chapter twenty-six

Characteristics, properties, and velocity of sound waves

CHARACTERISTICS OF NOTES

NOTES may be similar to or different from each other in three respects: (i) *pitch*, (ii) *loudness*, (iii) *quality*; so that if each of these three quantities of a particular note is known, the note is completely defined or "characterised".

Pitch

Pitch is analogous to colour in light, which is characterised by the wavelength, or by the frequency, of the electromagnetic vibrations (p. 690). Similarly, the pitch of a note depends only on the frequency of the vibrations, and a high frequency gives rise to a high-pitched note. A low frequency produces a low-pitched note. Thus the high-pitched whistle of a boy may have a frequency of several thousand Hz, whereas the low-pitched hum due to A.C. mains frequency when first switched on may be a hundred Hz. The range of sound frequencies is about 15 to 20000 Hz and depends on the observer.

Musical Intervals

If a note of frequency 300 Hz, and then a note of 600 Hz, are sounded by a siren, the pitch of the higher note is recognised to be an upper octave of the lower note. A note of frequency 1000 Hz is recognised to be an upper octave of a note of frequency 500 Hz, and thus the *musical interval* between two notes is an upper octave if the ratio of their frequencies is 2 : 1. It can be shown that the musical interval between two notes depends on the *ratio* of their frequencies, and not on the actual frequencies. The table below shows the various

Note	C doh	D ray	E me	F fah	G soh	A lah	B te	c doh
Natural (Diatonic) Scale Frequency	256	288	320	341	384	427	480	512
Intervals between notes ..	9/8	10/9	16/15	9/8	10/9	9/8	16/15	
Intervals above C ..	1·000	1·125	1·250	1·333	1·500	1·667	1·875	2·000
Equal Temperament Scale intervals above C ..	1·000	1·122	1·260	1·335	1·498	1·682	1·888	2·000

Note 1.—There are 12 semitones to the octave in the scale of equal temperament; each semitone has a frequency ratio of $2^{1/12}$.

Note 2.—The frequency of C is 256 on the scale of Helmholtz above; in music it is 261·2.

musical intervals and the corresponding ratio of the frequencies of the notes.

Intensity and Loudness

The *intensity* of a sound at a place is defined as the energy per second flowing through one square metre held normally at that place to the direction along which the sound travels. As we go farther away from a source of sound the intensity diminishes, since the intensity decreases as the square of the distance from the source (see also p. 565).

Suppose the displacement y of a vibrating layer of air is given by $y = a \sin \omega t$, where $\omega = 2\pi/T$ and a is the amplitude of vibration, see equation (1), p. 577. The velocity, v, of the layer is given by

$$v = \frac{dy}{dt} = \omega a \cos \omega t,$$

and hence the kinetic energy, W, is given by

$$W = \tfrac{1}{2} mv^2 = \tfrac{1}{2} m\omega^2 a^2 \cos^2 \omega t \quad . \quad . \quad \text{(i)}$$

where m is the mass of the layer. The layer also has potential energy as it vibrates. Its total energy, W_0, which is constant, is therefore equal to the maximum value of the kinetic energy. From (i), it follows that

$$W_0 = \tfrac{1}{2} m\omega^2 a^2 \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

In 1 second, the air is disturbed by the wave over a distance V cm., where V is the velocity of sound in m s^{-1}; and if the area of cross-section of the air is 1 m^2, the volume of air disturbed is V m^3. The mass of air disturbed per second is thus $V\rho$ kg, where ρ is the density of air in kg m^{-3}, and hence, from (ii),

$$W = \tfrac{1}{2} V\rho\omega^2 a^2 \quad . \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

It therefore follows that *the intensity of a sound due to a wave of given frequency is proportional to the square of its amplitude of vibration.*

It can be seen from (ii) that the greater the mass m of air in vibration, the greater is the intensity of the sound obtained. For this reason the sound set up by the vibration of the diaphragm of a telephone earpiece cannot be heard except with the ear close to the earpiece. On the other hand, the cone of a loudspeaker has a large surface area, and thus disturbs a large mass of air when it vibrates, giving rise to a sound of much larger intensity than the vibrating diaphragm of the telephone earpiece. It is difficult to hear a vibrating tuning-fork a small distance away from it because its prongs set such a small mass of air vibrating. If the fork is placed with its end on a table, however, a much louder sound is obtained, which is due to the large mass of air vibrating by contact with the table.

Loudness is a sensation, and hence, unlike intensity, it is difficult to measure because it depends on the individual observer. Normally, the greater the intensity, the greater is the loudness of the sound (see p. 607).

The Decibel

We are already familiar with the fact that when the frequency of a note is doubled its pitch rises by an octave. Thus the increase in pitch sounds the same to the ear when the frequency increases from 100 to 200 Hz as from 500 to 1000 Hz. Similarly, it is found that increases in loudness depend on the *ratio* of the intensities, and not on the absolute differences in intensity.

If the power of a source of sound increases from 0·1 watt to 0·2 watt, and then from 0·2 watt to 0·4 watt, the loudness of the source to the ear increases in equal steps. The equality is thus dependent on the equality of the *ratio* of the powers, not their difference, and in commercial practice the increase in loudness is calculated by taking the *logarithm of the ratio of the powers to the base* 10, which is $\log_{10} 2$, or 0·3, in this case.

Relative intensities or powers are expressed in *bels*, after Graham Bell, the inventor of the telephone. If the power of a source of sound changes from P_1 to P_2, then

$$\text{number of bels} = \log_{10}\left(\frac{P_2}{P_1}\right).$$

In practice the bel is too large a unit, and the **decibel (db)** is therefore adopted. This is defined as one-tenth of a bel, and hence in the above case

$$\text{number of decibels} = 10\log_{10}\left(\frac{P_2}{P_1}\right).$$

The minimum change of power which the ear is able to detect is about 1 db, which corresponds to an increase in power of about 25 per cent.

Calculation of Decibels

Suppose the power of a sound from a loudspeaker of a radio receiver is 50 milliwatts, and the volume control is turned so that the power increases to 500 milliwatts. The increase in power is then given by

$$10\log_{10}\left(\frac{P_2}{P_1}\right) = 10 \times \log\frac{500}{50} = 10 \text{ db}.$$

If the volume control is turned so that the power increases to 1000 milliwatts, the increase in power compared with the original sound

$$= 10\log_{10}\left(\frac{1000}{50}\right) = 10\log_{10} 20 = 13 \text{ db}.$$

If the volume control is turned down so that the power decreases from 1000 to 200 milliwatts, the change in power

$$= 10\log_{10}\left(\frac{200}{1000}\right) = 10\log_{10}2 - 10\log_{10}10 = -7 \text{ db}.$$

The minus indicates a decrease in power. Besides its use in acoustics the decibel is used by radio and electrical engineers in dealing with changes in electrical power.

Intensity Levels. Threshold of Hearing

Since the intensity of sound is defined as the energy per second crossing 1 metre^2 normal to the direction of the sound, the unit of intensity is "watt metre^{-2}", symbol "W m^{-2}". The *intensity level* of a source is its intensity relative to some agreed 'zero' intensity level. If the latter has an intensity of P_0 watt metre^{-2}, a sound of intensity P watt metre^{-2} has an intensity level defined as:

$$10 \log_{10} \left(\frac{P}{P_0}\right) \text{ db.}$$

The lowest audible sound at a frequency of 1000 Hz, which is called the *threshold of hearing*, corresponds to an intensity P_0 of 10^{-12} watt m^{-2} or 10^{-10} microwatt cm^{-2}. This is chosen as the 'zero' of sound intensity level. An intensity level of a low sound of + 60 db is 60 decibels or 6 bels higher than 10^{-12} watt metre^{-2}. The intensity is thus 10^6 times as great, and is therefore equal to $10^6 \times 10^{-12}$ or 10^{-6} W m^{-2}.

Calculation of intensity level. The difference in intensity levels of two sounds of intensities P_1, P_2 watt metre^{-2} respectively is $10 \log_{10}(P_2/P_1)$ db. Thus the difference in intensity levels of a sound of intensity 8×10^{-5} W m^{-2} due to a person talking, and one of an intensity 10^{-1} W m^{-2} due to an orchestra playing,

$$= 10 \log_{10} \left(\frac{8 \times 10^{-5}}{10^{-1}}\right) = -40 + 10 \log_{10} 8 = -31 \text{ db.}$$

The negative sign indicates a decrease in intensity level. Similarly, if two intensity levels differ by 20 decibels, the ratio P'/P of the two intensities is given by

$$10 \log_{10} \left(\frac{P'}{P}\right) = 20,$$

or

$$\frac{P'}{P} = 10^2 = 100.$$

A source of sound such as a small loudspeaker produces a sound intensity round it proportional to $1/d^2$, where d is the distance from the loudspeaker (p. 607). Suppose the intensity level is 10 db at a distance of 20 m from the speaker. At a distance of 40 m the intensity will be four times less than at 20 m, a reduction of $10 \log_{10} 4$ db or about 6 db.

$$\therefore \quad \text{intensity level here} = 10 \text{ db} - 6 \text{ db} = 4 \text{ db.}$$

At a point 10 m from the speaker the intensity will be four times greater than at 20 m. The intensity level here is thus 10 + 6 or 16 db.

If the electrical power supplied to the loudspeaker is doubled, the sound intensity at each point is doubled. Thus if the original intensity level was 16 db, the new intensity level is higher by $10 \log_{10} 2$ db or about 3 db. The new intensity level is hence 19 db.

Loudness. The Phon

The loudness of a sound is a sensation, and thus depends on the observer, whereas power, or intensity, of a sound is independent of the observer. Observations show that sounds which appear equally loud to a person have different intensities or powers, depending on the frequency, f, of the sound. The curves a, b, c represent respectively three values of *equal loudness*, and hence the intensity at X, when the frequency is 1000 Hz, is less than the intensity at Y, when the frequency is 500, although the loudness is the same, Fig. 26.1.

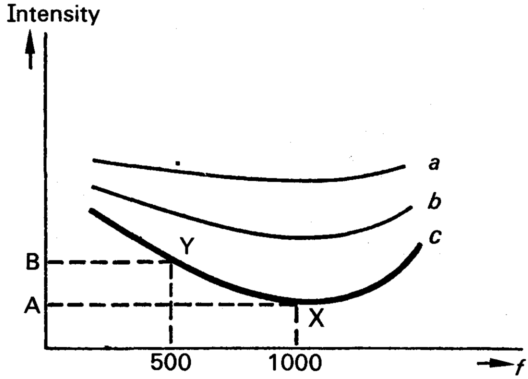

FIG. 26.1. Curves of equal loudness.

In order to measure loudness, therefore, scientists have adopted a "standard" source having a frequency of 1000 Hz, with which all other sounds are compared. The source H whose loudness is required is placed near the standard source, and the latter is then altered until the loudness is the same as H. The intensity or power level of the standard source is then measured, and if this is *n* decibels above the threshold value (10^{-10} microwatt per sq cm, p. 609) the loudness is said to be *n phons*. The phon, introduced in 1936, is thus a unit of loudness, whereas the decibel is a unit of intensity or power. *Noise meters*, containing a microphone, amplifier, and meter, are used to measure loudness, and are calibrated directly in phons. The "threshold of feeling", when sound produces a painful sensation to the ear, corresponds to a loudness of about 120 phons.

Quality or Timbre

If the same note is sounded on the violin and then on the piano, an untrained listener can tell which instrument is being used, without seeing it. We say that the *quality* or *timbre* of the note is different in each case.

The waveform of a note is never simple harmonic in practice; the nearest approach is that obtained by sounding a tuning-fork, which thus produces what may be called a "pure" note, Fig. 26.2 (i). If the same note is played on a violin and piano respectively, the waveforms produced might be represented by Fig. 26.2 (ii), (iii), which have the same frequency and amplitude as the waveform in Fig. 26.2 (i). Now curves of the shape of Fig. 26.2 (ii), (iii) can be analysed mathematically into the sum of a number of *simple harmonic* curves, whose frequencies are multiples of f_0, the frequency of the original waveform; the amplitudes of these curves diminish as the frequency increases. Fig. 26.2 (iv), for example, might be an analysis of a curve similar to Fig. 26.2 (iii), corresponding to a note on a piano. The ear is able to detect simple harmonic waves (p. 595), and thus registers the presence of notes of frequencies $2f_0$ and $3f_0$, in addition to f_0, when the note is sounded on the piano. The amplitude of the curve corresponding to f_0 is greatest,

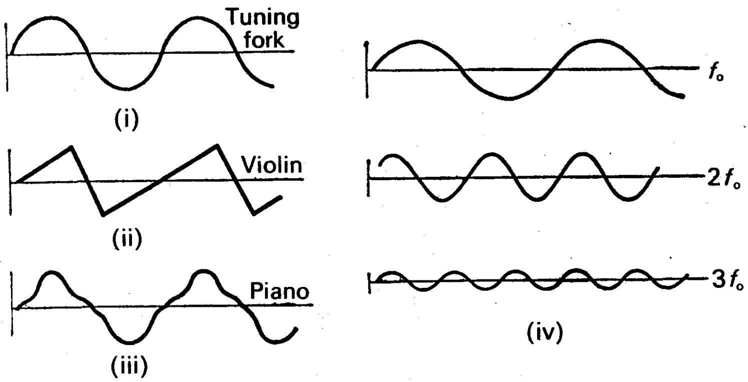

FIG. 26.2. Wave-forms of notes.

Fig. 26.2 (iv), and the note of frequency f_0 is heard predominantly because the intensity is proportional to the square of the amplitude (p. 607). In the background, however, are the notes of frequencies $2f_0$, $3f_0$, which are called the *overtones*. The frequency f_0 is called the *fundamental*.

As the waveform of the same note is different when it is obtained from different instruments, it follows that the analysis of each will differ; for example, the waveform of a note of frequency f_0 from a violin may contain overtones of frequencies $2f_0$, $4f_0$, $6f_0$. The musical "background" to the fundamental note is therefore different when it is sounded on different instruments, and hence *the overtones present in a note determine its quality or timbre.*

A *harmonic* is the name given to a note whose frequency is a simple multiple of the fundamental frequency f_0. The latter is thus termed the "first harmonic"; a note of frequency $2f_0$ is called the "second harmonic", and so on. Certain harmonics of a note may be absent from its overtones; for example, the only possible notes obtained from an organ-pipe closed at one end are f_0, $3f_0$, $5f_0$, $7f_0$, and so on (p. 647).

Helmholtz Resonators

HELMHOLTZ, one of the greatest scientists of the nineteenth century, devised a simple method of detecting the overtones accompanying the fundamental note. He used vessels, P, Q, of different sizes, containing air which "responded" or *resonated* (see p. 653) to a note of a particular frequency, Fig. 26.3. When a sound wave entered a small cavity or neck

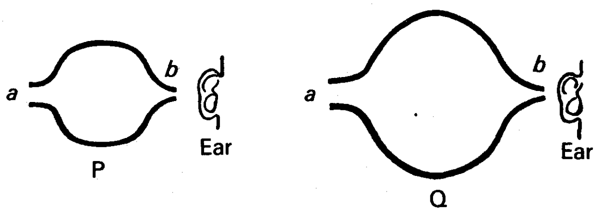

FIG. 26.3. Helmholtz resonators.

a in the resonator, as the vessel was called, an observer at *b* on the other side heard a note if the wave contained the frequency to which the resonator responded. By using resonators of various sizes, which were themselves singularly free from overtones, Helmholtz analysed the notes obtained from different instruments.

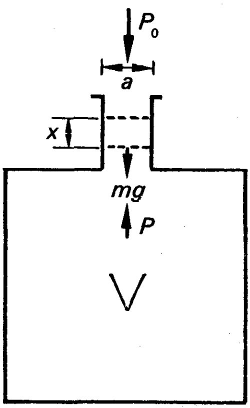

FIG. 26.4. Theory of a Resonator.

Theory of Resonator

We shall now see how the frequency of a resonator depends on the volume *V* of air inside it; and to define the situation, suppose we have a bottle with a narrow neck of cross-sectional area *a* and containing air of mass *m*, Fig. 26.4.

If the pressure outside is p_0, and the air-pressure inside is *p*, then, for equilibrium,

$$p_0 a + mg = pa \quad . \quad . \quad \text{(i)}$$

When the air in the vessel is resonating to a particular note, the air in the neck moves up and down, acting like a damper or piston on the large mass of air of volume *V* beneath the neck. Suppose the air in the neck moves downward through a distance *x* at an instant. Then, assuming an adiabatic contraction, the increased pressure p_1 in the vessel is given by

$$p_1 (V - ax)^\gamma = pV^\gamma.$$

$$\therefore \quad p_1 = p\left[\frac{V}{V - ax}\right]^\gamma = p\left[1 + \frac{ax}{V - ax}\right]^\gamma$$

$$= p\left[1 + \frac{\gamma ax}{V - ax}\right],$$

by binomial expansion, assuming *ax* is small compared with $(V - ax)$; this is true for a narrow neck connected to a large volume *V*.

$$\therefore \quad p_1 - p = \frac{\gamma pax}{V - ax} \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

The net downward force, *P*, on the air in the resonator

$$= p_0 a + mg - p_1 a = pa - p_1 a, \text{ from (i).}$$

Hence, from (ii)

$$P = -\frac{\gamma pax}{V - ax} \times a = \frac{-\gamma pa^2 x}{V},$$

neglecting *ax* compared with *V*. From the relationship "force = mass × acceleration", it follows that

$$\frac{-\gamma pa^2 x}{V} = m \times \text{accn.},$$

or

$$\text{accn.} = -\frac{\gamma pa^2}{mV} \times x.$$

Thus the motion of the air in the neck is simple harmonic, and the period T is given by

$$T = 2\pi \sqrt{\frac{mV}{\gamma p a^2}}.$$

Hence the frequency, f, is given by

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{\gamma p a^2}{mV}} \quad . \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

The velocity of sound, v, is given by $v = \sqrt{\gamma p/\rho}$, where ρ is the density of the air (p. 624), or $\gamma p = v^2 \varrho$. If l is the length of the neck, the mass $m = al\varrho$. Thus the frequency f can also be expressed by

$$f = \frac{1}{2\pi}\sqrt{\frac{v^2 \rho a^2}{al\rho V}} = \frac{v}{2\pi}\sqrt{\frac{a}{lV}} \quad . \quad . \quad . \quad . \quad \text{(iv)}$$

From these formulae for f, it follows that

$$f^2 V = constant.$$

The adiabatic changes at the neck are not perfect, and this result is thus only approximately true. In practice, the law more nearly obeyed is that given by

$$f^2 (V + c) = constant.$$

where c is a "correction" to V.

Experiment. In an experiment to verify the law, tuning-forks of known frequency, a bottle with a narrow neck, and a pipette and burette, are required. Water is run slowly into the bottle until resonance is obtained with the lowest note, for example. The volume of air V which is resonating is then found by subtracting the volume of the bottle below the neck, determined in a preliminary experiment, from the water run in. This is repeated for the various forks, and a graph of V is plotted against $1/f^2$. A straight line passing close to the origin is obtained, thus showing that $V + c = d/f^2$, where d is a constant, or $f^2(V + c) = $ constant.

PROPERTIES OF SOUND WAVES

Reflection

Like light waves, sound waves are reflected from a plane surface so that the angle of incidence is equal to the angle of reflection. This can be demonstrated by placing a tube T_1 in front of a plane surface AB and blowing a whistle gently at S, Fig. 26.5. Another tube T_2, directed towards N, is placed on the other side of the normal NQ, and moved until a sensitive flame (see p. 650), or a microphone connected to a cathode-ray tube, is considerably affected at R, showing that the reflected wave passes along NR. It will then be found that angle RNQ = angle SNQ.

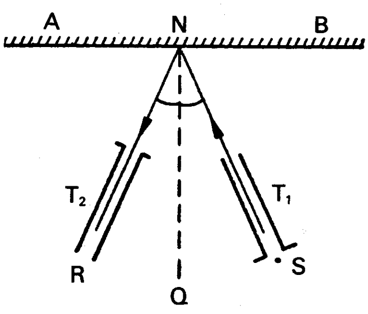

FIG. 26.5. Reflection of sound.

It can also be demonstrated that sound waves come to a focus when they are incident on a curved concave mirror. A surface shaped like a parabola reflects sound waves to long distances if the source of sound is placed at its focus (see also p. 404). The famous whispering gallery of St. Paul's is a circular-shaped chamber whose walls repeatedly reflect sound waves round the gallery, so that a person talking quietly at one end can be heard distinctly at the other end.

Acoustics of Rooms. Reverberation

A concert-hall, lecture-room, or a broadcasting studio requires special design to be acoustically effective. The technical problems concerned were first investigated in 1906 by SABINE in America, who was consulted about a hall in which it was difficult for an audience to hear the lecturer.

Generally, an audience in a hall hears sound from different directions at different times. They hear (*a*) sound *directly* from the speaker or orchestra, as the case may be, (*b*) sound from *echoes* produced by walls and ceilings, (*c*) sound *diffused* from the walls and ceilings and other objects present. The echoes are due to regular reflection at a plane surface (p. 391), but the diffused sound is scattered in different directions and reflection takes place repeatedly at other surfaces. When reflection occurs some energy is absorbed from the sound wave, and after a time the sound diminishes below the level at which it can be heard. The perseverance of the sound after the source ceases is known as **reverberation.** In the case of the hall investigated by Sabine the time of reverberation was about $5\frac{1}{2}$ seconds, and the sound due to the first syllable of a speaker thus overlapped the sound due to the next dozen or so syllables, making the speech difficult to comprehend. The quality of a sound depends on the time of reverberation. If the time is very short, for example 0·5 second, the music from an orchestra sounds thin or lifeless; if the time too long the music sounds muffled. The reverberation time at a B.B.C. concert-hall used for orchestral performances is about $1\frac{3}{4}$ seconds, whereas the reverberation time for a dance-band studio is about 1 second.

Sabine's Investigations. Absorptive Power

Sabine found that the time T of reverberation depended on the volume V of the room, its surface area A, and the *absorptive power*, a, of the surfaces. The time T is given approximately by

$$T = \frac{kV}{aA},$$

where k is a constant. In general some sound is absorbed and the rest is reflected; if too much sound is reflected T is large. If many thick curtains are present in the room too much sound is absorbed and T is small.

Sabine chose the absorptive power of unit area of an open window as the unit, since this is a perfect absorber. On this basis the absorptive power of a person in an audience, or of thick carpets and rugs, is 0·5, linoleum has an absorptive power of 0·12, and polished wood and glass have an absorptive power of 0·01. The absorptive power of a material depends on its pores to a large extent; this is shown by the fact that an unpainted brick has a high absorptive power, whereas the painted brick has a low absorptive power.

From Sabine's formula for *T* it follows that the time of reverberation can be shortened by having more spectators in the hall concerned, or by using felt materials to line some of the walls or ceiling. The seats in an acoustically-designed lecture-room have plush cushions at their backs to act as an absorbent of sound when the room is not full. B.B.C. studios used for plays or news talks should have zero reverberation time, as clarity is all-important, and the studios are built from special plaster or cork panels which absorb the sound completely. The structure of a room also affects the acoustics. Rooms with large curved surfaces tend to focus echoes at certain places, which is unpleasant aurally to the audience, and a huge curtain was formerly hung from the roof of the Albert Hall to obscure the dome at orchestral concerts.

Refraction

Sound waves can be refracted as well as reflected. TYNDALL placed a watch in front of a balloon filled with carbon dioxide, which is heavier than air, and found that the sound was heard at a definite place on the other side of the balloon. The sound waves thus converged to a focus on the other side of the balloon, which therefore has the same effect on sound waves as a convex lens has on light waves (see Fig. 28.12, p. 684). If the balloon is filled with hydrogen, which is lighter than air, the sound waves diverge on passing through the balloon. The latter thus acts similarly to a concave lens when light waves are incident on it (see p. 683).

The refraction of sound explains why sounds are easier to hear at night than during day-time. In the latter case the upper layers of air are colder than the layers near the earth. Now sound travels faster the higher the temperature (see 624), and sound waves are hence refracted in a direction away from the earth. The intensity of the sound waves thus diminish. At night-time, however, the layers of air near the earth are colder than those higher up, and hence sound waves are now refracted towards the earth, with a consequent increase in intensity.

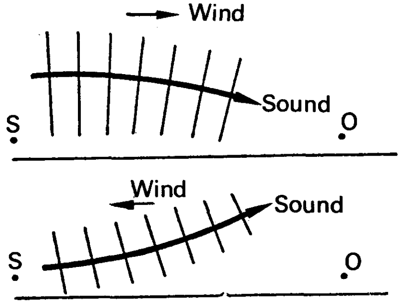

FIG. 26.6. Refraction of sound.

For a similar reason, a distant observer O hears a sound from a source S more easily when the wind is blowing towards him than away from him, Fig. 26.6. When the wind is blowing towards O, the bottom of the sound wavefront is moving slower than the upper part, and hence the wavefronts veer towards the observer, who therefore hears the sound easily. When the wind is blowing in the opposite direction the reverse is the case, and the wavefronts veer upwards away from the ground and O. The sound intensity thus diminishes. This phenomenon is hence another example of the refraction of sound.

Interference of Sound Waves

Besides reflection and refraction, sound waves can also exhibit the phenomenon of *interference*, whose principles we shall now discuss.

Suppose two sources of sound, A, B, have exactly the same frequency and amplitude of vibration, and that their vibrations are always in phase with each other, Fig. 26.7. Such sources are called "coherent" sources. *Their combined effect at a point is obtained by adding algebraically the displacements at the point due to the sources individually;* this is known as the *Principle of Superposition.* Thus their resultant effect at X, for example, is the algebraic sum of the vibrations at X due to the source A alone and the vibrations at X due to the source B alone. If X is equidistant from A and B, the vibrations at X due to the two sources are *always* in phase as (i) the distance AX travelled by the wave originating at A is equal to the distance BX travelled by the wave originating at B, (ii) the sources A, B are assumed to have the same frequency and to be always in phase with each other. Fig. 26.8 (i), (ii) illustrate the vibrations at X due to A, B, which have the same amplitude. The resultant vibration at X is obtained by adding the two curves, and has an amplitude double that of either curve and a frequency the same as either, Fig. 26.8 (iii). Now the energy of a vibrating source is proportional to the square of its amplitude (p. 607). Consequently the sound energy at X is four times that due to A or B alone, and a loud sound is thus heard at X. As A and B are coherent sources, the loud sound is *permanent.*

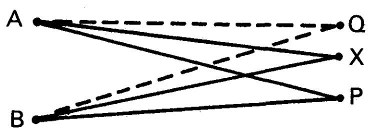

FIG. 26.7. Interference of sound.

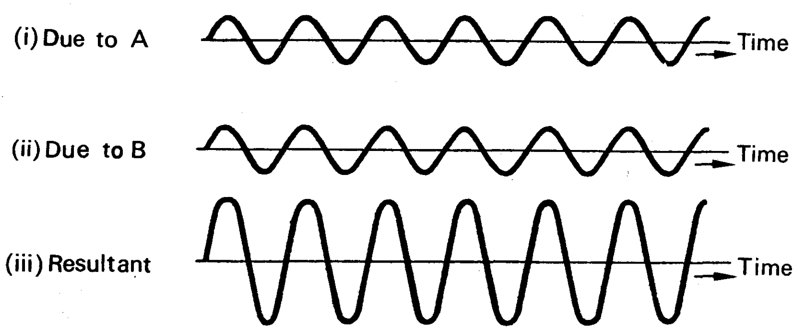

FIG. 26.8. Vibrations at X.

If Q is a point such that BQ is greater than AQ by a whole number of wavelengths (Fig. 26.7), the vibration at Q due to A is in phase with the vibration there due to B (see p. 593). A permanent loud sound is then obtained at Q. Thus a permanent loud sound is obtained at any point *Y* if the *path difference*, BY $-$ AY, is given by

$$BY - AY = m\lambda,$$

where λ is the wavelength of the sources A, B, and m is an integer.

Destructive Interference

Consider now a point P in Fig. 26.7 whose distance from B is half a wavelength longer than its distance from A, i.e., AP $-$ BP $= \lambda/2$. The vibration at P due to B will then be 180° out of phase with the vibration there to A (see p. 586), Fig. 26.9 (i), (ii). The resultant effect at P is thus zero, as the displacements at any instant are equal and opposite to each other, Fig. 26.9 (iii). No sound is therefore heard at P, and the permanent silence is said to be due to "destructive interference" between the sound waves from A and B.

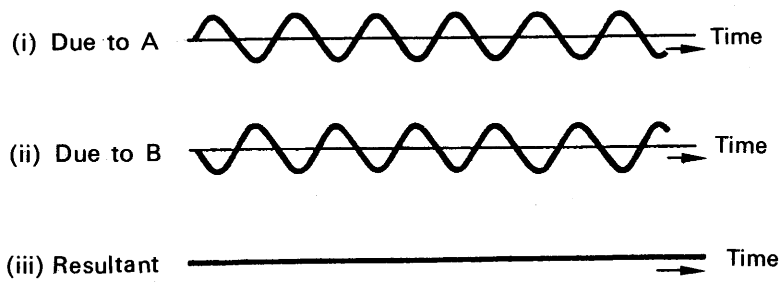

FIG. 26.9. Vibrations at P.

If the path difference, AP $-$ BP, were $3\lambda/2$ or $5\lambda/2$, instead of $\lambda/2$, permanent silence would also exist at P as the vibrations there due to A, B would again be 180° out of phase. Summarising, then,

silence occurs if the path-difference is an odd number of half wavelengths, and

a loud sound occurs if the path-difference is a whole number of wavelengths

The total sound energy in all the positions of loud sound discussed above is equal to the total sound energy of the two sources A, B, from the principle of the conservation of energy. The extra sound at the positions of loud sound thus makes up for the absent sound in the positions of silence.

Quincke's Tube. Measurement of Velocity of Sound in a Tube

QUINCKE devised a simple method of obtaining permanent interference between two sound waves. He used a closed tube SAEB which had openings at S, E, and placed a source of sound at S, Fig. 26.10. A wave then travelled in the direction SAE round the tube, while another wave travelled in the opposite direction SBE; and since these waves are due to the same source, S, they always set out in phase, i.e., they are coherent.

Like a trombone, one side, B, of the tube can be pulled out, thus making SAE, SBE of different lengths. When SAE and SBE are equal in length an observer at E hears a loud sound, since the paths of the two waves are then equal. As B is pulled out the sound dies away and becomes a minimum when the path difference, SBE − SAE, is $\lambda/2$, where λ is the wavelength. In this case the two waves arrive 180° out of phase (p. 617). If the tube is pulled out farther, the sound increases in loudness to a maximum; the path difference is then λ. If k is the distance moved from one position of minimum sound, MN say, to the next position of minimum sound, PQ say, then $2k = \lambda$, Fig. 26.10. Thus the wavelength of the sound can be simply obtained by measuring k.

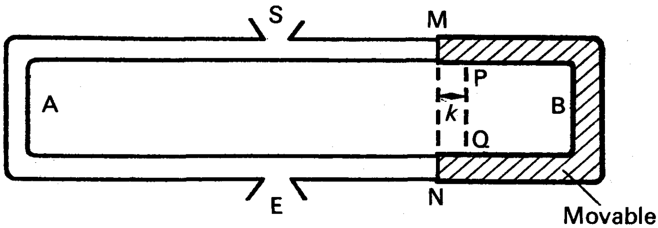

FIG. 26.10. Quincke's tube.

The velocity of sound in the tube is given by $V = f\lambda$, where f is the frequency of the source S, and thus V can be found when a source of known frequency is used. In a particular experiment with Quincke's tube, the tube B was moved a distance 4·28 cm between successive minima of sound, and the frequency of the source was 4000 Hz.

Thus $\lambda = 2 \times 4{\cdot}28$ cm,

and $V = f\lambda = 4000 \times 2 \times 4{\cdot}28 = 34240\ \text{cm s}^{-1} = 342{\cdot}4\ \text{m s}^{-1}$

It can be seen that, unlike reflection and refraction, the phenomenon of interference can be utilised to measure the wavelength of sound waves. We shall see later that interference is also utilised to measure the wavelength of light waves (p. 689).

Velocity of Sound in Free Air. Hebb's Method

In 1905 HEBB performed an accurate experiment to measure the velocity of sound in free air which utilised a method of interference. He carried out his experiment in a large hall to eliminate the effect of wind, and obtained the temperature of the air by placing thermometers at different parts of the room. Two parabolic reflectors, R_1, R_2, are placed at each end of the hall, and microphones, M_1, M_2, are positioned at the respective foci, S_1, S_2 to receive sound reflected from R_1, R_2, Fig. 26.11. By means of a transformer, the currents in the microphones are induced into a telephone earpiece P, so that the *resultant* effect of the sound waves received by M_1, M_2 respectively can be heard.

A source of sound of known constant frequency is placed at the focus S_1. The sound waves are reflected from R_1 in a parallel direction (p. 613), and travel to R_2 where they are reflected to the focus S_2 and received by M_2. The microphone M_1 receives sound waves directly from the source, and hence the sound heard in the telephone earpiece is due to the resultant effect of two coherent sources. With the source

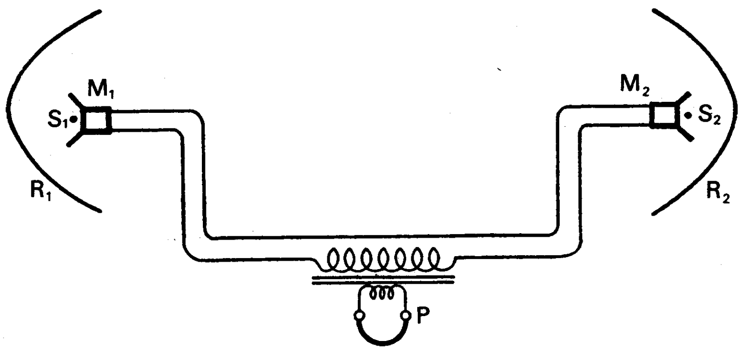

FIG. 26.11. Hebb's method.

and microphone maintained at its focus S_1, R_1 is moved along its axis in one direction. The positions of R_1 are noted when minima of sound are heard; and since the distance between successive minima corresponds to one wavelength, λ, the velocity of sound can be calculated from the relation $V = f\lambda$, as f and λ are known.

Other Velocity of Sound Determinations

The velocity of sound in *air* has been determined by many scientists. One of the first accurate determinations was carried out in 1738 by French scientists, who observed the time between the flash and the hearing of a cannon report about 30 km away. Their results confirmed that the velocity of sound increased as the temperature of the air increased (p. 624), and they obtained the result of 362 metres per second for the velocity at 0° C. Similar experiments were carried out by French scientists in 1822. In 1844 experiments carried out in the Tyrol district, several thousand metres above sea-level, showed that the velocity of sound was independent of the pressure of the air (p. 624).

REGNAULT, the eminent French experimental scientist of the nineteenth century, carried out an accurate series of measurements on the velocity of sound in 1864. Guns were fired at one place, breaking an electrical circuit automatically, and the arrival of the sound at a distant place was recorded by a second electrical circuit. Both circuits actuated a pen or style pressing against a drum rotating at a steady speed round its axis, which is known as a *chronograph*. Thus marks were made on the drum at the instant the sound occurred and the instant it was received. The small interval corresponding to the distance between the marks was determined from a wavy trace made on the drum by a style attached to an electrically-maintained tuning-fork whose frequency was known, and the speed of sound was thus calculated.

The velocity of sound in *water* was first accurately determined in 1826. The experiment was carried out by immersing a bell in the Lake of Geneva, and arranging to fire gunpowder at the instant the bell was struck. Miles away, the interval was recorded between the flash and the later arrival of the sound in the water, and the velocity was then calculated. This and other experiments have shown that the velocity in water is about 1435 m s^{-1}, more than four times the speed in air.

An objection to all these methods of determining velocity is the unknown time lag between the receipt of the sound by an observer and his recording of the sound. The observer has, as it were, a "personal equation" which must be taken into account to determine the true time of travel of the sound. In Hebb's method, however, which utilises interference, no such personal equation enters into the considerations, which is an advantage of the method.

Beats

If two notes of nearly equal frequency are sounded together, a periodic rise and fall in intensity can be heard. This is known as the phenomenon of *beats*, and the frequency of the beats is the number of intense sounds heard per second.

Consider a layer of air some distance away from two pure notes of nearly equal frequency, say 48 and 56 Hz respectively, which are sounding. The variation of the displacement, y_1, of the layer due to one fork alone is shown in Fig. 26.12 (i); the variation of the displacement y_2,

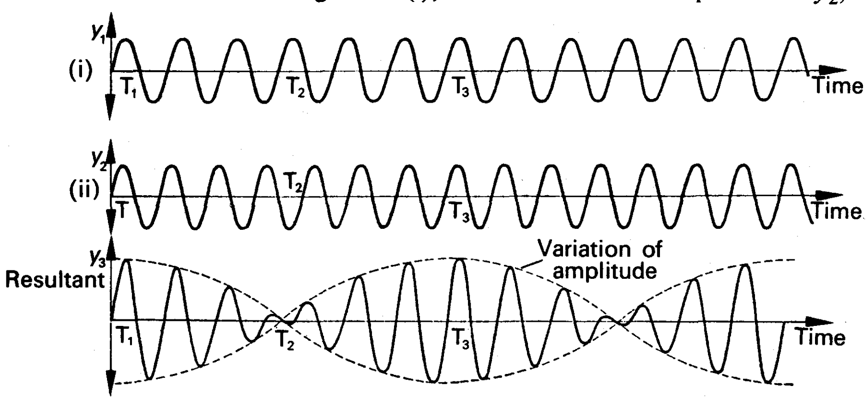

FIG. 26.12. Beats (*not to scale*).

of the layer due to the second fork alone is shown in Fig. 26.12 (ii). According to the Principle of Superposition (p. 588), the variation of the resultant displacement, y, of the layer is the algebraic sum of the two curves, which varies in amplitude in the way shown in Fig. 26.12 (iii). To understand the variation of y, suppose that the displacements y_1, y_2 are in phase at some instant T_1, Fig. 26.12. Since the frequency of the curve in Fig. 26.12 (i) is 48 cycles per sec the variation y_1 undergoes 3 complete cycles in $\frac{1}{16}$th second; in the same time, the variation y_2 undergoes $3\frac{1}{2}$ cycles, since its frequency is 56 cycles per second. Thus y_1 and y_2 are 180° out of phase with each other at this instant, and their resultant y is then a minimum at some instant T_2. Thus T_1T_2 represents $\frac{1}{16}$th of a second in Fig. 26.12 (iii). In $\frac{1}{8}$th of a second from T_1, y_1 has undergone 6 complete cycles and y_2 has undergone 7 complete cycles. The two waves are hence in phase again at T_3, where T_1T_3 represents $\frac{1}{8}$th of a second, and their resultant at their instant is again a maximum, Fig. 26.12 (iii). In this way it can be seen that a loud sound is heard after every $\frac{1}{8}$ second, and thus the beat frequency is 8 cycles per second. This is the difference between the frequencies, 48, 56, of the two notes, and it is shown soon that *the beat frequency is always equal to the difference of the two nearly equal frequencies.*

It can now be seen that beats are a phenomenon of repeated interference. Unlike the cases in sound previously considered, however, the two sources are not coherent ones.

Beat Frequency Formula

Suppose two sounding tuning-forks have frequencies f_1, f_2 cycles per second close to each other. At some instant of time the displacement of a particular layer of air near the ear due to each fork will be a maximum to the right. The resultant displacement is then a maximum, and a loud sound or beat is heard. After this, the vibrations of air due to each fork go out of phase, and t seconds later the displacement due to each fork is again a maximum to the right, so that a loud sound or beat is heard again. One fork has then made exactly one cycle more than the other. But the number of cycles made by each fork in t seconds is f_1t and f_2t respectively. Assuming f_1 is greater than f_2,

$$\therefore \quad f_1t - f_2t = 1$$

$$\therefore \quad f_1 - f_2 = \frac{1}{t}$$

Now 1 beat has been made in t seconds, so that $1/t$ is the number of beats per second or beat frequency.

$$\therefore \quad f_1 - f_2 = \textit{beat frequency}.$$

Mathematical derivation of beat frequency. Suppose y_1, y_2 are the displacements of a given layer of air due to two tuning-forks of frequencies f_1, f_2 respectively. If the amplitudes of each vibration are equal to a, then $y_1 = a \sin \omega_1 t$, $y_2 = a \sin (\omega_2 t + \theta)$, where $\omega_1 = 2\pi f_1$, $\omega_2 = 2\pi f_2$, and θ is the constant phase angle between the two variations.

$$\therefore \quad y = y_1 + y_2 = a\,[\sin \omega_1 t + \sin (\omega_2 t + \theta)]$$

$$\therefore \quad y = 2a \sin \left(\frac{\omega_1 + \omega_2}{2}\,t + \frac{\theta}{2}\right) . \cos \left(\frac{\omega_1 - \omega_2}{2}\,t - \frac{\theta}{2}\right)$$

$$\therefore \quad y = A \sin \left(\frac{\omega_1 + \omega_2}{2}\,t + \frac{\theta}{2}\right)$$

where $A = 2a \cos \left(\frac{\omega_1 - \omega_2}{2}\,t - \frac{\theta}{2}\right)$.

We can regard A as the *amplitude* of the variation of y. The intensity of the resultant note is proportional to A^2, the square of the amplitude (p. 607) and

$$A^2 = 4a^2 \cos^2 \left(\frac{\omega_1 - \omega_2}{2}\,t - \frac{\theta}{2}\right) = 2a^2\,[1 + \cos \overline{(\omega_1 - \omega_2)\,t - \theta}]$$

since $2 \cos^2 \alpha = 1 + \cos 2\alpha$. It then follows that the intensity varies at a frequency f given by

$$2\pi f = \omega_1 - \omega_2.$$

But $\omega_1 = 2\pi f_1, \omega_2 = 2\pi f_2$.

$$\therefore \quad 2\pi f = 2\pi f_1 - 2\pi f_2$$

$$\therefore \quad f = f_1 - f_2$$

The frequency f of the beats is thus equal to the difference of the frequencies.

Uses of Beats

The phenomenon of beats can be used to measure the unknown frequency, f_1, of a note. For this purpose a note of known frequency f_2 is used to provide beats with the unknown note, and the frequency f of the beats is obtained by counting the number made in a given time. Since f is the difference between f_2 and f_1, it follows that $f_1 = f_2 - f$, or $f_1 = f_2 + f$. Thus suppose $f_2 =$ 1000 Hz, and the number of beats per second made with a tuning-fork of unknown frequency f_1 is 4. Then $f_1 =$ 1004 or 996 Hz.

To decide which value of f_1 is correct, the end of the tuning-fork prong is loaded with a small piece of plasticine, which diminishes the frequency a little, and the two notes are sounded again. If the beats are *increased*, a little thought indicates that the frequency of the note must have been originally 996 Hz. If the beats are decreased, the frequency of the note must have been originally 1004 Hz. The tuning-fork must not be overloaded, as the frequency may decrease, if it was 1004 Hz, to a frequency such as 995 Hz, in which case the significance of the beats can be wrongly interpreted.

Beats are also used to "tune" an instrument to a given note. As the instrument note approaches the given note, beats are heard, and the instrument can be regarded as "tuned" when the beats are occurring at a very slow rate.

Velocity of Sound in a Medium

When a sound wave travels in a medium, such as a gas, a liquid, or a solid, the particles in the medium are subjected to varying stresses, with resulting strains (p. 585). The velocity of a sound wave is thus partly governed by the *modulus of elasticity*, *E*, of the medium, which is defined by the relation

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\text{force per unit area}}{\text{change in length (or volume)/ original length (or volume)}} \quad . \quad . \quad . \quad . \quad \text{(i)}$$

The velocity, V, also depends on the density, ρ, of the medium, and it can be shown that

$$V = \sqrt{\frac{E}{\rho}} \quad . \quad . \quad . \quad . \quad (1)$$

When E is in newton per metre^2 (N m^{-2}) and ρ in kg m^{-3}, then V is in metre per second (m s^{-1}). The relation (1) was first derived by Newton.

For a solid, E is Young's modulus of elasticity. The magnitude of E

for steel is about 2×10^{11} N m^{-2}, and the density ρ of steel is 7800 kg m^{-3}. Thus the velocity of sound in steel is given by

$$V = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{7800}} = 5060 \text{ m s}^{-1}$$

For a liquid, E is the bulk modulus of elasticity. Water has a bulk modulus of $2{\cdot}04 \times 10^9$ N m^{-2}, and a density of 1000 kg m^{-3}. The calculated velocity of sound in water is thus given by

$$V = \sqrt{\frac{2{\cdot}04 \times 10^9}{1000}} = 1430 \text{ m s}^{-1}$$

The proof of the velocity formula requires advanced mathematics, and is beyond the scope of this book. It can partly be verified by the method of dimensions, however. Thus since density, ρ, = mass/volume, the dimensions of ρ are given by ML^{-3}. The dimensions of force (mass × acceleration) are MLT^{-2}, the dimensions of area are L^2; and the denominator in (i) has zero dimensions since it is the ratio of two similar quantities. Thus the dimensions of modulus of elasticity, E, are given by

$$\frac{ML}{T^2L^2} \text{ or } ML^{-1}T^{-2}$$

Suppose the velocity, V, $= kE^x\rho^y$, where k is a constant. The dimensions of V are LT^{-1}

$$\therefore \quad LT^{-1} = (ML^{-1}T^{-2})^x \times (ML^{-3})^y$$

using the dimensions of E and ρ obtained above. Equating the respective indices of M, L, T on both sides, then

$$x + y = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

$$-x - 3y = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

$$-2x = -1 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iv)}$$

From (iv), $x = 1/2$, from (ii), $y = -1/2$. Thus, as $V = kE^x\rho^y$,

$$V = kE^{\frac{1}{2}}\rho^{-\frac{1}{2}}$$

$$\therefore \quad V = k\sqrt{\frac{E}{\rho}}$$

It is not possible to find the magnitude of k by the method of dimensions, but a rigid proof of the formula by calculus shows that $k = 1$ since $V = \sqrt{\frac{E}{\rho}}$.

Velocity of Sound in a Gas. Laplace's Correction

The velocity of sound in a *gas* is also given by $V = \sqrt{\frac{E}{\rho}}$ where E is the *bulk modulus* of the gas and ρ is its density. Now it is shown on p. 162 that $E = p$, the pressure of the gas, if the stresses and strains in the gas take place isothermally. The formula for the velocity then becomes $V = \sqrt{\frac{p}{\rho}}$ and as the density, ρ, of air is $1{\cdot}29$ kg per m^3 at S.T.P. and

$p = 0{\cdot}76 \times 13600 \times 9{\cdot}8$ N m^{-2};

$$V = \sqrt{\frac{0{\cdot}76 \times 13600 \times 9{\cdot}8}{1{\cdot}29}} = 280 \text{ m s}^{-1} \text{ (approx.)}.$$

This calculation for V was first performed by Newton, who saw that the above theoretical value was well below the experimental value of about 330 m s^{-1}. The discrepancy remained unexplained for more than a century, when LAPLACE suggested in 1816 that E should be the *adiabatic* bulk modulus of a gas, not its isothermal bulk modulus as Newton had assumed. Alexander Wood in his book *Acoustics* (Blackie) points out that adiabatic conditions are maintained in a gas because of the relative slowness of sound wave oscillations.* It is shown later that the adiabatic bulk modulus of a gas is γp where γ is the ratio of the principal specific heats of a gas (i.e., $\gamma = c_p/c_V$). The formula for the velocity of sound in a gas thus becomes

$$V = \sqrt{\frac{\gamma p}{\rho}} \quad . \quad . \quad . \quad . \quad (2)$$

The magnitude of γ for air is 1·40, and *Laplace's correction*, as it is known, then amends the value of the velocity in air at 0° C to

$$V = \sqrt{\frac{1{\cdot}40 \times 0{\cdot}76 \times 13600 \times 9{\cdot}8}{1{\cdot}29}} = 331 \text{ m s}^{-1}$$

This is in good agreement with the experimental value.

Effect of Pressure and Temperature on Velocity of Sound in a Gas

Suppose that the mass of a gas is m, and its volume is v. Its density, ρ, is then m/v, and hence the velocity of sound, V, is given by

$$V = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma p v}{m}}.$$

But $pv = mRT$, where R is the gas constant for unit mass of the gas and T is its absolute temperature. Thus $pv/m = RT$, and hence

$$V = \sqrt{\gamma RT} \quad . \quad . \quad . \quad . \quad . \quad (i)$$

Since γ and R are constants for a given gas, it follows that *the velocity of sound in a gas is independent of the pressure* if the temperature remains constant. This has been verified by experiments which showed that the velocity of sound at the top of a mountain is about the same as at the bottom, p. 619. It also follows from (i) that *the velocity of sound is proportional to the square root of its absolute temperature.* Thus if the velocity in air at 16° C is 338 m s^{-1} by experiment, the velocity, V, at 0° C is calculated from

$$\frac{V}{338} = \sqrt{\frac{273}{289}},$$

from which

$$V = 338\sqrt{\frac{273}{289}} = 328{\cdot}5 \text{ m s}^{-1}$$

* It was supposed for many years that the changes are so rapid that there is no time for transfer of heat to occur. The reverse appears to be the case. At ultrasonic (very high) frequencies adiabatic conditions no longer hold.

Ultrasonics

There are sound waves of higher frequency than 20000 Hz, which are inaudible to a human being. These are known as *ultrasonics*; and since velocity = wavelength × frequency, ultrasonics have short wavelengths compared with sound waves in the audio-frequency range.

In recent years ultrasonics have been utilised for a variety of industrial purposes. They are used on board coasting vessels for depth sounding, the time taken by the wave to reach the bottom of the sea from the surface and back being determined. Ultrasonics are also used to kill bacteria in liquids, and they are used extensively to locate faults and cracks in metal castings, following a method similar to that of radar. Ultrasonic waves are sent into the metal under investigation, and the beam reflected from the fault is picked up on a cathode-ray tube screen together with the reflection from the other end of the metal. The position of the fault can then easily be located.

Production of Ultrasonics

In 1881 Curie discovered that a thin plate of quartz increased or decreased in length if an electrical battery was connected to its opposite faces. By correctly cutting the plate, the expansion or contraction could be made to occur along the axis of the faces to which the battery was applied. When an alternating voltage of ultrasonic frequency was connected to the faces of such a crystal the faces vibrated at the same frequency, and thus ultrasonic sound waves were produced.

Another method of producing ultrasonics is to place an iron or nickel rod inside a solenoid carrying an alternating current of ultrasonic frequency. Since the length of a magnetic specimen increases slightly when it is magnetised, ultrasonic sound waves are produced by the vibrations of the rod.

EXAMPLES

1. How does the velocity of sound in a medium depend upon the elasticity and density? Illustrate your answer by reference to the case of air and of a long metal rod. The velocity of sound in air being 330·0 m s^{-1} at 0° C and the coefficient of expansion 1/273 per degree, find the change in velocity per degree Centigrade rise of temperature. (*L.*)

First part. The velocity of sound, V, is given by $V = \sqrt{E/\rho}$, where E is the modulus of elasticity of the medium and ρ is its density. In the case of air, a gas, E represents the bulk of modulus of the air under adiabatic conditions, and $E = \gamma p$ (see p. 624). Thus $V = \sqrt{\gamma p/\rho}$ for air.

For a long metal rod, E is Young's modulus for the metal, assuming the sound travels along the length of the rod.

Second part. Since the coefficient of expansion is 1/273 per degree Centigrade, the absolute temperature corresponding to t° C is given by $(273 + t)$. The velocity of sound in a gas is proportional to the square root of its absolute temperature, and hence

$$\frac{V}{V_0}=\sqrt{\frac{274}{273}},$$

where V is the velocity at 1° C and V_0 is the velocity at 0° C.

$$\therefore\ V=V_0\sqrt{\frac{274}{273}}=330\times\sqrt{\frac{274}{273}}=330{\cdot}6\text{ m s}^{-1}$$

$$\therefore\quad \text{change in velocity}=0{\cdot}6\text{ m s}^{-1}$$

2. How would you find by experiment the velocity of sound in air? Calculate the velocity of sound in air in metre second^{-1} at 100° C if the density of air at S.T.P. is 1·29 kg m^{-3}, the density of mercury at 0° C 13600 kg m^{-3}, the specific heat capacity of air at constant pressure 1·02, and the specific heat capacity of air at constant volume 0·72, in kJ kg^{-1}K^{-1}. (*L.*)

First part. See Hebb's method, p. 618 or p. 587.

Second part. The velocity of sound in air is given by

$$V=\sqrt{\frac{\gamma p}{\rho}}$$

with the usual notation. The density, ρ, of air is 1·29 kg m^{-3}. The pressure p is given by

$$p=h\rho g$$
$$=0{\cdot}76\times 13600\times 9{\cdot}8\text{ N m}^{-2}$$

since S.T.P. denotes 76 cm mercury pressure and 0° C. Also,

$$\gamma=\frac{C_p}{C_v}=\frac{1{\cdot}02}{0{\cdot}72}.$$

$$\therefore\quad V=\sqrt{\frac{1{\cdot}02\times 0{\cdot}76\times 13600\times 9{\cdot}8}{0{\cdot}72\times 1{\cdot}293}},$$

where V is the velocity at 0° C.

But $\qquad$ velocity $\propto\sqrt{T}$,

where T is the absolute temperature of the air. Thus if V' is the velocity at 100° C,

$$\frac{V'}{V}=\sqrt{\frac{273+100}{273}}=\sqrt{\frac{373}{273}}$$

$$\therefore\quad V'=\sqrt{\frac{373}{273}}\,V=\sqrt{\frac{373\times 1{\cdot}02\times 0{\cdot}76\times 13600\times 9{\cdot}8}{273\times 0{\cdot}72\times 1{\cdot}293}}$$

$$\therefore\quad V'=388\text{ m s}^{-1}$$

3. State briefly how you would show by experiment that the characteristics of the transmission of sound are such that (*a*) a finite time is necessary for transmission, (*b*) a material medium is necessary for propagation, (*c*) the disturbance may be reflected and refracted. The wavelength of the note emitted by a tuning-fork, frequency 512 Hz, in air at 17° C is 66·5 cm. If the density of air at S.T.P. is 1·293 kg m^{-3}, calculate the ratio of the two specific heat capacities of air. Assume that the density of mercury is 13600 kg m^{-3}. (*N.*)

First part. See text.

Second part. Since $V=f\lambda$, the velocity of sound at 17° C. is given by

$$V=512\times 0{\cdot}665\text{ m s}^{-1}\quad . \quad . \quad . \quad \text{(i)}$$

Now
$$\frac{V_o}{V} = \sqrt{\frac{273}{290}},$$

where V_0 is the velocity at 0° C, since the velocity is proportional to the square root of the absolute temperature.

$$\therefore \quad V_o = \sqrt{\frac{273}{290}} \times V = \sqrt{\frac{273}{290}} \times 512 \times 0{\cdot}665 \quad . \quad . \quad \text{(ii)}$$

But
$$V_o = \sqrt{\frac{\gamma p}{\rho}},$$

where $p = 0{\cdot}76$ m of mercury $= 0{\cdot}76 \times 13600 \times 9{\cdot}8$ N m^{-2}, and $\rho = 1{\cdot}293$ kg m^{-3}.

$$\therefore \quad \gamma = \frac{V_o^2 \times \rho}{p}$$
$$= \frac{272 \times 512^2 \times 0{\cdot}665 \times 1{\cdot}293}{290 \times 0{\cdot}76 \times 13600 \times 9{\cdot}8}$$
$$= 1{\cdot}39$$

Doppler Effect

The whistle of a train or a jet aeroplane appears to increase in pitch as it approaches a stationary observer; as the moving object passes the observer, the pitch changes and becomes lowered. The apparent alteration in frequency was first predicted by DOPPLER in 1845, who stated that a change of frequency of the wave-motion should be observed when a source of sound or light was moving, and it is accordingly known as the *Doppler effect.*

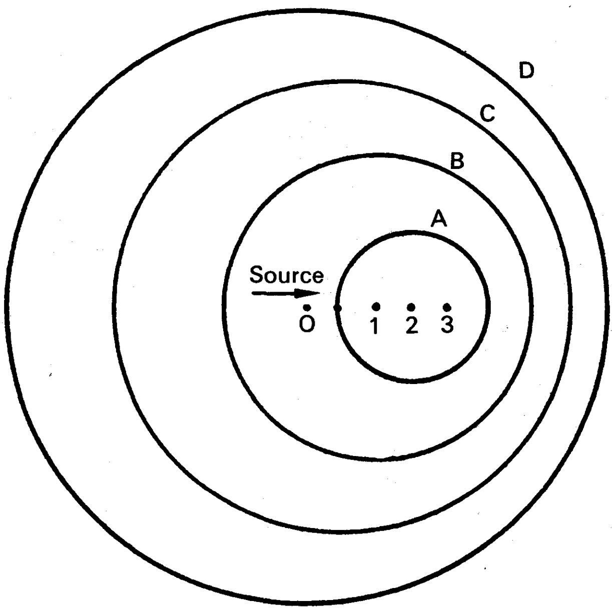

FIG. 26.13. Doppler effect.

The Doppler effect occurs whenever there is a *relative velocity* between the source of sound or light and an observer. In light, this effect was observed when measurements were taken of the wavelength of the colour of a moving star; they showed a marked variation. In sound, the Doppler effect can be demonstrated by placing a whistle in the end of a long piece of rubber tubing, and whirling the tube in a horizontal circle above the head while blowing the whistle. The open end of the tube acts as a moving source of sound, and an observer hears a rise and fall in pitch as the end approaches and recedes from him.

A complete calculation of the apparent frequency in particular cases is given shortly, but Fig. 26.13 illustrates how the change of wavelengths, and hence frequency, occurs when a source of sound is moving towards a stationary observer. At a certain instant the position of the moving source is at 4. At four successive seconds *before* this instant the source had been at the positions 3, 2, 1, 0 respectively. If V is the velocity of sound, the wavefront from the source when in the position 3 reaches the surface A of a sphere of radius V and centre 3 when the source is just at 4. In the same way, the wavefront from the source when it was in the position 2 reaches the surface B of a sphere of radius 2V and centre 2. The wavefront C corresponds to the source when it was in the position 1, and the wavefront D to the source when it was in the position O. Thus if the observer is on the right of the source S, he receives wavefronts which are relatively more crowded together than if S were stationary; the frequency of S thus appears to increase. When the observer is on the left of S, in which case the source is moving away from him, the wavefronts are farther apart than if S were stationary, and hence the observer receives correspondingly fewer waves per second. The apparent frequency is thus lowered.

Calculation of Apparent Frequency

Suppose V is the velocity of sound in air, u_s is the velocity of the source of sound S, u_o is the velocity of an observer O, and f is the true frequency of the source.

(i) *Source moving towards stationary observer.* If the source S were stationary, the f waves sent out in one second towards the observer O

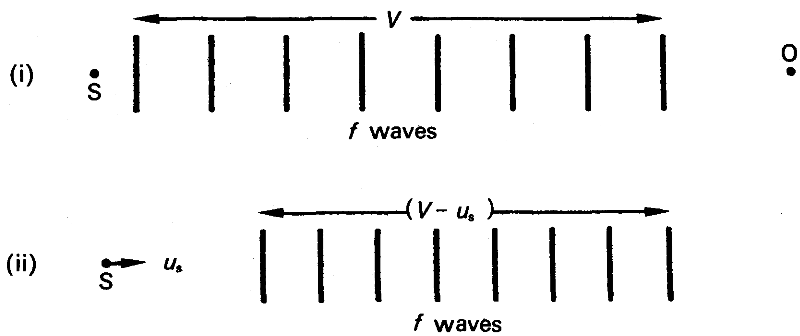

FIG. 26.14. Source moving towards stationary observer.

would occupy a distance V, and the wave length would be V/f, Fig. 26.14. (i). If S moves with a velocity u_s towards O, however, the f waves sent out occupy a distance $(V - u_s)$, because S has moved a distance u_s towards O in 1 sec, Fig. 26.14 (ii). Thus the wavelength λ' of the waves reaching O is now $(V - u_s)/f$.

But velocity of sound waves = V.

$$\therefore \text{ apparent frequency, } f' = \frac{V}{\lambda'} = \frac{V}{(V - u_s)/f}$$

$$\therefore \quad f' = \frac{V}{V - u_s} f \quad . \quad . \quad . \quad (3)$$

Since $(V - u_s)$ is less than V, f' is greater than f; the apparent frequency thus appears to increase when a source is moving towards an observer.

(ii) *Source moving away from stationary observer.* In this case the f waves sent out towards O in 1 sec occupy a distance $(V + u_s)$, Fig. 26.15.

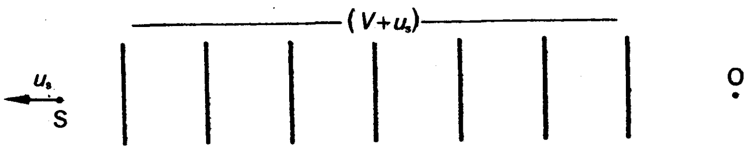

FIG. 26.15. Source moving away from stationary observer.

The wavelength λ' of the waves reaching O is thus $(V + u_s)/f$, and hence the apparent frequency f' is given by

$$f' = \frac{V}{\lambda'} = \frac{V}{(V + u_s)/f}$$

$$\therefore \quad f' = \frac{V}{V + u_s} \cdot f \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Since $(V + u_s)$ is greater than V, f' is less than f, and hence the apparent frequency decreases when a source moves away from an observer.

(iii) *Source stationary, and observer moving towards it.* Since the source is stationary, the f waves sent out by S towards the moving observer O occupies a distance V, Fig. 26.16. The wavelength of the waves reaching O is hence V/f, and thus unlike the cases already considered, the wavelength is unaltered.

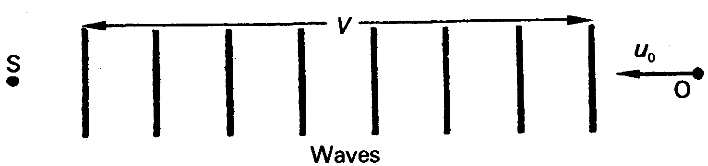

FIG. 26.16. Observer moving towards stationary source.

The velocity of the sound waves relative to O is not V, however, as O is moving relative to the source. The velocity of the sound waves relative to O is given by $(V + u_o)$ in this case, and hence the apparent frequency f' is given by

$$f' = \frac{(V + u_o)}{\text{wavelength}} = \frac{V + u_o}{V/f}$$

$$\therefore \quad f' = \frac{V + u_o}{V} \cdot f \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Since $(V + u_o)$ is greater than V, f' is greater than f; thus the apparent frequency is increased.

(iv) *Source stationary, and observer moving away from it,* Fig. 26.17. As in the case just considered, the wavelength of the waves reaching O is unaltered, and is given by V/f.

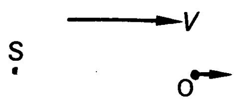

FIG. 26.17. Observer moving away from stationary source.

The velocity of the sound waves relative to O $= V - u_o$, and hence

$$\text{apparent frequency, } f', = \frac{V - u_o}{\text{wavelength}} = \frac{V - u_o}{V/f}$$

$$\therefore \quad f' = \frac{V - u_o}{V} \cdot f \quad . \quad . \quad . \quad . \quad (6)$$

Since $(V - u_o)$ is less than V, the apparent frequency f' appears to be decreased.

Source and Observer Both Moving

If the source and the observer are both moving, the apparent frequency f' can be found from the formula

$$f' = \frac{V'}{\lambda'}$$

where V' is the velocity of the sound waves relative to the observer, and λ' is the wavelength of the waves reaching the observer. This formula can also be used to find the apparent frequency in any of the cases considered before.

Suppose that the observer has a velocity u_o, the source a velocity u_s, and that both are moving in the *same* direction. Then

$$V' = V - u_o$$

and

$$\lambda' = (V - u_s)/f$$

as was deduced in case (i), p. 628.

$$\therefore \quad f' = \frac{V'}{\lambda'} = \frac{V - u_o}{(V - u_s)/f} = \frac{V - u_o}{V - u_s} \cdot f \quad . \quad . \quad (i)$$

If the observer is moving towards the source, $V' = V + u_o$, and the apparent frequency f' is given by

$$f' = \frac{V + u_o}{V - u_s} \cdot f \quad . \quad . \quad . \quad . \quad (ii)$$

From (i), it follows that $f' = f$ when $u_o = u_s$, in which case there is no relative velocity between the source and the observer. It should also be noted that the motion of the observer affects only V', the velocity of the waves reaching the observer, while the motion of the source affects only λ', the wavelength of the waves reaching the observer.

The effect of the wind can also be taken into account in the Doppler effect. Suppose the velocity of the wind is u_W, in the direction of the line SO joining the source S to the observer O. Since the air has then a velocity u_W relative to the ground, and the velocity of the sound waves relative to the air is V, the velocity of the waves relative to ground is $(V + u_W)$ if the wind is blowing in the same direction as SO. All our previous expressions for f' can now be adjusted by replacing the velocity V in it by $(V + u_W)$. If the wind is blowing in the opposite direction to SO, the velocity V must be replaced by $(V - u_W)$.

When the source is moving at an angle to the line joining the source and observer, the apparent frequency changes continuously. Suppose the source is moving along AB with a velocity v, while the observer is stationary at O, Fig. 229. At S, the component velocity of v along OS is $v \cos \theta$, and is *towards* O. The observer thus hears a note of higher pitch whose frequency f' is given by

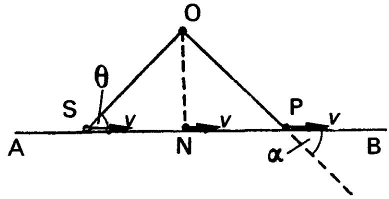

FIG. 26.18. Source direction perpendicular to observer.

$$f' = \frac{V}{V - v\cos\theta} f,$$

where V is the velocity of sound and f is the frequency of the source of sound. See equation (3), in which u_s now becomes $v \cos \theta$. When the source reaches P, Fig. 26.18, the component of v is $v \cos \alpha$ *away* from O, and the apparent frequency f'' is given by

$$f'' = \frac{V}{V + v\cos\alpha} f,$$

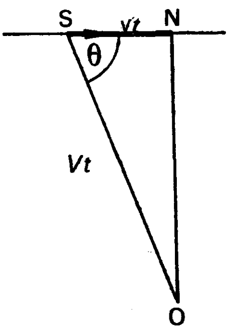

FIG. 26.19. Frequency heard when source at N.

from equation (4). The apparent frequency is thus lower than the frequency f of the source. When the source reaches N, the foot of the perpendicular from O to AB, the velocity v is perpendicular to ON and has thus no component towards the observer O. If the waves reach O shortly after, the observer hears a note of the same frequency f as the source.

Before the source S reaches N, however, it emits waves, travelling with a velocity V in air which reach O, Fig. 26.19. If S reaches N at the same instant as the waves reach O, the observer hears the note corresponding to the instant when the source was at S. In this case SN = vt and SO = Vt, where t is the time-interval concerned. Thus:

$$\cos\theta = \frac{vt}{Vt} = \frac{v}{V}.$$

The frequency f' of the note heard by O when S just reaches N is hence given by

$$f' = \frac{V}{V - v\cos\theta} \cdot f = \frac{V}{V - v^2/V} \cdot f = \frac{V^2}{V^2 - v^2} \cdot f$$

Doppler's Principle in Light

The speed of distant stars and planets has been estimated from measurements of the wavelengths of the spectrum lines which they emit. Suppose a star or planet is moving with a velocity v away from the earth and emits light of wavelength λ. If the frequency of the vibrations is f cycles per second, then f waves are emitted in one second, where $c = f\lambda$ and c is the velocity of light *in vacuo*. Owing to the velocity v, the f waves occupy a distance $(c + v)$. Thus the *apparent wavelength* λ' to an observer on the earth in line with the star's motion is

$$\lambda' = \frac{c+v}{f} = \frac{c+v}{c} \cdot \lambda = \left(1 + \frac{v}{c}\right)\lambda$$

$$\therefore \quad \lambda' - \lambda = \text{"shift" in wavelength} = \frac{v}{c}\lambda, \quad . \quad . \quad \text{(i)}$$

and hence $$\frac{\lambda' - \lambda}{\lambda} = \text{fractional change in wavelength} = \frac{v}{c}. \quad \text{(ii)}$$

From (i), it follows that λ' is greater than λ when the star or planet is moving away from the earth, that is, there is a "shift" or displacement *towards the red*. The position of a particular wavelength in the spectrum of the star is compared with that obtained in the laboratory, and the difference in the wavelengths, $\lambda' - \lambda$, is measured. From (i), knowing λ and c, the velocity v can be calculated.

If the star is moving *towards* the earth with a velocity u, the apparent wavelength λ'' is given by

$$\lambda'' = \frac{c-u}{f} = \frac{c-u}{c} \cdot \lambda = \left(1 - \frac{u}{c}\right)\lambda.$$

$$\therefore \quad \lambda - \lambda'' = \frac{u}{c}\lambda.$$

Since λ'' is less than λ, there is a displacement towards the blue in this case.

In measuring the speed of a star, a photograph of its spectrum is taken. The spectral lines are then compared with the same lines obtained by photographing in the laboratory an arc or spark spectrum of an element present in the star. If the former are displaced towards the red, the star is receding from the earth; if it is displaced towards the violet, the star is approaching the earth. By this method the velocities of the stars have been found to be between about 10 km s^{-1} and 300 km s^{-1}. The Doppler effect has also been used to measure the speed of rotation of the sun. Photographs are taken of the east and west edges of the sun; each contains absorption lines due to elements such as iron vaporised in the sun, and also some absorption lines due to oxygen in the earth's atmosphere. When the two photographs are put together so that the oxygen lines coincide, the iron lines in the two photographs are displaced relative to each other. In one case the edge of the sun approaches the earth, and in the other the opposite edge recedes from the earth. Measurements show a rotational speed of about 2 km s^{-1}.

Measurement of Plasma Temperature

In very hot gases or plasma, used in thermonuclear fusion experiments, the temperature is of the order of millions of degrees Celsius. At these high temperatures molecules of the glowing gas are moving away and towards the observer with very high speeds and, owing to the Doppler effect, the wavelength λ of a particular spectral line is apparently changed. One edge of the line now corresponds to an apparently increased wavelength λ_1 due to molecules moving directly towards the observer, and the other edge to an apparent decreased wavelength λ_2 due to molecules moving directly away from the observer. The line is thus observed to be *broadened*.

From our previous discussion, if v is the velocity of the molecules,

$$\lambda_1 = \frac{c + v}{c} \cdot \lambda$$

and

$$\lambda_2 = \frac{c - v}{c} \cdot \lambda$$

$$\therefore \text{ breadth of line, } \lambda_1 - \lambda_2 = \frac{2v}{c} \cdot \lambda \qquad \text{(i)}$$

The breadth of the line can be measured by a diffraction grating, and as λ and c are known, the velocity v can be calculated. By the kinetic theory of gases, the velocity v of the molecules is roughly the root-mean-square velocity, or $\sqrt{3RT}$, where T is the absolute temperature and R is the gas constant per gram of the gas. Consequently T can be found.

Doppler Effect and Radio Waves

A radio wave is an electromagnetic wave, like light, and travels with the same velocity, c, in free space of $3{\cdot}0 \times 10^5$ km s^{-1}. The Doppler effect with radio waves can be utilised for finding the speed of aeroplanes and satellites.

As an illustration, suppose an aircraft C sends out two radio beams at a frequency of 10^{10} Hz; one in a forward direction, and the other in a backward direction, each beam being inclined downward at an angle of 30° to the horizontal, Fig. 26.20. A Doppler effect is obtained when the radio waves are scattered at the ground at A, B, and when the

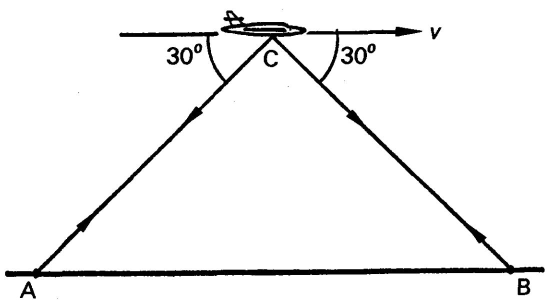

FIG. 26.20. Doppler effect and radio waves.

returning waves to C are combined, a beat frequency equal to their difference is measured. Suppose the beat frequency is 3×10^4 Hz.

If the velocity of the aircraft C is v, the velocity of radio waves is c and the frequency of the emitted beams is f, the apparent frequency f' of the waves reaching A is given by

$$f' = \frac{c}{c + v\cos\theta} \cdot f, \quad \ldots \quad \text{(i)}$$

where θ is 30°. The frequency f_1 of the wave received back at C from A is given by

$$f_1 = \frac{V'}{\lambda'}$$

where V' is the velocity of the wave relative to C and λ' is the wavelength of the waves reaching C. Since $V' = c - v\cos\theta$ and $\lambda' = c/f'$,

$$\therefore \quad f_1 = \frac{c - v\cos\theta}{c} f' = \frac{c - v\cos\theta}{c + v\cos\theta} \cdot f, \quad \text{(ii)}$$

from (i). Similarly, the frequency f_2 of the waves received back at C from B is given by

$$f_2 = \frac{c + v\cos\theta}{c - v\cos\theta} \cdot f \quad \ldots \quad \text{(iii)}$$

$$\therefore \quad \text{beat frequency at C} = f_2 - f_1 = \frac{4cv\cos\theta}{c^2 - v^2\cos^2\theta} \cdot f$$

Now $c = 3 \times 10^5$ km s^{-1}, $\theta = 30°$, $f_2 - f_1 = 3 \times 10^4$ Hz, $f = 10^{10}$ Hz, and $v^2\cos^2\theta$ is negligible compared with c^2.

$$\therefore \quad 3 \times 10^4 = \frac{4cv\cos\theta}{c^2} \cdot f = \frac{4v\cos\theta}{c} \cdot f$$

$$\therefore \quad v = \frac{3 \times 10^4 \times 3 \times 10^5}{4\cos 30° \times 10^{10}}$$

$$= 0{\cdot}26 \text{ km s}^{-1}$$

$$= 936 \text{ km h}^{-1} \text{ (approx.)}$$

The speed of the aircraft relative to the ground is thus nearly 940 km h^{-1}

EXAMPLES

1. Obtain the formula for the Doppler effect when the source is moving with respect to a stationary observer. Give examples of the effect in sound and light. A whistle giving out 500 Hz moves away from a stationary observer in a direction towards and perpendicular to a flat wall with a velocity of 1·5 m s^{-1}. How many beats per sec will be heard by the observer? [Take the velocity of sound as 336 m s^{-1} and assume there is no wind.] (*C.*)

First part. See text.

Second part. The observer hears a note of apparent frequency f' from the whistle directly, and a note of apparent frequency f'' from the sound waves reflected from the wall.

Now $$f' = \frac{V'}{\lambda'}$$

where V' is the velocity of sound in air relative to the observer and λ' is the wavelength of the waves reaching the observer. Since $V' = 336 \text{ m s}^{-1}$ and

$$\lambda' = \frac{336 + 1{\cdot}5}{500} \text{ m}$$

$$\therefore f' = \frac{336 \times 500}{337{\cdot}5} = 497{\cdot}8 \text{ Hz}$$

The note of apparent frequency f'' is due to sound waves moving towards the observer with a velocity of $1{\cdot}5 \text{ m s}^{-1}$

$$\therefore \quad f'' = \frac{V'}{\lambda'} = \frac{336}{(336 - 1{\cdot}5)/500}$$

$$= \frac{336 \times 500}{334{\cdot}5} = 502{\cdot}2 \text{ Hz}$$

$$\therefore \text{ beats per second} = f'' - f' = 502{\cdot}2 - 497{\cdot}8 = 4{\cdot}4$$

2. Two observers A and B are provided with sources of sound of frequency 500. A remains stationary and B moves away from him at a velocity of $1{\cdot}8 \text{m s}^{-1}$ How many beats per sec are observed by A and by B, the velocity of sound being 330 m s^{-1}? Explain the principles involved in the solution of this problem. (*L.*)

Beats observed by A. A hears a note of frequency 500 due to its own source of sound. He also hears a note of apparent frequency f' due to the moving source B. With the usual notation,

$$f' = \frac{V'}{\lambda'} = \frac{330}{(330 + 1{\cdot}8)/500}$$

since the velocity of sound, V', relative to A is 330 m s^{-1} and the wavelength λ' of the waves reaching him is $(330 + 1{\cdot}8)/500$ m.

$$\therefore \quad f' = \frac{330 \times 500}{331{\cdot}8} = 497{\cdot}3$$

$$\therefore \quad \text{beats observed by A} = 500 - 497{\cdot}29 = 2{\cdot}71 \text{ Hz.}$$

Beats observed by B. The apparent frequency f' of the sound from A is given by

$$f' = \frac{V'}{\lambda'}.$$

In this case V' = velocity of sound relative to B $= 330 - 1{\cdot}8 = 328{\cdot}2 \text{ m s}^{-1}$ and the wavelength λ' of the waves reaching B is unaltered. Since $\lambda' = 330/500$ m, it follows that

$$f' = \frac{328{\cdot}2}{330/500} = \frac{328{\cdot}2 \times 500}{330} = 497{\cdot}27$$

$$\therefore \quad \text{beats heard by B} = 500 - 497{\cdot}27 = 2{\cdot}73 \text{ Hz}$$

EXERCISES 26

1. If the velocity of sound in air at 15°C is 342 metres per second calculate the velocity at (*a*) 0°C, (*b*) 47°C. What is the velocity if the pressure of the air changes from 76 cm to 75 cm mercury, the temperature remaining constant at 15°C?

2. Describe a determination (other than resonance) of the velocity of sound in air. How is the velocity dependent upon atmospheric conditions? Give Newton's expression for the velocity of sound in a gas, and Laplace's correction. Hence calculate the velocity of sound in air at 27° C. (Density of air at S.T.P. = 1·29 kg m^{-3}; C_p = 1·02 kJ kg^{-1} K^{-1}; C_v = 0·72 kJ kg^{-1} K^{-1}.) (*L.*)

3. Describe the factors on which the velocity of sound in a gas depends. A man standing at one end of a closed corridor 57 m long blew a short blast on a whistle. He found that the time from the blast to the sixth echo was two seconds. If the temperature was 17° C, what was the velocity of sound at 0° C? (*C.*)

4. Describe an experiment to find the velocity of sound in air at room temperature.

A ship at sea sends out simultaneously a wireless signal above the water and a sound signal through the water, the temperature of the water being 4° C. These signals are received by two stations, *A* and *B*, 40 km apart, the intervals between the arrival of the two signals being 16½ s at *A* and 22 s at *B*. Find the bearing from *A* of the ship relative to *AB*. The velocity of sound in water at t° C m s^{-1} = 1427 + 3·3t. (*N.*)

5. Write down an expression for the speed of sound in an ideal gas. Give a consistent set of units for the quantities involved.

Discuss the effect of changes of pressure and temperature on the speed of sound in air.

Describe an experimental method for finding a *reliable* value for the speed of sound in free air. (*N.*)

6. Describe an experiment to measure the velocity of sound in the open air. What factors may affect the value obtained and in what way may they do so?

It is noticed that a sharp tap made in front of a flight of stone steps gives rise to a ringing sound. Explain this and, assuming that each step is 0·25 m deep, estimate the frequency of the sound. (The velocity of sound may be taken to be 340 m s^{-1}.) (*L.*)

7. Explain why sounds are heard very clearly at great distances from the source (*a*) on still mornings after a clear night, and (*b*) when the wind is blowing from the source to the observer. (*W.*)

8. Describe one or two experiments to test each of the following statements: (*a*) If two notes are recognised by ear to be of the same pitch their sources are making the same number of vibrations per sec. (*b*) The musical interval between two notes is determined by the ratio of the frequencies of the vibrating sources of the notes. (*L.*)

9. Give a brief account of any important and characteristic wave phenomena which occur in sound. Why are sound waves in air regarded as longitudinal and not transverse?

An observer looking due north sees the flash of a gun 4 seconds before he records the arrival of the sound. If the temperature is 20° C and the wind is blowing from east to west with a velocity of 48 km per hour, calculate the distance between the observer and the gun. The velocity of sound in air at 0° C is 330 m s^{-1}. Why does the velocity of sound in air depend upon the temperature but not upon the pressure? (*N.*)

10. Explain upon what properties and conditions of a gas the velocity of sound through it depends.

Describe, and explain in detail, a laboratory method of measuring the velocity of sound in air. (*L.*)

Beats

11. Explain how *beats* are produced by two notes sounding together and obtain an expression for the number of beats heard per second.

A whistle of frequency 1000 Hz is sounded on a car travelling towards a cliff with a velocity of 18 $m s^{-1}$, normal to the cliff. Find the apparent frequency of the echo as heard by the car driver. Derive any relations used. (Assume velocity of sound in air to be 330 $m s^{-1}$.) (*L.*)

12. What is meant by (*a*) the *amplitude*, (*b*) the *frequency* of a vibration in the atmosphere? What are the corresponding characteristics of the musical sound associated with the vibration? How would you account for the difference in quality between two notes of the same pitch produced by two different instruments, e.g., by a violin and by an organ pipe?

What are 'beats'? Given a set of standard forks of frequencies 256, 264, 272, 280, and 288, and a tuning-fork whose frequency is known to be between 256 and 288, how would you determine its frequency to four significant figures? (*W.*)

13. Explain the origin of the beats heard when two tuning-forks of slightly different frequency are sounded together. Deduce the relation between the frequency of the beats and the difference in frequency of the forks. How would you determine which fork had the higher frequency?

A simple pendulum set up to swing in front of the 'seconds' pendulum ($T = 2$ s) of a clock is seen to gain so that the two swing in phase at intervals of 21 s. What is the time of swing of the simple pendulum? (*L.*)

Doppler's Principle

14. An observer beside a railway line determines the speed of a train by observing the change in frequency of the note of its whistle as it passes him. Explain why a change of frequency occurs and derive the relation from which the speed may be calculated. Describe an example of the same principle in another branch of physics.

Find the lowest velocity that can be measured in this way, if the true frequency of the whistle is 1000 Hz and the observer is unable to detect departures from this frequency of less than 20 Hz. (Assume the velocity of sound to be 340 $m s^{-1}$.) (*L.*)

15. Explain what is meant by the *Doppler effect* in sound. Does an observer hear the same pitch from a given source of sound irrespective of whether the source approaches the stationary observer at a certain velocity or the observer approaches the stationary source at the same velocity? Explain how you arrived at your answer.

The light of the H (calcium) line of the spectrum is deviated through an angle of 45° 12′ by a certain prism. When observed in the light of a distant nebula, the deviation is 44° 15′. Calculate the velocity of the nebula in the line of sight, taking the velocity of light in vacuo to be $3{\cdot}00 \times 10^8$ $m s^{-1}$ and the deviation to be inversely proportional to the wavelength of the light over the range of values to be considered. (*L.*)

16. Explain in each case the change in the apparent frequency of a note brought about by the motion of (i) the source, (ii) the observer, relative to the transmitting medium.

Derive expressions for the ratio of the apparent to the real frequency in the cases where (*a*) the source, (*b*) the observer, is at rest, while the other is moving along the line joining them.

The locomotive of a train approaching a tunnel in a cliff face at 95 km.p.h. is sounding a whistle of frequency 1000 Hz. What will be the apparent frequency of the echo from the cliff face heard by the driver? What would be the apparent frequency of the echo if the train were emerging from the tunnel at the same speed? (Take the velocity of sound in air as 330 m s^{-1}.) (*L.*)

17. (*a*) State the conditions necessary for 'beats' to be heard and derive an expression for their frequency.

(*b*) A fixed source generates sound waves which travel with a speed of 330 m s^{-1}. They are found by a distant stationary observer to have a frequency of 500 Hz. What is the wavelength of the waves? From first principles find (i) the wavelength of the waves in the direction of the observer, and (ii) the frequency of the sound heard if (1) the source is moving towards the stationary observer with a speed of 30 m s^{-1}, (2) the observer is moving towards the stationary source with a speed of 30 m s^{-1}, (3) both source and observer move with a speed of 30 m s^{-1} and approach one another. (*N.*)

18. What is the *Doppler effect*? Find an expression for it when the observer is at rest and there is no wind.

A whistle is whirled in a circle of 100 cm radius and traverses the circular path twice per second. An observer is situated outside the circle but in its plane. What is the musical interval between the highest and lowest pitch observed if the velocity of sound is 332 m s^{-1}? (*L.*)

19. Explain why the frequency of a wave motion appears, to a stationary observer, to change as the component of the velocity of the source along the line joining the source and observer changes. Describe two illustrations of this effect, one with sound and one with light.

A stationary observer is standing at a distance l from a straight railway track and a train passes with uniform velocity v sounding a whistle with frequency n_0. Taking the velocity of sound as V, derive a formula giving the observed frequency n as a function of the time. At which position of the train will $n = n_0$? Give a physical interpretation of the result. (*C.*)

Sound Intensity. Acoustics

20. Explain what is meant by (*a*) an intensity level in sound, (*b*) the statement that two intensity levels differ by 5 decibels. What considerations have determined the choice of a zero level in connection with the specification of *loudness*?

A loudspeaker produces a sound intensity level of 8 decibels above a certain reference level at a point P, 40 m from it. Find (*a*) the intensity level at a point 30 m from the loudspeaker, (*b*) the intensity level at *P* if the electrical power to the loudspeaker is halved. (*L.*)

21. Distinguish between the *intensity* and *loudness* of a sound. In what units would the intensity be measured? Define in each instance a unit employed to compare (*a*) the intensity and (*b*) the loudness of two sounds.

A source of sound is situated midway, between an observer and a flat wall. If the absorption coefficient of the wall is 0·25 find the ratio of the intensities of sound heard by the observer directly and by reflexion. Give the answer in decibels. (*L.*)

22. Describe a method for the accurate measurement of the velocity of sound in *free* air.

Indicate the factors which influence the velocity and how they are allowed for or eliminated in the experiment you describe.

At a point 20 m from a small source of sound the intensity is 0·5 microwatt cm^{-2}. Find a value for the rate of emission of sound energy from the source, and state the assumptions you make in your calculation. (*N.*)

23. Distinguish between *intensity* and *intensity level* of a sound.

The time taken for a sound to decay to one-millionth of its previous intensity after the source has been cut off is called the reverberation time. For a pure tone which gives an intensity level of 83 decibels in an empty lecture theatre the reverberation time was found to be 3·8 seconds. Calculate the sound intensity 7·6 seconds after the note was switched off. (Assume that the reference zero of intensity was 10^{-12} watt m^{-2}.)

Explain what was meant by a listener who stated that the note had a loudness of 70 phons.

Discuss how the acoustic properties of this lecture theatre might be improved. (*N.*)

24. (*a*) Discuss the relation between the *intensity level* and the *loudness* of a sound. Define suitable units in which each may be expressed.

(*b*) Give an account of the effect on the acoustics of a concert hall of such factors as: the design and material of the walls: the size of the audience; the frequency of the note. (*L.*)

25. A hall is 25 m long, 8 m wide and has walls 8 m high. The ceiling is a barrel vault of radius 5 m and the ends of the hall are plane. The floor is wood block and the walls are hard plaster, wood panelling and glass. The ceiling is also of hard plaster.

Indicate and give reasons for *three* defects of this hall as an auditorium and show how you would attempt to correct them.

Diagrams are essential in the answer to this question. (*N.*)