convention on p. 407, we have

$$h\left(\frac{n_1}{+\text{OP}} + \frac{n_2}{+\text{PI}}\right) = h\,\frac{(n_2 - n_1)}{\text{PC}},$$

since O is a real object and I is a real image.

$$\therefore \quad \frac{n_1}{\text{OP}} + \frac{n_2}{\text{PI}} = \frac{n_2 - n_1}{\text{PC}}.$$

If the object distance, OP, from P $= u$, the image distance, IP, from P $= v$, and PC $= r$, then

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{r} \quad . \quad . \quad \text{(1)}$$

Sign Convention for Radius of Curvature

Equation (1) is the general relation between the object and image distances, u, v, from the middle or pole of the refracting surface, its radius of curvature r, and the refractive indices of the media, n_2, n_1. The quantity $(n_2 - n_1)/r$ is known as the **power** of the surface. If a ray is made to converge by a surface, as in Fig. 21.1, the power will be assumed *positive* in sign; if a ray is made to diverge by a surface, the power will be assumed negative. Since refractive index is a ratio of velocities (p. 421), n_1 *and* n_2 *have no sign.* $(n_2 - n_1)$ on the right side of equation (1) will be taken always as a *positive* quantity, and thus denotes the smaller refractive index subtracted from the greater refractive index. The sign convention for the radius of curvature, r, of a spherical surface is now as follows: if the surface is *convex to the less dense* medium, its radius is *positive*; if it is concave to the less dense medium, its radius is negative. We have thus to view the surface from a point in the less dense medium. In Fig. 21.3 (i), the surface A is convex to the less dense

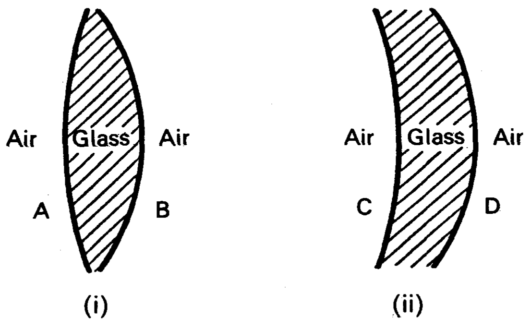

FIG. 21.3. Sign convention for radius of curvature.

medium air, and hence its radius is positive. The surface C is concave to the less dense medium air, and its radius is thus negative, Fig. 21.3 (ii). The radii of the surfaces B and D are both positive.

Special Cases

The general formula $\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 \sim n_1}{r}$, can easily be remembered on account of its symmetry. The object distance u corresponds to the refractive index n_1 of the medium in which the object is situated; while the image distance v corresponds to the medium of refractive index n_2 in which the image is situated.

Suppose an object O in air is x cm from a curved spherical surface, and the image I is real and in glass of refractive index n, at a distance of y cm from the surface, Fig. 21.4 (i). Then $u = +x$, $v = +y$, $n_1 = 1$, $n_2 = n$. If the surface is convex to the less dense medium, as shown in Fig. 21.4 (i), the radius of curvature, a cm, is given by $r = +a$.

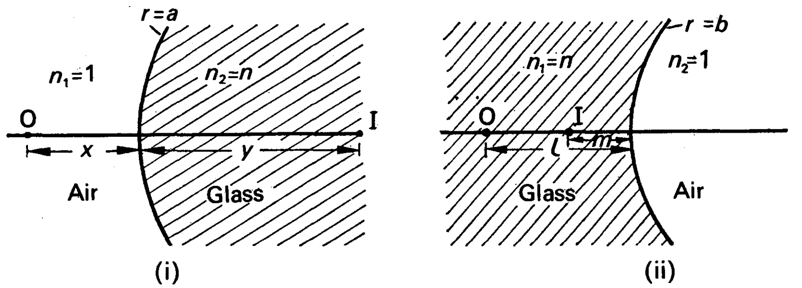

FIG. 21.4. Special cases of refraction formula.

Substituting in $$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 \sim n_1}{r}$$

$$\therefore \quad \frac{1}{x} + \frac{n}{y} = \frac{n-1}{a}$$

Fig. 21.4 (ii) illustrates the case of an object O in glass of refractive index n, the surface being concave to the less dense surface air. The radius, b cm, is then given by $r = -b$. If the image I is virtual, its distance $v = -m$. If l is the distance of O, then $u = +l$.

Substituting in $$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 \sim n_1}{r}$$

$$\therefore \quad \frac{n}{l} + \frac{1}{-m} = \frac{n-1}{-b}$$

If a surface is *plane*, its radius of curvature, r, is infinitely large.

Hence $\frac{n_2 \sim n_1}{r}$ is zero, whatever different values n_1 and n_2 may have.

Deviation of Light by Sphere

Suppose a ray AO in air is incident on a sphere of glass or a drop of water, Fig. 21.5 (i). The light is refracted at O, then reflected inside B,

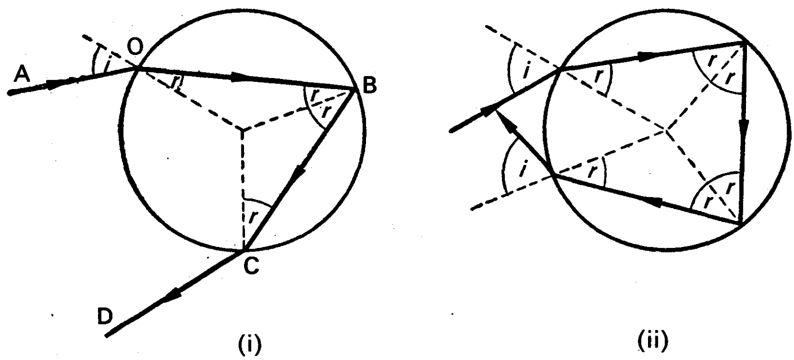

FIG. 21.5. Deviation of light by sphere.

and finally emerges into the air along CD. If i, r are the angles of incidence and refraction at O, the deviation of the light at O and C is $(i - r)$ each time; it is $(180° - 2r)$ at B. The total deviation, δ, in a clockwise direction is thus given by

$$\delta = 2(i - r) + 180° - 2r = 180° + 2i - 4r \quad . \quad . \quad \text{(i)}$$

It can be seen that the deviation at each reflection inside the sphere is $(180° - 2r)$ and that the deviation at each refraction is $(i - r)$. Thus if a ray undergoes two reflections inside the sphere, and two refractions, as shown in Fig. 21.5 (ii), the total deviation in a clockwise direction $=$ $2(i - r) + 2(180° - 2r) = 360° + 2i - 6r$. After m internal reflections,

$$\text{the total deviation} = 2(i - r) + m(180° - 2r).$$

The Rainbow

The explanation of the colours of the rainbow was first given by Newton about 1667. He had already shown that sunlight consisted of a mixture of colours ranging from red to violet, and that glass could disperse or separate the colours (p. 454). In the same way, he argued, water droplets in the air dispersed the various colours in different directions, so that the colours of the spectrum were seen.

The curved appearance of the rainbow was first correctly explained about 1611. It was attributed to refraction of light at a water drop, followed by reflection inside the drop, the ray finally emerging into the air as shown in Fig. 21.6. The *primary bow* is the rainbow usually seen, and is obtained by two refractions and one reflection at the drops, as in Fig. 21.6. Sometimes a *secondary bow* is seen higher in the sky, and it is formed by rays undergoing two refractions and two reflections at the drop, as in Fig. 21.6.

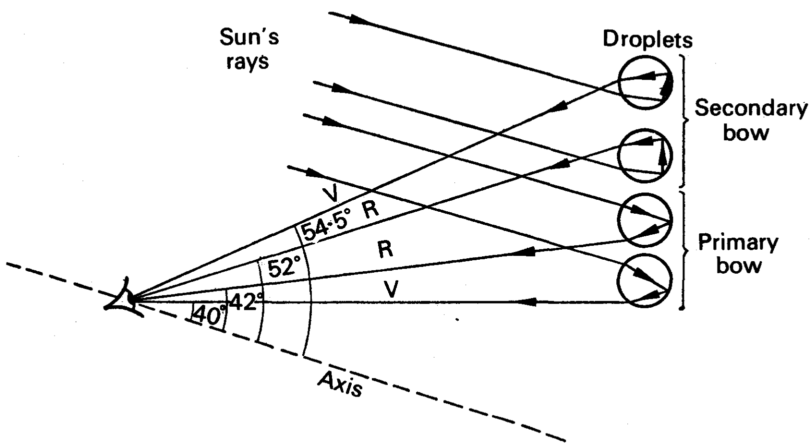

FIG. 21.6. The Rainbow.

The total deviation δ of the light when one reflection occurs in the drop, Fig. 21.6, is given by $\delta = 180° + 2i - 4r$, as proved before. Now the light emerging from the drop will be intense at those angles of incidence corresponding to the *minimum* deviation position, since a considerable number of rays have about the same deviation at the minimum value and thus emerge almost parallel. Now for a minimum value, $\frac{d\delta}{di} = 0$.

Differentiating the expression for δ, we have $2 - 4\frac{dr}{di} = 0$.

$$\therefore \quad \frac{dr}{di} = \frac{1}{2}$$

But $$\sin i = n \sin r,$$
where n is the refractive index of water.

$$\therefore \quad \cos i = n \cos r \frac{dr}{di} = \frac{n}{2} \cos r$$

$$\therefore \quad 4\cos^2 i = n^2 \cos^2 r = n^2 - n^2 \sin^2 r = n^2 - \sin^2 i$$
$$= n^2 - (1 - \cos^2 i)$$

$$\therefore \quad 3\cos^2 i = n^2 - 1$$

$$\therefore \quad \cos i = \sqrt{\frac{n^2 - 1}{3}} \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

The refractive index of water for red light is 1·331. Substituting this value in (ii) i can be found, and thus r is obtained. The deviation δ can then be calculated, and the acute angle between the incident and emergent red rays, which is the supplement of δ, is about 42·1°. By substituting the refractive index of water for violet light in (ii), the acute angle between the incident and emergent violet rays is found to be about 40·2°. Thus if a shower of drops is illuminated by the sun's rays, an observer standing with his back to the sun sees a brilliant red light at an angle of 42·1° with the line joining the sun to him, and a brilliant violet light at an angle of 40·2° with this line, Fig. 21.6. Since the phenomenon is the same in all planes passing through the line, the brightly coloured drops form an arc of a circle whose centre is on the line.

The secondary bow is formed by two internal reflections in the water drops, as illustrated in Fig. 21.5 (ii) and Fig. 21.6. The minimum deviation occurs when $\cos i = \sqrt{(n^2 - 1)/8}$ in this case. The acute angle between the incident and emergent red rays is then found to be about 51·8°, and that for the violet rays is found to be about 54·5°. Thus the secondary bow has red on the inside and violet on the outside, whereas the primary bow colours are the reverse, Fig. 21.6.

EXAMPLES

1. Obtain a formula connecting the distances of object and image from a spherical refracting surface. A small piece of paper is stuck on a glass sphere of 5 cm radius and viewed through the glass from a position directly opposite. Find the position of the image. Find also the position of the image formed, by the sphere, of an object at infinity. (*O. & C.*)

First part. See text.

Second part. Suppose O is the piece of paper, Fig. 21.7 (i). The refracting surface of the glass is at P, and $u = +10$. Now

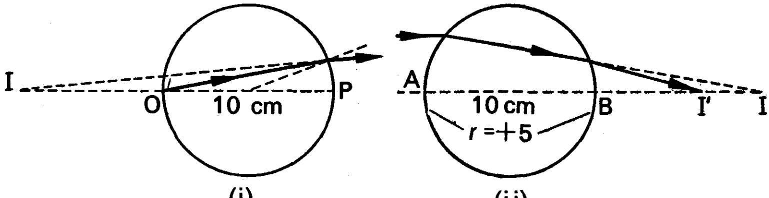

FIG. 21.7. Example

$$\frac{n_2}{v}+\frac{n_1}{u}=\frac{n_2 \sim n_1}{r}$$

where $n_1 = 1{\cdot}5$, $n_2 = 1$, $r = +5$ p. 472 and v is the image distance from P.

Substituting, $$\frac{1}{v}+\frac{1{\cdot}5}{10}=\frac{1{\cdot}5-1}{5}$$

$$\therefore \quad \frac{1}{v}=0{\cdot}1-0{\cdot}15=-0{\cdot}05$$

$$\therefore \quad v=-20 \text{ cm.}$$

Thus the image is virtual, i.e., it is 20 cm from P on the same side as O.

Third part. Suppose I is the position of the image by refraction at the first surface, A, Fig. 21.7 (ii). Now $\frac{n_2}{v}+\frac{n_1}{u}=\frac{n_2 \sim n_1}{r}$, where $u = \infty$, $n_1 = 1$ $n_2 = 1{\cdot}5$, $r = +5$.

$$\therefore \quad \frac{1{\cdot}5}{v}=\frac{1{\cdot}5-1}{5}$$

$$\therefore \quad v = 15 \text{ cm} = \text{AI, or BI} = 5 \text{ cm.}$$

I is a virtual object for refraction at the curved surface B. Since $u = -$ BI $= -5$ cm, $n_1 = 1{\cdot}5$, $n_2 = 1$, $r = +5$, it follows from

$$\frac{n_2}{v}+\frac{n_1}{u}=\frac{n_2 \sim n_1}{r}$$

that $$\frac{1}{v}+\frac{1{\cdot}5}{(-5}=\frac{1{\cdot}5-1}{5}$$

from which $$v = 2{\cdot}5 \text{ cm} = \text{BI}'.$$

2. An object is placed in front of a spherical refracting surface. Derive an expression connecting the distances from the refracting surface of the object and the image produced. The apparent thickness of a thick plano-convex lens is measured with (*a*) the plane face uppermost (*b*) the convex face uppermost, the values being 2 cm and $2\frac{2}{9}$ cm respectively. If its real thickness is 3 cm, calculate the refractive index of the glass and the radius of curvature of the convex face. (*L.*)

First part. See text.

Second part. With the plane face uppermost, the image I of the lowest point O is obtained by considering refraction at the plane surface D, Fig. 21.8 (i). Now

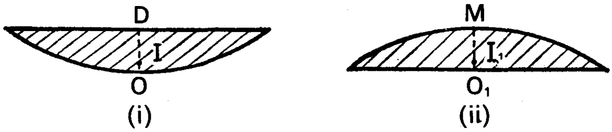

FIG. 21.8. Example

$$n = \frac{\text{real depth}}{\text{apparent depth}}$$

$$\therefore \quad n = \frac{3}{2} = 1{\cdot}5$$

With the curved surface uppermost, the image I_1 of the lowest point O_1 is obtained by considering refraction at the curved surface M, Fig. 21.8 (ii). In this case $MI_1 = v =$ apparent thickness $= -2\frac{2}{9}$ cm, the image I_1 being virtual. Now $u = MO_1 = 3$ cm, $n_2 = 1$, $n_1 = n = 1{\cdot}5$. Substituting in

$$\frac{n_2}{v} + \frac{n_1}{u} = \frac{n_2 \sim n_1}{r}$$

we have $$\frac{1}{-2\frac{2}{9}} + \frac{1{\cdot}5}{3} = \frac{1{\cdot}5 - 1}{r}.$$

Simplifying, $$r = 10 \text{ cm}.$$

REFRACTION THROUGH THIN LENSES

Converging and Diverging Lenses

At the beginning of the chapter we defined a lens as an object, usually of glass, bounded by one or two spherical surfaces. Besides the converging (convex) lens shown in Fig. 21.1 (i) on p. 471, Fig. 21.9 (i) illustrates two other types of converging lenses, which are thicker in the middle than at the edges. Fig. 84 (ii) illustrates two types of diverging (concave) lenses, a diverging lens being also shown in Fig. 21.1 (ii) on p. 471.

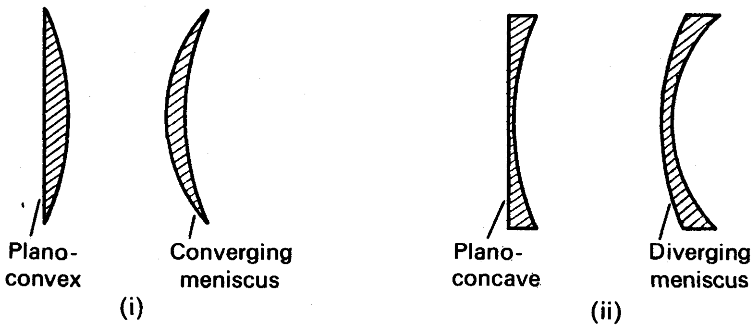

FIG. 21.9. (i). Convex (converging) lenses. (ii). Concave (diverging) lenses.

The *principal axis* of a lens is the line joining the centres of curvature of the two surfaces, and passes through the middle of the lens. Experiments with a ray-box show that a thin convex lens brings an incident parallel beam of rays to a *principal focus*, F, on the other side of the lens when the beam is narrow and incident close to the principal axis, Fig. 21.10 (i). On account of the convergent beam contained with it, the convex lens is better described as a "converging" lens. If a similar parallel beam is incident on the other (right) side of the lens, it converges to a focus F′, which is at the same distance from the lens as F when the lens is thin. To distinguish F from F′ the latter is called the "first principal focus"; F is known as the "second principal focus".

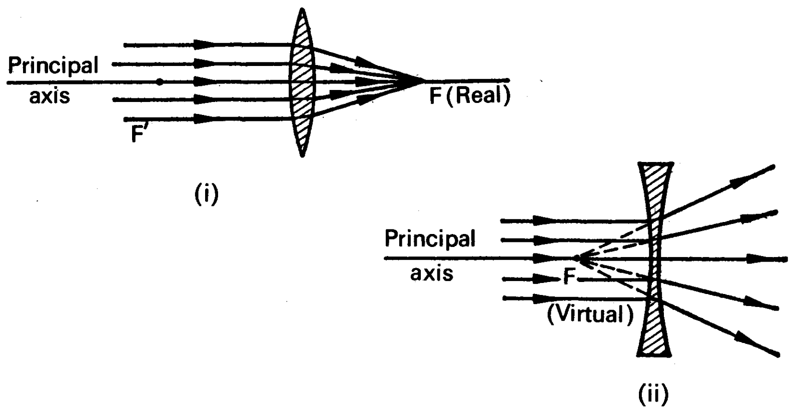

FIG. 21.10. Focus of converging (convex) and diverging (concave) lenses.

When a narrow parallel beam, close to the principal axis, is incident on a thin concave lens, experiment shows that a beam is obtained which appears to diverge from a point F on the same side as the incident beam, Fig. 21.10 (ii). F is known as the principal "focus" of the concave lens. Since a divergent beam is obtained, the concave lens is better described as a "diverging" lens.

Explanation of Effects of Lenses

A thin lens may be regarded as made up of a very large number of *small-angle prisms* placed together, as shown in the exaggerated sketches of Fig. 21.11. If the spherical surfaces of the various truncated prisms are

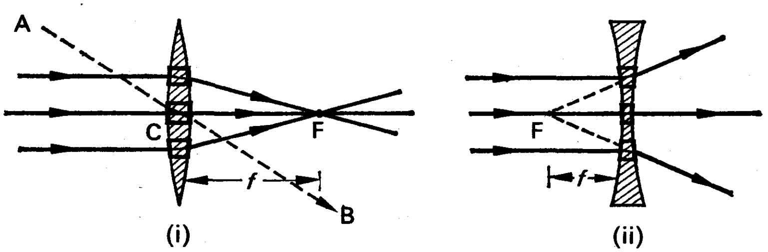

FIG. 21.11. Action of converging (convex) and diverging (concave) lenses.

imagined to be produced, the angles of the prisms can be seen to increase from zero at the middle to a small value at the edge of the lens. Now the deviation, d, of a ray of light by a small-angle prism is given by $d = (n - 1) A$, where A is the angle of the prism, see p. 457. Consequently the truncated prism corresponding to a position farther away from the middle of the lens deviates an incident ray more than those prisms nearer the middle. Thus, for the case of the converging lens, the refracted rays converge to the same point or focus F, Fig. 21.11 (i). It will be noted that a ray AC incident on the middle, C, of the lens emerges parallel to AC, since the middle acts like a rectangular piece of glass (p. 422). This fact is utilised in the drawing of images in lenses (p. 485).

Since the diverging lens is made up of truncated prisms pointing the opposite way to the converging lens, the deviation of the light is in the opposite direction, Fig. 21.11 (ii). A divergent beam is hence obtained when parallel rays are refracted by the lens.

The Signs of Focal Length, f

From Fig. 21.11 (i), it can be seen that a convex lens has a real focus; the focal length, f, of a *converging* lens is thus *positive* in sign. Since the focus of a diverging lens is virtual, the focal length of such a lens is negative in sign, Fig. 21.11 (ii). The reader must memorise the sign of f for a converging and diverging lens respectively, as this is always required in connection with lens formulæ.

Relations Between Image and Object Distances for Thin Lens

We can now derive a relation between the object and image distances when a lens is used. We shall limit ourselves to the case of a *thin* lens, i.e., one whose thickness is small compared with its other dimensions, and consider narrow beams of light incident on its central portion.

Suppose a lens of refractive index n_2 is placed in a medium of refractive index n_1, and a point object O is situated on the principal axis, Fig. 21.12.

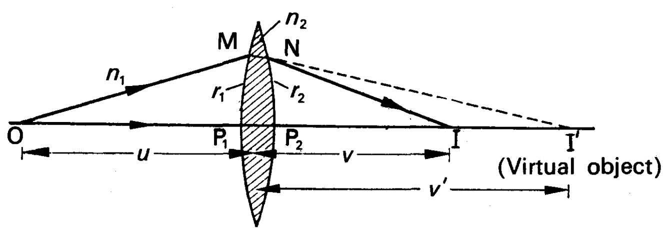

FIG. 21.12. Lens proof (exaggerated for clarity).

A ray from O through the middle of the lens passes straight through as it is normal to both lens surfaces. A ray OM from O, making a *small* angle with the principal axis, is refracted at the first surface in the direction MNI′, and then refracted again at N at the second surface so that it emerges along NI.

Refraction at first surface, MP_1. Suppose u is the distance of the object from the lens, i.e., $u = OP_1$, and v' is the distance of the image I' by refraction at the first surface, MP_1, of the lens, i.e., $v' = I'P_1$. Then, since I' is situated in the medium of refractive index n_2 (I' is on the ray MN produced), we have, if $n_2 > n_1$,

$$\frac{n_2}{v'} + \frac{n_1}{u} = \frac{n_2 - n_1}{r_1} \quad . \quad . \quad . \quad . \quad . \quad (i)$$

where r_1 is the radius of the spherical surface MP_1, see p. 472.

Refraction at second surface, NP_2. Since MN and P_1P_2 are the incident rays on the second surface NP_2. it follows that I' is a *virtual object* for refraction at this surface (see p. 412). Hence the object distance $I'P_2$ is negative; and as we are dealing with a thin lens, $I'P_2 = -v'$. The corresponding image distance, IP_2 or v, is positive since I is a real image. Substituting in the formula for refraction at a single spherical surface,

$$\frac{n_2}{-v'} + \frac{n_1}{v} = \frac{n_2 - n_1}{r_2} \quad . \quad . \quad . \quad . \quad (ii)$$

where r_2 is the radius of curvature of the surface NP_2 of the lens.

Lens equation. Adding (i) and (ii) to eliminate v', we have

$$\frac{n_1}{v} + \frac{n_1}{u} = (n_2 - n_1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right);$$

and dividing throughout by n_1,

$$\frac{1}{v} + \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad . \quad . \quad . \quad (iii)$$

Now parallel rays incident on the lens are brought to a focus. In this case, $u = \infty$ and $v = f$. From (iii),

$$\frac{1}{f} + \frac{1}{\infty} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\therefore \quad \frac{1}{\mathbf{f}} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{\mathbf{r_1}} + \frac{1}{\mathbf{r_2}}\right) \quad . \quad . \quad (2)$$

Substituting $\frac{1}{f}$ for the right-hand side of (iii), we obtain the important equation

$$\frac{1}{\mathbf{v}} + \frac{1}{\mathbf{u}} = \frac{1}{\mathbf{f}} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This is the "lens equation", and it applies equally to converging and diverging lenses if the sign convention is used (see also p. 407).

Focal Length of Lens. Small-angle Prism Method

The focal length f of a lens can also be found by using the deviation formula due to a small-angle prism. Consider a ray PQ parallel to the principal axis at a height h above it. Fig. 21.13 (i). This ray is refracted to the principal focus, and thus undergoes a small deviation through an angle d given by

$$d = \frac{h}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$

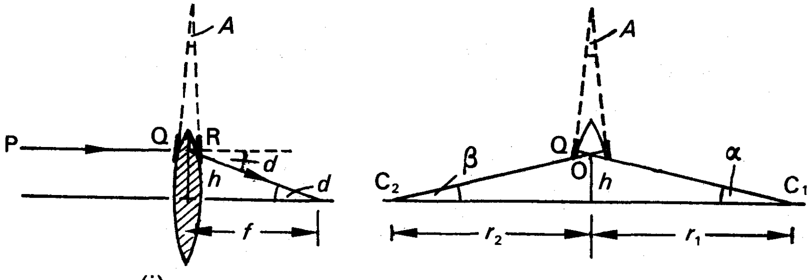

FIG. 21.13. Focal length by small angle prism.

This is the deviation through a prism of small angle A formed by the tangents at Q, R to the lens surfaces, as shown. Now for a small angle of incidence, which is the case for a thin lens and a ray close to the principal axis, $d = (n - 1) A$. See p. 457.

From (i), $$\frac{h}{f} = (n - 1) A$$

$$\therefore \quad \frac{1}{f} = (n - 1)\frac{A}{h} \qquad . \quad . \quad . \qquad \text{(ii)}$$

The normals at Q, R pass respectively through the centres of curvatures C_1, C_2 of the lens surfaces. From the geometry, angle $ROC_1 = A = \alpha + \beta$, where α, β, are the angles with the principal axis at C_1, C_2 respectively, as shown. But $\alpha = h/r_1$, $\beta = h/r_2$.

$$\therefore \quad A = \alpha + \beta = \frac{h}{r_1} + \frac{h}{r_2} \qquad \text{(iii)}$$

$$\therefore \quad \frac{A}{h} = \frac{1}{r_1} + \frac{1}{r_2}.$$

Substituting in (ii), $$\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right).$$

Focal Length Values

Since $\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$, it follows that the focal length of a lens depends on the refractive index, n_2, of its material, the refractive index, n_1, of the medium in which it is placed, and the radii of curvature, r_1, r_2, of the lens surfaces. The quantity $\frac{n_2}{n_1}$ may be termed the "relative refractive index" of the lens material; if the lens is made of glass of $n_2 = 1{\cdot}5$, and it is placed in water of $n_1 = 1{\cdot}33$, then the relative refractive index $= \frac{1{\cdot}5}{1{\cdot}33} = 1{\cdot}13$.

In practice, however, lenses are usually situated in air; in which case $n_1 = 1$. If the glass has a refractive index, n_2, equal to n, the relative refractive index, $\frac{n_2}{n_1} = \frac{n}{1} = n$. Substituting in (22), then

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad . \quad . \quad . \quad (4)$$

Fig. 21.14 illustrates four different types of glass lenses in air, whose refractive indices, n, are each 1·5. Fig. 21.14 (i) is a biconvex lens, whose radii of curvature, r_1, r_2, are each 10 cm. Since a spherical surface convex to a less dense medium has a positive sign (see p. 473), $r_1 = +10$ and $r_2 = +10$. Substituting in (4).

$$\therefore \quad \frac{1}{f} = (1 \cdot 5 - 1)\left(\frac{1}{(+10)} + \frac{1}{(+10)}\right) = 0 \cdot 5 \times \frac{2}{10} = 0 \cdot 1$$

$$\therefore \quad f = +10 \text{ cm.}$$

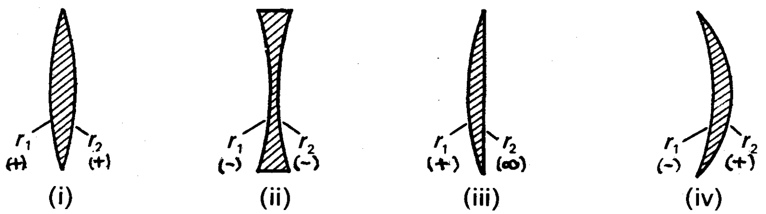

FIG. 21.14. Signs of radius of lens surface.

Fig. 21.14 (ii) is a biconcave lens in air. Since its surfaces are both concave to the less dense medium, $r_1 = -10$ and $r_2 = -10$, assuming the radii are both 10 cm. Substituting in (24),

$$\therefore \quad \frac{1}{f} = (1 \cdot 5 - 1)\left(\frac{1}{(-10)} + \frac{1}{(-10)}\right) = 0 \cdot 5 \times -\frac{2}{10} = -0 \cdot 1$$

$$\therefore \quad f = -10 \text{ cm.}$$

In the case of a plano-convex lens, suppose the radius is 8 cm. Then $r_1 = +8$ and $r_2 = \infty$, Fig. 21.14 (iii). Hence $\frac{1}{f} = (1 \cdot 5 - 1)\left(\frac{1}{(+8)} + \frac{1}{\infty}\right)$ $= 0 \cdot 5 \times \frac{1}{8} = \frac{1}{16}$. Thus $f = +16$ cm.

In Fig. 21.14 (iv), suppose the radii r_1, r_2 are numerically 16 cm, 12 cm respectively. Then $r_1 = -16$, but $r_2 = +12$. Hence

$$\frac{1}{f} = (1 \cdot 5 - 1)\left(\frac{1}{(-16)} + \frac{1}{(+12)}\right) = 0 \cdot 5\left(-\frac{1}{16} + \frac{1}{12}\right) = +\frac{1}{96}.$$

Thus $f = +96$ cm, confirming that the lens is a converging one.

Some Applications of the Lens Equation

The following examples should assist the reader in understanding how to apply correctly the lens equation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$:

1. An object is placed 12 cm from a converging lens of focal length 18 cm. Find the position of the image.

Since the lens is converging, $f = +18$ cm. The object is real, and therefore $= +12$ cm. Substituting in $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$\therefore \quad \frac{1}{v}+\frac{1}{(+12)}=\frac{1}{(+18)}$$

$$\therefore \quad \frac{1}{v}=\frac{1}{18}-\frac{1}{12}=-\frac{1}{36}$$

$$\therefore \quad v=-36$$

Since v is negative in sign the image is *virtual*, and it is 36 cm from the lens. See Fig. 21.17 (ii).

2. A beam of light, converging to a point 10 cm behind a converging lens, is incident on the lens. Find the position of the point image if the lens has a focal length of 40 cm.

If the incident beam converges to the point O, then O is a *virtual object*, Fig. 21.15. See p. 412. Thus $u = -10$ cm. Also, $f = +40$ cm since the lens is converging. Substituting in $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$\frac{1}{v}+\frac{1}{(-10)}=\frac{1}{(+40)}$$

$$\therefore \quad \frac{1}{v}=\frac{1}{40}+\frac{1}{10}=\frac{5}{40}$$

$$\therefore \quad v=\frac{40}{5}=8$$

Since v is positive in sign the image is *real*, and it is 8 cm from the lens. The image is I in Fig. 21.15.

3. An object is placed 6 cm in front of a diverging lens of focal length 12 cm. Find the image position.

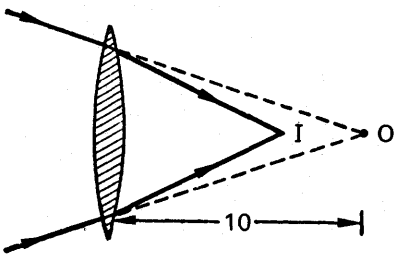

FIG. 21.15. Virtual object.

Since the lens is concave, $f = -12$ cm. The object is real, and hence $u = +6$cm. Substituting in $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$\therefore \quad \frac{1}{v}+\frac{1}{(+6)}=\frac{1}{(-12)}$$

$$\therefore \quad \frac{1}{v} = -\frac{1}{12} - \frac{1}{6} = -\frac{3}{12}$$

$$\therefore \quad v = -\frac{12}{3} = -4$$

Since v is negative in sign the image is virtual, and it is 4 cm from the lens. See Fig. 21.18 (i).

4. A converging beam of light is incident on a diverging lens of focal length 15 cm. If the beam converges to a point 3 cm behind the lens, find the position of the point image.

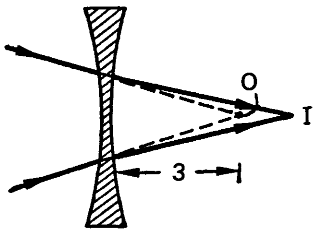

FIG. 21.16. Virtual object.

If the beam converges to the point O, then O is a virtual object, as in example 3, Fig. 21.15. Thus $u = -3$ cm. Since the lens is diverging, $f = -15$ cm. Substituting in $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$,

$$\therefore \quad \frac{1}{v} + \frac{1}{(-3)} = \frac{1}{(-15)}$$

$$\therefore \quad \frac{1}{v} = -\frac{1}{15} + \frac{1}{3} = \frac{4}{15}$$

$$\therefore \quad v = \frac{15}{4} = 3\tfrac{3}{4}$$

Since v is positive in sign the point image, I, is *real*, and it is $3\frac{3}{4}$ cm from the lens, Fig. 21.16.

Images in Lenses

Converging lens. (i) When an object is a very long way from this lens, i.e., at infinity, the rays arriving at the lens from the object are parallel. Thus the image is formed at the focus of the lens, and is real and inverted.

(ii) Suppose an object OP is placed at O perpendicular to the principal axis of a thin converging lens, so that it is farther from the lens than its principal focus, Fig. 21.17 (i). A ray PC incident on the middle, C, of the lens is very slightly displaced by its refraction through the lens, as the opposite surfaces near C are parallel (see Fig. 21.11, which is an exaggerated sketch of the passage of the ray). We therefore consider that PC passes *straight through* the lens, and this is true for any ray incident on the middle of a thin lens.

A ray PL parallel to the principal axis is refracted so that it passes through the focus F. Thus the image, Q, of the top point P of the object

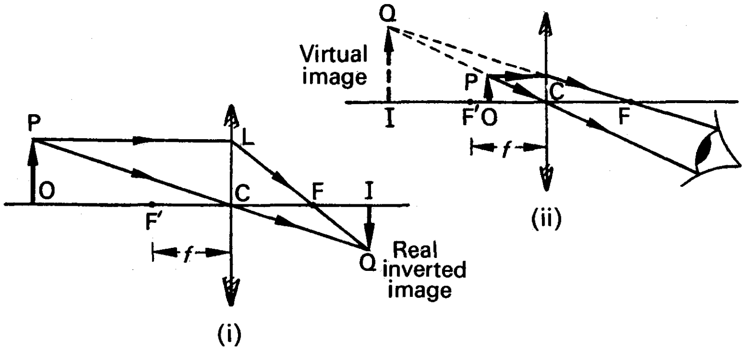

FIG. 21.17. Images in converging lenses.

is formed below the principal axis, and hence the whole image IQ is real and inverted. In making accurate drawings the lens should be represented by a straight line, as illustrated in Fig. 21.17, as we are only concerned with thin lenses and a narrow beam incident close to the principal axis.

(iii) The image formed by a converging lens is always real and inverted until the object is placed nearer the lens than its focal length, Fig. 21.17 (ii). In this case the rays from the top point P *diverge* after refraction through the lens, and hence the image Q is *virtual.* The whole image, IQ, is erect (the same way up as the object) and magnified, besides being virtual, and hence the converging lens can be used as a simple "magnifying glass" (see p. 527).

Diverging lens. In the case of a converging lens, the image is sometimes real and sometimes virtual. In a diverging lens, the image is

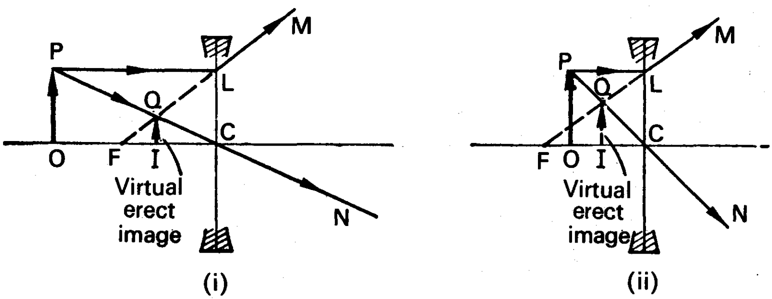

FIG. 21.18. Images in diverging lenses.

always virtual; in addition, the image is always erect and diminished. Fig. 21.18 (i), (ii) illustrate the formation of two images. A ray PL appears to diverge from the focus F after refraction through the lens, a ray PC passes straight through the middle of the lens and emerges along CN, and hence the emergent beam from P appears to diverge from Q on the same side of the lens as the object. The image IQ is thus *virtual.*

The rays entering the eye from a point on an object viewed through a lens can easily be traced. Suppose L is a converging lens, and IQ is the image of the object OP, drawn as already explained, Fig. 21.19. If the eye E observes the top point P of the object through the lens, the cone of rays entering E are those bounded by the image Q of P and the pupil of the eye. If these rays are produced back to meet the lens L, and the

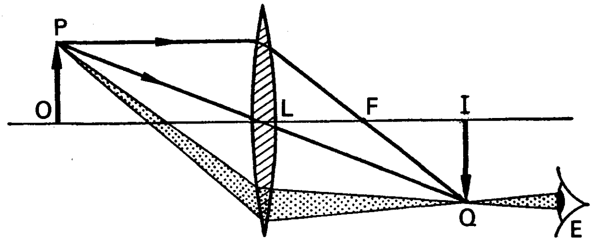

FIG. 21.19. Rays entering the eyes.

points of incidence are joined to P, the rays entering E are shown shaded in the beam. The method can be applied to trace the beam of light entering the eye from any other point on the object; the important thing to remember is to *work back from the eye.*

Another proof of $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$. We have already shown how the lens equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ can be derived by considering refraction in turn at the two curved surfaces (p. 480). A proof of the equation can also be obtained from Fig. 21.17 or Fig. 21.18, but it is not as rigid a proof as that already given on page 481.

In Fig. 21.18, triangles CQI, CPO are similar. Hence IQ/PO = CI/CO. Since triangles FQI, FLC are similar, IQ/CL = FI/FC. Now CL = PO. Thus the left sides of the two ratios are equal.

$$\therefore \quad \frac{CI}{CO}=\frac{FI}{FC}$$

But CI = $-v$; CO = $+u$; FI = FC − IC = $-f-(-v)=v-f$; and FC = $-f$.

$$\therefore \quad \frac{-v}{+u}=\frac{v-f}{-f}$$

$$\therefore \quad vf=uv-uf$$

$$\therefore \quad uf+vf=uv$$

Dividing throughout by uvf and simplifying each term,

$$\therefore \quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

The same result can be derived by considering similar triangles in Fig. 21.17, a useful exercise for the student.

Lateral Magnification

The lateral or transverse or linear magnification, m, produced by a lens is defined by

$$m=\frac{\textit{height of image}}{\textit{height of object}} \quad . \quad . \quad . \quad . \quad (5)$$

Thus $m = \frac{IQ}{OP}$ in Fig. 21.17 or Fig. 21.18. Since triangles QIC, POC are similar in either of the diagrams,

$$\frac{IQ}{OP} = \frac{CI}{CO} = \frac{v}{u},$$

where v, u are the respective image and object distances from the lens.

$$\therefore \quad m = \frac{v}{u} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Equation (6) provides a simple formula for the magnitude of the magnification; there is no need to consider the signs of v and u in this case.

Other formulæ for magnification. Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we have, by multiplying throughout by v,

$$1 + \frac{v}{u} = \frac{v}{f}$$

$$\therefore \quad 1 + m = \frac{v}{f}$$

$$\therefore \quad m = \frac{v}{f} - 1 \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Thus if a real image is formed 25 cm from a converging lens of focal length 10 cm, the magnification, m, $= \frac{+25}{+10} - 1 = 1{\cdot}5$.

By multiplying both sides of the lens equation by u, we have

$$\frac{u}{v} + 1 = \frac{u}{f}$$

$$\therefore \quad \frac{1}{m} + 1 = \frac{u}{f}$$

Object at Distance $2f$ from Converging Lens

When an object is placed at a distance of $2f$ from a convex lens, drawing shows that the real image obtained is the same size as the image

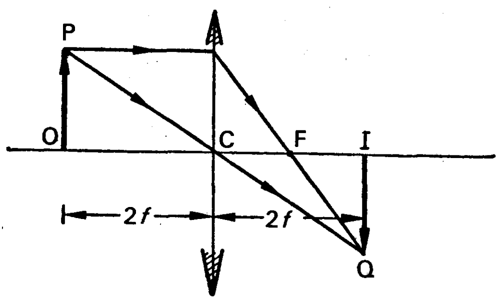

FIG. 21.20. Object and image of same size.

and is also formed at a distance $2f$ from the lens, Fig. 21.20. This result can be accurately checked by using the lens equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.

Substituting $u = +2f$, and noting that the focal length, f, of a converging lens is positive, we have

$$\frac{1}{v}+\frac{1}{2f}=\frac{1}{f}$$

$$\therefore \quad \frac{1}{v}=\frac{1}{f}-\frac{1}{2f}=\frac{1}{2f}$$

$$\therefore \quad v = 2f = \text{image distance.}$$

$\therefore$ lateral magnification, $m = \frac{v}{u}=\frac{2f}{2f}=1,$

showing that the image is the same size as the object.

Least Possible Distance Between Object and Real Image with Converging Lens

It is not always possible to obtain a real image on a screen, although the object and the screen may both be at a greater distance from a converging lens than its focal length. The theory below shows that the distance between an object and a screen must be equal to, or greater than, *four times the focal length* if a real image is required.

Theory. Suppose I is the real image of a point object O in a converging lens. If the image distance $= x$, and the distance OI $= d$, the object distance $=$

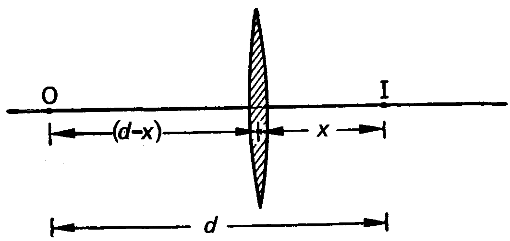

FIG. 21.21. Minimum distance between object and image.

$(d-x)$, Fig. 21.21. Thus $v = +x$, and $u = +(d-x)$. Substituting in the lens equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, in which f is positive, we have

$$\frac{1}{x}+\frac{1}{d-x}=\frac{1}{f}$$

$$\therefore \quad \frac{d}{x(d-x)}=\frac{1}{f}$$

$$\therefore \quad x^2-dx+df=0 \qquad . \quad . \quad . \quad . \quad (i)$$

For a real image, the roots of this quadratic equation for x must be real roots. Applying to (i) the condition $b^2 - 4\,ac >$ for 0 the general quadratic $ax^2 + bx + c = 0$, then

$$d^2 - 4df > 0$$
$$\therefore\ d^2 > 4d,$$
$$\therefore\ d > 4f$$

Thus the distance OI between the object and screen must be greater than $4f$, otherwise no image can be formed on the screen. Hence $4f$ is the minimum distance between object and screen; the latter case is illustrated by Fig. 21.20, in which $u = 2f$ and $v = 2f$. If it is difficult to obtain a real image on a screen when a converging lens is used, possible causes may be (i) the object is nearer to the lens than its focal length, Fig. 21.17 (ii), or (ii) the distance between the screen and object is less than four times the focal length of the lens.

Conjugate Points. Newton's Relation

Suppose that an object at a point O in front of a lens has its image

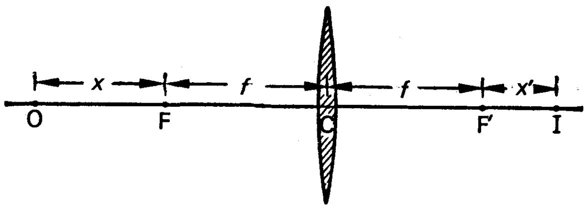

FIG. 21.22. Newton's relation.

formed at a point I. Since light rays are reversible, it follows an object placed at I will give rise to an image at O. The points O, I are thus "interchangeable", and are hence called *conjugate points* (or conjugate foci) with respect to the lens. Newton showed that conjugate points obey the relation $xx' = f^2$, where x, x' are their respective distances from the focus on the same side of the lens.

The proof of this relation can be seen by taking the case of the converging lens in Fig. 21.22, in which $OC = u = x + f$, and $CI = v = x' + f$.

Substituting in the lens equation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$,

$$\frac{1}{x' + f} + \frac{1}{x + f} = \frac{1}{f}$$
$$\therefore\ f(x' + x + 2f) = (x' + f)(x + f)$$
$$\therefore\quad xx' = f^2 \qquad (8)$$

Since $x' = f^2/x$, it follows that x' increases as x decreases. The image I thus recedes from the focus F′ away from the lens when the object O approaches the lens.

The property of conjugate points stated above, namely that an object and an image at these points are interchangeable, can also be derived from the lens equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$. Thus if u = 15 cm and v = 10 cm satisfies this equation, so must u = 10 cm and v = 15 cm.

Displacement of Lens when Object and Screen are Fixed

Suppose that an object O, in front of a converging lens A, gives rise to an image on a screen at I, Fig. 21.23. Since the image distance AI (v) is

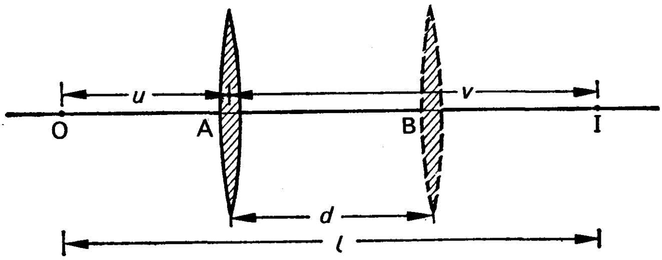

FIG. 21.23. Displacement of lens.

greater than the object distance AO (u), the image is larger than the object. If the object and the screen are kept fixed at O, I respectively, another clear image can be obtained on the screen by moving the lens from A to a position B. This time the image is smaller than the object, as the new image distance BI is less than the new object distance OB.

Since O and I are conjugate points with respect to the lens, it follows that OB = IA and IB = OA. (If this is the case the lens equation will be satisfied by $\frac{1}{IB}+\frac{1}{OB}=\frac{1}{f}$ and by $\frac{1}{IA}+\frac{1}{OA}=\frac{1}{f}$.) If the *displacement*, AB, of the lens = d, and the constant distance OI = l, then OA + BI = $l - d$. But, from above, OA = IB. Hence OA = $(l - d)/2$. Further, AI = AB + BI = OA + AB = $(l - d)/2 + d = (l + d)/2$.

But u = OA, and v = AI for the lens in the position A. Substituting for OA and AI in $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$\frac{1}{(l+d)/2}+\frac{1}{(l-d)/2}=\frac{1}{f}$$

$$\therefore \quad \frac{2}{l+d}+\frac{2}{l-d}=\frac{1}{f}$$

$$\therefore \quad \frac{4l}{l^2-d^2}=\frac{1}{f}$$

$$\therefore \quad f=\frac{l^2-d^2}{4l} \quad . \quad . \quad . \quad . \quad (9)$$

Thus if the displacement d of the lens, and the distance l between the object and the screen, are measured, the focal length f of the lens can be found from equation (9). This provides a very useful method of measuring the focal length of a lens whose surfaces are inaccessible (for example, when the lens is in a tube), when measurements of v and u cannot be made (see p. 445).

Magnification. When the lens is in the position A, the lateral magnification m_1 of the object $= \frac{v}{u} = \frac{AI}{OA}$, Fig. 21.23.

$$\therefore \quad \frac{h_1}{h} = \frac{AI}{AO} \quad . \quad . \quad . \quad . \quad . \quad (i)$$

where h_1 is the length of the image and h is the length of the object.

When the lens is in the position B, the image is smaller than the object. The lateral magnification, $m_2 = \frac{BI}{OB}$.

$$\therefore \quad \frac{h_2}{h} = \frac{BI}{OB} \quad . \quad . \quad . \quad . \quad (ii)$$

where h_2 is the length of the image. But, from our previous discussion, AI = OB and OA = BI. From (i) and (ii) it follows that, by inverting (i),

$$\frac{h}{h_1} = \frac{h_2}{h}$$

$$\therefore \quad h^2 = h_1 h_2$$

$$\therefore \quad h = \sqrt{h_1 h_2} \quad . \quad . \quad . \quad (10)$$

The length, h, of an object can hence be found by measuring the lengths h_1, h_2 of the images for the two positions of the lens. This method of measuring h is most useful when the object is inaccessible, for example, when the width of a slit in a tube is required.

EXAMPLES

1. A converging lens of focal length 30 cm is 20 cm away from a diverging lens of focal length 5 cm. An object is placed 6 metres distant from the former lens (which is the nearer to it) and on the common axis of the system. Determine the position, magnification, and nature of the image formed. (*O. & C.*)

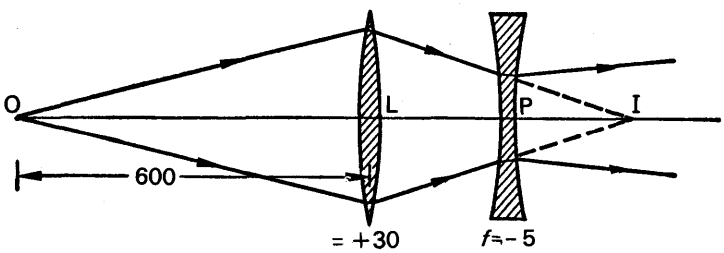

FIG. 21.24.

Suppose O is the object, Fig. 21.24.

For the converging lens,

$$u = +600 \text{ cm}, f = +30 \text{ cm}.$$

Substituting in the lens equation,

$$\therefore \quad \frac{1}{v} + \frac{1}{(+600)} = \frac{1}{(+30)}$$

from which $$v = \frac{600}{19} = 31\tfrac{11}{19} \text{ cm} = \text{LI}$$

$$\therefore \quad \text{PI} = \text{LI} - \text{LP} = 31\tfrac{11}{19} - 20 = 11\tfrac{11}{19} \text{ cm}.$$

For the diverging lens, I is a virtual object. Thus $u = \text{PI} = -11\frac{11}{19}$. Also $f = -5$. Substituting in the lens equation, we have

$$\frac{1}{v} + \frac{1}{(-11\frac{11}{19})} = \frac{1}{(-5)}$$

from which $$v = -8{\cdot}8 \text{ cm}.$$

The image is thus virtual, and hence the rays diverge after refraction through P, as shown. The image is 8·8 cm to the left of P.

The magnification, m, is given by $m = m_1 \times m_2$, where m_1, m_2 are the magnifications produced by the converging and diverging lens respectively.

But $$m_1 = \frac{v}{u} = 31\tfrac{11}{19}/600$$

and $$m_2 = 8{\cdot}8/11\tfrac{11}{19}$$

$$\therefore \quad m = 31\tfrac{11}{19}/600 \times 8\tfrac{4}{5}/11\tfrac{11}{19} = \frac{1}{25}$$

2. Establish a formula connecting object-distance and image-distance for a simple lens. A small object is placed at a distance of 30 cm from a converging lens of focal length 10 cm. Determine at what distances from this lens a second converging lens of focal length 40 cm must be placed in order to produce (i) an erect image, (ii) an inverted image, in each case of the same size as the object. (*L.*)

First part. See text.

Second part. Suppose O is the object, Fig. 21.25. The image I in the convex lens L is formed at a distance v from L given by

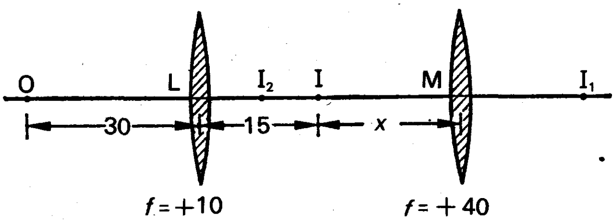

FIG. 21.25.

$$\frac{1}{v} + \frac{1}{(+30)} = \frac{1}{(+10)}$$

from which $$v = +15.$$

For an erect image. Since $\frac{v}{u} = \frac{15}{30} = \frac{1}{2}$, the image at I is half the object size; also, the image is inverted, since it is real (see Fig. 21.17 (i)). If an erect image is required, the second lens, M, must invert the image at I. Further, if the new image, I_1, say, is to be the same size as the object at O, the magnification produced by M of the image at I must be 2. Suppose IM $= x$ numerically; then, since the magnification $\left(\frac{v}{u}\right) = 2$, $MI_1 = 2x$. As I and I_1 are both real, we have, from

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

$$\frac{1}{(+2x)} + \frac{1}{(+x)} = \frac{1}{(+40)}$$

$$\therefore \quad \frac{3}{2x} = \frac{1}{40}$$

from which $x = 60$ cm.

Thus M must be placed 75 cm from L for an erect image of the same size as O.

For an inverted image. Since the image at I is inverted, the image I_2 of I in M must be erect with respect to I. The lens M must thus act like a magnifying glass which produces a magnification of 2, and the image I_2 is *virtual* in this case. Suppose IM $= x$ numerically; then $I_2M = 2x$ numerically. Substituting in

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore \quad \frac{1}{(-2x)} + \frac{1}{(+x)} = \frac{1}{(+40)}$$

$$\therefore \quad x = 20 \text{ cm.}$$

Thus M must be placed 35 cm from L for an inverted image of the same size as O.

SOME METHODS OF MEASURING FOCAL LENGTHS OF LENSES, AND THEIR RADII OF CURVATURE

Converging Lens

(1) *Plane mirror method.* In this method a plane mirror M is placed on a table, and the lens L is placed on the mirror, Fig. 21.26. A pin O is then moved along the axis of the lens until its image I is observed to coincide with O when they are both viewed from above, the method of no parallax being used. The distance from the pin O to the lens is then the focal length, f, of the lens, which can thus be measured.

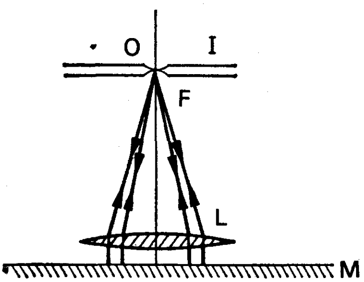

FIG. 21.26. Plane mirror method.

The explanation of the method is as follows. In general, rays from O pass through the lens, are reflected from the mirror M, and then pass through the lens again to form an image at some place. When O and the image coincide in position, the rays from O incident on M must have returned along their incident path after reflection from the mirror.

This is only possible if the rays are incident *normally* on M. Consequently the rays entering the lens after reflection are all parallel, and hence the point to which they converge must be the focus, F, Fig. 21.26. It will thus be noted that the mirror provides a simple method of obtaining parallel rays incident on the lens.

(2) *Lens formula method.* In this method five or six values of u and v are obtained by using an illuminated object and a screen, or by using two pins and the method of no parallax. The focal length, f, can then be calculated from the equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, and the average of the values obtained. Alternatively, the values of $\frac{1}{u}$ can be plotted against $\frac{1}{v}$, and a straight line drawn through the points. When $\frac{1}{u}=0$, $\frac{1}{v}=\text{OA}=\frac{1}{f}$, from the lens equation; thus $f=\frac{1}{\text{OA}}$, and hence can be calculated, Fig. 21.27. Since $\frac{1}{u}=\frac{1}{f}$ when $\frac{1}{v}=0$, from the lens equation, $\text{OB}=\frac{1}{f}$. Thus f also be evaluated from $\frac{1}{\text{OB}}$.

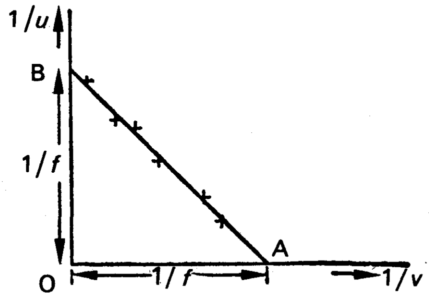

FIG. 21.27. Graph of $1/u$ against $1/v$.

(3) *Displacement method.* In this method, an illuminated object O is placed in front of the lens, A, and an image I is obtained on a screen. Keeping the object and screen fixed, the lens is then moved to a position B so that a clear image is again obtained on the screen, Fig. 21.28. From our discussion on p. 491, it follows that a magnified sharp image is obtained at I when the lens is in the position A, and a diminished sharp image when the lens is in the position B. If the displacement of the lens is d, and the distance between the object and the screen is l, the focal length, f, is given by $f=\frac{l^2-d^2}{4l}$, from p. 491. Thus f can be calculated. The experiment can be repeated by altering the distance between the object and the screen, and the average value of f is then calculated. It should be noted that the screen must be at a distance from the object of at least four times the focal length of the lens, otherwise an image is unobtainable on the screen (p. 490).

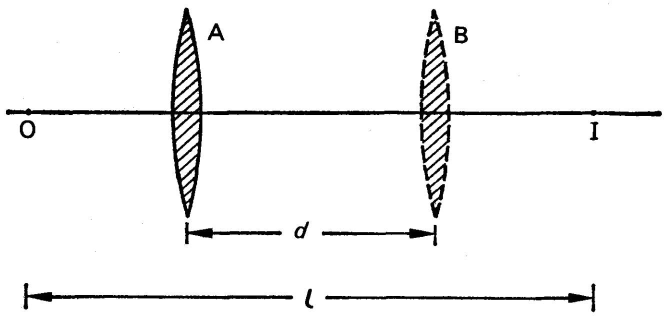

FIG. 21.28. Displacement method for focal length.

Since no measurements need be made to the surfaces of the lens (the "displacement" is simply the distance moved by the holder of the lens), this method can be used for finding the focal length of (i) a thick lens, (ii) an inaccessible lens, such as that fixed inside an eye-piece or telescope tube. Neither of the two methods previously discussed could be used for such a lens.

Lateral Magnification Method of Measuring Focal Length

On p. 488, we showed that the lateral magnification, m, produced by a lens is given by

$$m = \frac{v}{f} - 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

where f is the focal length of the lens and v the distance of the image.

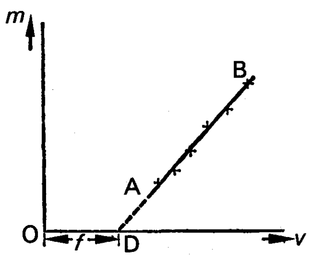

FIG. 21.29. Graph of m against v.

If an illuminated glass scale is set up as an object in front of a lens, and the image is received on a screen, the magnification, m, can be measured directly. From (i) a straight line graph BA is obtained when m is plotted against the corresponding image distance v, Fig. 21.29. Further, from (i), $v/f - 1 = 0$ when $m = 0$; thus $v = f$ in this case. Hence, by producing BA to cut the axis of v in D, it follows that OD $= f$; the focal length of the lens can thus be found from the graph.

Diverging Lens

(1) *Converging lens method.* By itself, a diverging lens always forms a virtual image of a real object. A real image may be obtained, however, if a *virtual object* is used, and a converging lens can be used to provide such an object, as shown in Fig. 21.30. An object S is placed at a distance from M greater than its focal length, so that a beam converging to a point O is obtained. O is thus a virtual object for the diverging lens L placed as shown in Fig. 21.30, and a real image I can now be obtained.

I is farther away from L and O, since the concave lens makes the incident beam on it diverge more.

The image distance, v, from the diverging lens is CI and can be measured; v is + ve in sign as I is real. The object distance, u, from this

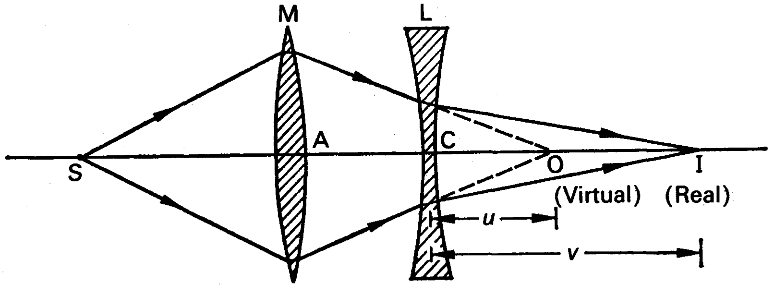

FIG. 21.30. Focal length of diverging lens.

lens = CO = AO − AC, and AC can be measured. The length AO is obtained by removing the lens L, leaving the converging lens, and noting the position of the real image now formed at O by the lens M. Thus u (= CO) can be found; it is a − ve distance, since O is a virtual object for the diverging lens. Substituting for u and v in $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, the focal length of the diverging lens can be calculated.

(2) *Concave mirror method.* In this method a real object is placed in front of a diverging lens, and the position of the virtual image is located with the aid of a concave mirror. An object O is placed in front of the lens L, and a concave mirror M is placed behind the lens so that a divergent beam is incident on it, Fig. 21.31. With L and M in the same position, the object O is moved until an image is obtained coincident with it in position, i.e., beside O. The distances CO, CM are then measured.

As the object and image are coincident at O, the rays must be incident *normally* on the mirror M. The rays BA, ED thus pass through the centre of curvature of M, and this is also the position of the virtual image I. The image distance, v, from the lens = IC = IM − CM =

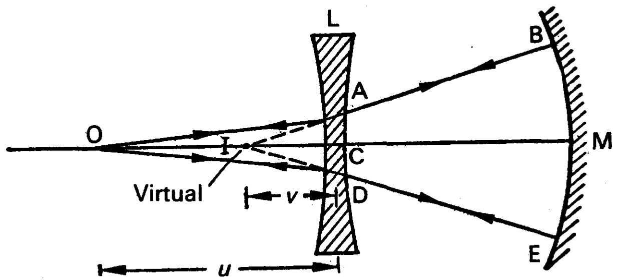

FIG. 21.31. Focal length of diverging lens.

r − CM, where r is the radius of curvature; CM can be measured, while r can be determined by means of a separate experiment, as described on p. 413. The object distance, u, from the lens = OC, and by substituting for u and v in the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, the focal length f can be calculated. Of course, v is negative as I is a virtual image for the lens.

Measurement of Radii of Curvature of Lens Surfaces

Diverging lens. The radius of curvature of a lens concave surface A can easily be measured by moving an object O in front of it until the image by reflection at A coincides with the object. Since the rays from

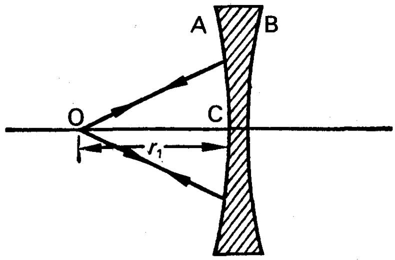

FIG. 21.32. Radius of diverging lens surface.

O are now incident normally on A, its radius of curvature, r_1, = OC, the distance from O to the lens, Fig. 21.32. If the radius of curvature of the surface B is required, the lens is turned round and the experiment is repeated.

Converging lens: Boys' method. Since a convex surface usually gives a virtual image, it is not an easy matter to measure the radius of curvature of such a lens surface. C. V. BOYS, however, suggested an ingenious method which is now known by his name, and is illustrated in Fig. 21.33. In Boys' method, an object O is placed in front of a converging lens, and is then moved until an image *by reflection at the back surface* NA is formed beside O. To make the image brighter O should be a well-illuminated object, and the lens can be floated with NA on top of mercury to provide better reflection from this surface.

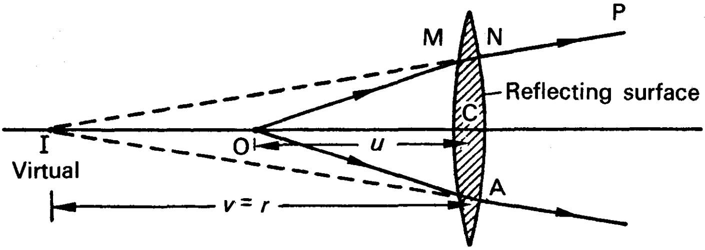

FIG. 21.33. Boys' method for radius of converging lens surface.

Since the image is coincident with O, the rays are incident *normally* on NA. A ray OM from O would thus pass *straight through* the lens along NP after refraction at M. Further, as PN produced passes through I, I is a virtual image of O by *refraction in the lens*. On account of the latter fact, we can apply the lens equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, where v = IC = r, the radius of curvature of NA, and u = OC. Thus knowing OC and the focal length, f, of the lens, r can be calculated. The same method can be used to measure the radius of curvature of the surface M of the lens, in which case the lens is turned round.

Although reflection from the lens back surface is utilised, the reader should take special pains to note that Boys' method uses the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ to calculate the radius of curvature. This is a formula for the *refraction* of light through the lens.

The refractive index of the material of a lens can be found by measuring the radii of curvature, r_1, r_2, of its surfaces and its focal length f. Since $\frac{1}{f}=(n-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$, where n is the refractive index, the latter can then be calculated.

Combined Focal Length of Two Thin Lenses in Contact

In order to diminish the colouring of the image due to dispersion when an object is viewed through a single lens, the lenses of telescopes and microscopes are made by placing two thin lenses together (see p. 515). The combined focal length, F, of the lenses can be found by considering a point object O placed on the principal axis of two *thin lenses in contact*, which have focal lengths f_1, f_2 respectively, Fig. 21.34. A ray OC from O passes through the middle, C, of both lenses undeviated.

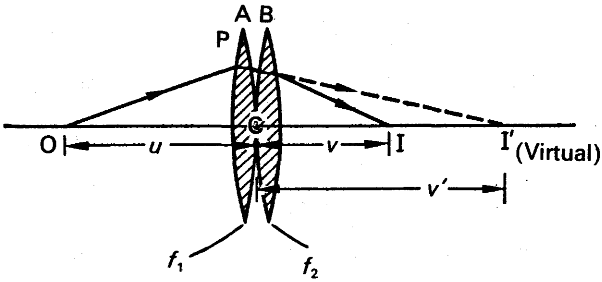

FIG. 21.34. Focal length of combined lenses.

A ray OP from O is refracted through the first lens A to intersect OC at I′, which is therefore the image of O in A. If OC = u, CI′ = v',

$$\therefore\ \frac{1}{v'}+\frac{1}{u}=\frac{1}{f_1} \qquad \text{(i)}$$

The beam of light incident on the second lens B converges to I′, which is therefore a *virtual* object for this lens. The image is formed at I at a distance CI, or v, from the lens. Thus since the object distance

CI′ is virtual, $u = -v'$ for refraction in this case. For lens B, therefore,

we have $$\frac{1}{v} + \frac{1}{(-v')} = \frac{1}{f_2},$$

or $$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2} \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

Adding (i) and (ii) to eliminate v',

$$\therefore \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Since I is the image of O by refraction through both lenses,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{F},$$

where F is the *focal length of the combined lenses*. Hence

$$\mathbf{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This formula for F applies to any two thin lenses in contact, such as two diverging lenses, or a converging and diverging lens. When the formula is used, the signs of the focal lengths must be inserted. As an illustration, suppose that a thin converging lens of 8 cm focal length is placed in contact with a diverging lens of 12 cm focal length. Then $f_1 = +8$, and $f_2 = -12$. The combined focal length, F, is thus given by

$$\frac{1}{F} = \frac{1}{(+8)} + \frac{1}{(-12)} = \frac{1}{8} - \frac{1}{12} = +\frac{1}{24}$$

$$\therefore \quad F = +24 \text{ cm}.$$

The positive sign shows that the combination acts like a converging lens.

The focal length of a diverging lens can be found by combining it with a converging lens of shorter focal length. The combination acts like a converging lens, as shown by the numerical example just considered, and its focal length F can be found by one of the methods described on p. 494. The diverging lens is then taken away, and the focal length f_1 of the convex lens alone is now measured. The focal length f_2 of the concave lens can then be calculated by substituting the values of F and f_1 in the formula $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$.

Refractive Index of a Small Quantity of Liquid

Besides the method given on p. 425, the refractive index of a small amount of liquid can be found by smearing it over a plane mirror and placing a converging lens on top, as shown in the exaggerated sketch of Fig. 21.35. An object O is then moved along the principal axis until the inverted image I seen looking down into the mirror is coincident with O in position. In this case the rays which pass through the lens and liquid are incident *normally* on the mirror, and the distance from O to the lens is now the focal length, F, of the *lens and liquid* combination (see p. 494). If the experiment is repeated with the convex lens alone on the mirror, the focal length f_1 of the lens can be measured. But $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$, where f_2 is the focal length of the liquid lens.

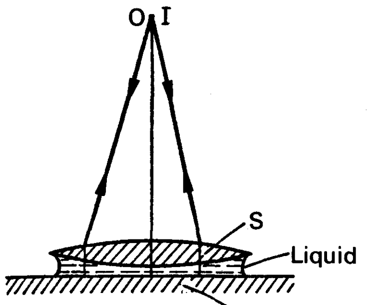

FIG. 21.35. Refractive index liquid.

Thus, knowing f_1 and F, f_2 can be calculated.

From Fig. 21.35, it can be seen that the liquid lens is a plano-concave type; its lower surface corresponds to the plane surface of the mirror, while the upper surface corresponds to the Surface S of the converging lens. If the latter has a radius of curvature r, then, from equation (4) on p. 483.

$$\frac{1}{f_2} = (n - 1)\left(\frac{1}{r} + \frac{1}{\infty}\right).$$

$$\therefore \quad \frac{1}{f_2} = (n - 1)\frac{1}{r}$$

$$\therefore \quad n - 1 = \frac{r}{f_2}$$

$$\therefore \quad n = 1 + \frac{r}{f_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

The radius of curvature r of the surface S of the lens can be measured by Boys' method (p. 498). Since f_2 has already been found, the refractive index n of the liquid can be calculated from (i). This method of measuring n is useful when only a small quantity of the liquid is available.

EXAMPLES

1. Draw a diagram to illustrate the principle of the convex driving mirror on a motor-car. A converging lens of focal length 24 cm is placed 12 cm in front of a convex mirror. It is found that when a pin is placed 36 cm in front of the lens it coincides with its own inverted image formed by the lens and mirror. Find the focal length of the mirror.

First part. See text.

Second part. Suppose O is the position of the pin, Fig. 21.36. Since an inverted image of the pin is formed at O, the rays from O strike the convex mirror normally. Thus the image, I, in the lens of O is at the centre of curvature of the mirror.

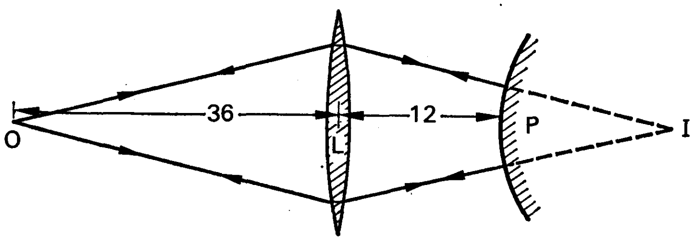

FIG. 21.36. Example.

Since u = OL = + 36, f = + 24, it follows from the lens equation that

$$\frac{1}{v}+\frac{1}{36}=\frac{1}{24}$$

from which $v = 72$ cm = LI

$\therefore$ PI = LI − LP = 72 − 12 = 60

$\therefore$ radius of curvature, r, of mirror = 60 cm.

$\therefore$ focal length of mirror $=\frac{r}{2}=30$ cm.

2. Give an account of a method of measuring the focal length of a diverging lens, preferably without the aid of an auxiliary converging lens. A luminous object and a screen are placed on an optical bench and a converging lens is placed between them to throw a sharp image of the object on the screen; the linear magnification of the image is found to be 2·5. The lens is now moved 30 cm nearer the screen and a sharp image again formed. Calculate the focal length of the lens. (*N.*)

First part. A concave mirror can be used, p. 497.

Second part. If O, I are the object and screen positions respectively, and L_1, L_2 are the two positions of the lens, then $OL_1 = IL_2$, Fig. 112. See *Displacement method for focal length*, p. 495. Suppose $OL_1 = x = L_2I$.

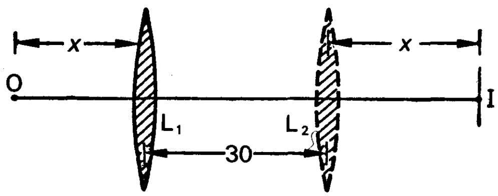

FIG. 21.37. Example.

For the lens in the position L_1, $u = OL_1 = x$, $v = L_1I = 30 + x$.

But magnification, $m=\frac{v}{u}=2{\cdot}5$

$$\frac{30+x}{x}=2{\cdot}5$$

$\therefore$ $x = 20$ cm.

$\therefore$ $u = OL_1 = 20$ cm.

$v = L_1I = 30 + x = 50$ cm.

Substituting in $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$\therefore \quad \frac{1}{20}+\frac{1}{50}=\frac{1}{f}$$

from which $f = 14\frac{2}{7}$ cm.

3. Describe two methods for the determination of the focal length of a diverging lens. A thin equiconvex lens is placed on a horizontal plane mirror, and a pin held 20 cm vertically above the lens coincides in position with its own image. The space between the under surface of the lens and the mirror is filled with water (refractive index 1·33) and then, to coincide with its image as before, the pin has to be raised until its distance from the lens is 27·5 cm. Find the radius of curvature of the surfaces of the lens. (*N.*)

First part. See text.

Second part. The focal length, f_1, of the lens = 20 cm, and the focal length, F, of the water and glass lens combination = 27·5 cm. See p. 501 and Fig. 21.35. The focal length, f, of the water lens is given by

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f_1}$$

$$\therefore \quad \frac{1}{(+27{\cdot}5)}=\frac{1}{f}+\frac{1}{(+20)}$$

Solving,

$$\therefore \quad \frac{1}{f}=\frac{1}{27{\cdot}5}-\frac{1}{20}=-\frac{3}{220}$$

the minus showing that the water lens is a diverging lens.

But

$$\frac{1}{f}=(n-1)\frac{1}{r},$$

where $n = 1{\cdot}33$, and r = radius of the curved face of the lens.

$$\therefore \quad -\frac{3}{220}=(1{\cdot}33-1)\frac{1}{r}$$

$$\therefore \quad r=-24{\cdot}2 \text{ cm.}$$

The glass lens is equiconvex, and hence the radii of its surfaces are the same.

4. Derive an expression for the equivalent focal length of a system of two thin lenses of focal lengths f_1 and f_2 in contact. Two equiconvex lenses of focal length 20 cm are placed in contact and the space between them filled with water. Find the focal length of the combination (${}_an_g = 3/2$, ${}_an_w = 4/3$). (*L.*)

First part. See text.

Second part. Since the lenses are equiconvex, the radii of curvature, r, of their surfaces are equal. Now

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$\therefore \quad \frac{1}{20}=\left(\frac{3}{2}-1\right)\left(\frac{1}{r}+\frac{1}{r}\right)$$

$$\therefore \quad r = 20 \text{ cm.}$$

The water between the lenses forms an equiconcave lens of refractive index 4/3 and radii 20 cm. Its focal length f_1 is thus given by

$$\frac{1}{f_1}=(n-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$=\left(\frac{4}{3}-1\right)\left(\frac{1}{-20}+\frac{1}{-20}\right)$$

$$\therefore \quad f_1=-30 \text{ cm.}$$

The focal length, F, of the combination is given by

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f_1}+\frac{1}{f}=\frac{2}{f}+\frac{1}{f_1}$$

where f is the focal length of a glass lens.

$$\therefore \quad \frac{1}{F}=\frac{2}{(+20)}+\frac{1}{(-30)}=\frac{2}{30}$$

$$\therefore \quad F=\frac{30}{2}=15 \text{ cm.}$$

EXERCISES 21

Refraction at a Single Curved Surface

1. A solid glass sphere has a radius of 10 cm and a refractive index of 1·5. Find the position from the centre, and nature, of the image of an object (i) 20 cm, (ii) 40 cm from the centre due to refraction at the nearest part of the sphere.

2. Obtain a formula connecting the distances of the object and the image from a spherical refracting surface. A transparent sphere of refractive index 4/3 has a radius of 12 cm. Find the positions of the image of a small object inside it 4 cm from the centre, when it is viewed first on one side and then on the other side of the sphere, in the direction of the line joining the centre to the object.

3. Viewed normally through its flat surface, the greatest thickness of a plano-convex lens appears to be 2·435 cm, and through its curved surface 2·910 cm. Actually it is 3·665 cm. Find (*a*) the refractive index of the glass, (*b*) the radius of curvature of the convex surface. Do you consider this is a satisfactory method of finding the radius of curvature? (*N.*)

4. A large glass sphere is placed immediately behind a small hole in an opaque screen, and a small filament lamp is placed at such a distance u in front of the hole that its image falls within the sphere, and at a distance v behind the hole. (*a*) Sketch the course taken by the light rays in the formation of this image. (*b*) Derive a formula connecting the quantities u and v with the refractive index n of the glass. (*c*) If $n = 1\cdot5$ and $u = 4r$, where r is the sphere's radius, find the image position due to refraction at the nearest part of the glass surface only.

Refraction Through Lenses

5. An object is placed (i) 12 cm, (ii) 4 cm from a converging (convex) lens of focal length 6 cm. Calculate the image position and the magnification in each case, and draw sketches illustrating the formation of the image.

6. What do you know about the image obtained with a diverging (concave) lens? The image of a real object in a diverging lens of focal length 10 cm is formed 4 cm from the lens. Find the object distance and the magnification. Draw a sketch to illustrate the formation of the image.

7. The image obtained with a converging lens is erect and three times the length of the object. The focal length of the lens is 20 cm. Calculate the object and image distances.

8. A beam of light converges to a point 9 cm behind (i) a converging lens of focal length 12 cm, (ii) a diverging lens of focal length 15 cm. Find the image position in each case, and draw sketches illustrating them.

9. (i) The surfaces of a biconvex lens are 8 cm and 12 cm radius of curvature. If the refractive index of the glass is 1·5, calculate the focal length. (ii) The curved surface of a plano-concave lens is 10 cm radius of curvature, and the refractive index of the glass is 1·6. Calculate the focal length.

10. Describe an experiment to obtain an accurate value for the focal length of a thin converging lens.

A thin converging lens is fixed inside a tube *AB*. A sharp image of an illuminated object is formed on a screen when the end *A* of the tube is 90·0 cm from the screen and again when it is 140·0 cm from the screen. If the distance between object and screen is 250 cm in each case, how far is the lens from *A*? (*L.*)

11. Why is a sign convention used in geometrical optics?

A thin equiconvex lens of glass of refractive index 1·50 whose surfaces have a radius of curvature of 24·0 cm is placed on a horizontal plane mirror. When the space between the lens and mirror is filled with a liquid, a pin held 40·0 cm vertically above the lens is found to coincide with its own image. Calculate the refractive index of the liquid. (*N.*)

13. State clearly the sign convention you employ in optics. Light from a point *O* in air on the axis of a simple spherical interface between air and glass is refracted so as to form an image at *I*. Derive a formula connecting the distances of *O* and *I* from the pole when *I* is (*a*) real, (*b*) virtual. Show that the two formulae can be reduced to a single formula by use of your sign convention.

Calculate the focal length in air of a thin converging meniscus lens with surfaces having radii of curvature 16·0 cm and 24·0 cm, the refractive index of the glass being 1·60.

Indicate briefly a method for measuring this focal length. (*L.*)

12. What do you understand by *a virtual object* in optics? Describe a direct method, not involving any calculation, of finding (*a*) the focal length of a double concave lens, and (*b*) the radius of curvature of one of its faces. You may use other lenses and mirrors if you wish.

A converging lens of 20 cm focal length is arranged coaxially with a diverging lens of focal length 8·0 cm. A point object lies on the same side as the converging lens and very far away on the axis. What is the smallest possible distance between the lenses if the combination is to form a real image of the object?

If the lenses are placed 6·0 cm apart, what is the position and nature of the final image of this distant object? Draw a diagram showing the passage of a wide beam of light through the system in this case. (*C.*)

14. Explain in detail how, with the aid of a pin and a plane mirror, you would determine the focal length of a thin biconvex lens.

Having found the focal length of this lens, explain how you would find the radius of curvature of one of its faces by Boys' method. Discuss whether or not this method can be used to find the radii of curvature of the faces of a thin converging meniscus lens.

The radii of curvature of the faces of a thin converging meniscus lens of material of refractive index 3/2 are 10 cm and 20 cm. What is the focal length of the lens (*a*) in air, (*b*) when completely immersed in water of refractive index 4/3? (*N*.)

15. A thin planoconvex lens is made of glass of refractive index 1·5. When an object is set up 10 cm from the lens, a virtual image ten times its size is formed. What is (*a*) the focal length of the lens, (*b*) the radius of curvature of the lens surface?

If the lens is floated on mercury with the curved side downwards and a luminous object placed vertically above it, how far must the object be from the lens in order that it may coincide with the image produced by reflection in the curved surface? (*L*.)

16. Deduce an expression for the focal length of a lens in terms of u and v, the object and image distance from the lens.

A lens is set up and produces an image of a luminous point source on a screen 25 cm away. *If the aperture of the lens is small*, where must the screen be placed to receive the image when a parallel slab of glass 6 cm thick is placed at right angles to the axis of the lens and between the lens and the screen, if the refractive index of the glass is 1·6? Deduce any formula you use. (*L*.)

17. Derive an expression for the focal length of a lens in terms of the radii of curvature of its faces and its refractive index.

Find the condition that the distance between the object and image is a minimum, and explain how you would verify your result experimentally. (*C*.)

18. Give an account of a method of finding the focal length of a thin concave lens using an auxiliary convex lens which is not placed in contact with it.

A thin equiconvex lens of refractive index 1·50 is placed on a horizontal plane mirror, and a pin fixed 15·0 cm above the lens is found to coincide in position with its own image. The space between the lens and the mirror is now filled with a liquid and the distance of the pin above the lens when the image and object coincide is increased to 27·0 cm. Find the refractive index of the liquid. (*N*.)

19. A thin converging lens is mounted coaxially inside a short cylindrical tube whose ends are closed by means of thin windows of parallel-sided glass. Explain the *principles* of two methods by which the focal length of the lens could be determined.

When a thin biconvex lens is placed on a horizontal mirror, a pin placed 14·0 cm above the lens on the axis is found to coincide with its own image. When a little water of refractive index 1·33 is inserted between lens and mirror the self-conjugate positions of the pin are respectively 17·2 cm and 26·2 cm above the lens, with first one and then the other face of the lens in contact with the water.

Deduce what information you can about the lens from these data. (*L*.)

20. Describe in detail how you would determine the focal length of a diverging lens with the help of (*a*) a converging lens, (*b*) a concave mirror.

A converging lens of 6 cm focal length is mounted at a distance of 10 cm from a screen placed at right angles to the axis of the lens. A diverging lens of 12 cm focal length is then placed coaxially between the converging lens and the screen so that an image of an object 24 cm from the converging lens is focused on the screen. What is the distance between the two lenses? Before commencing the calculation state the sign convention you will employ. (*N.*)

21. (*a*) Find an expression for the focal length of a combination of two thin lenses in contact. (*b*) A symmetrical convex glass lens, the radii of curvature of which are 3 cm, is situated just below the surface of a tank of water which is 40 cm deep. An illuminated scratch on the bottom of the tank is viewed vertically downwards through the lens and the water. Where is the image, and where should the eye of the observer be placed in order to see it? The refractive indices of glass and water may be taken as 3/2 and 4/3 respectively. (*O. & C*).

22. Find the relation between the focal lengths of two thin lenses in contact and the focal length of the combination.

The curved face of a planoconvex lens ($n = 1 \cdot 5$) is placed in contact with a plane mirror. An object at 20 cm distance coincides with the image produced by the lens and reflection by the mirror. A film of liquid is now placed between the lens and the mirror and the coincident object and image are at 100 cm distance. What is the index of refraction of the liquid? (*L.*)

23. Describe an optical method of finding the radius of curvature of a surface of a thin convex lens.

An object is placed on the axis of a thin planoconvex lens, and is adjusted so that it coincides with its own image formed by light which has been refracted into the lens at its first surface, internally reflected at the second surface, and refracted out again at the first surface. It is found that the distance of the object from the lens is 20·5 cm when the convex surface faces the object, and 7·9 cm when the plane surface faces the object. Calculate (*a*) the focal length of the lens, (*b*) the radius of curvature of the surface, (*c*) the refractive index of the glass. (*C*).

24. Define *focal length, conjugate foci, real image*. Obtain an expression for the transverse magnification produced by a thin converging lens.

Light from an object passes through a thin converging lens, focal length 20 cm, placed 24 cm from the object and then through a thin diverging lens, focal length 50 cm, forming a real image 62·5 cm from the diverging lens. Find (*a*) the position of the image due to the first lens, (*b*) the distance between the lenses, (*c*) the magnification of the final image. (*L.*)

25. Describe how you would determine the focal length of a diverging lens if you were provided with a converging lens (*a*) of shorter focal length, (*b*) of longer focal length.

An illuminated object is placed at right angles to the axis of a converging lens, of focal length 15 cm, and 22·5 cm from it. On the other side of the converging lens, and coaxial with it, is placed a diverging lens of focal length 30 cm. Find the position of the final image (*a*) when the lenses are 15 cm apart and a plane mirror is placed perpendicular to the axis 40 cm beyond the diverging lens, (*b*) when the mirror is removed and the lenses are 35 cm apart. (*N.*)