# chapter twenty

# Dispersion. Spectra

## Spectrum of White Light

IN 1666, NEWTON made a great scientific discovery. He found that sunlight, or white light, was made up of different colours, consisting of red, orange, yellow, green, blue, indigo, violet. Newton made a small hole in a shutter in a darkened room, and received a white circular patch of sunlight on a screen S in the path of the light, Fig. 20.1 (i). But on interposing a glass prism between the hole and the screen he observed a series of overlapping coloured patches in place of the white patch, the total length of the coloured images being several times their width, Fig. 20.1 (ii). By separating one colour from the rest, Newton demonstrated that the colours themselves could not be changed by refraction through a prism, and he concluded that the colours were not *introduced* by the prism, but were components of the white light. The *spectrum* (colours) of white light consists of red, orange, yellow, green, blue, indigo, and violet, and the separaion of the colours by the prism is known as *dispersion.*

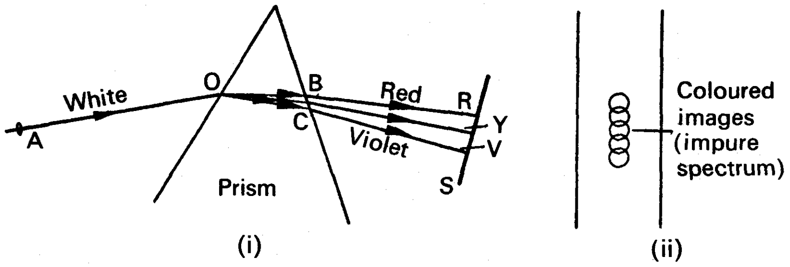

FIG. 20.1. Impure spectrum.

The red rays are the least deviated by the prism, and the violet rays are the most deviated, as shown in the exaggerated sketch of Fig. 20.1 (i). Since the angle of incidence at O in the air is the same for the red and violet rays, and the angle of refraction made by the red ray OB in the glass is greater than that made by the violet ray OC, it follows from sin $i$/sin $r$ that the refractive index of the prism material for red light is less than for violet light. Similarly, the refractive index for yellow light lies between the refractive index values for red and violet light (see also p. 458).

## Production of Pure Spectrum

Newton's spectrum of sunlight is an *impure spectrum* because the different coloured images overlap, Fig. 20.1 (ii). A *pure spectrum* is one in which the different coloured images contain light of one colour only, i.e., they are monochromatic images. In order to obtain a pure spectrum (i) the white light must be admitted through a very narrow opening, so as to assist in the reduction of the overlapping of the images, (ii) the beams of coloured rays emerging from the prism must be parallel, so that each beam can be brought to a separate focus.

The spectrometer can be used to provide a pure spectrum. The collimator slit is made very narrow, and the collimator C and the telescope T are both adjusted for parallel light, Fig. 20.2. A bright source of white light, S, is placed near the slit, and the prism P is usually set in the minimum deviation position for yellow light, although this is not essential. The rays refracted through P are now separated into a number of different coloured parallel beams of light, each travelling in slightly different directions, and the telescope brings each coloured beam to a separate focus. A pure spectrum can now be seen through T, consisting of a series of monochromatic images of the slit.

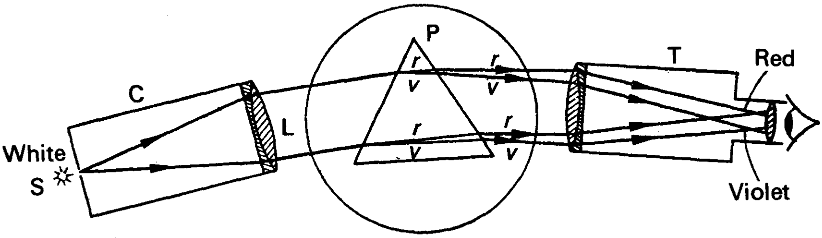

FIG. 20.2. Pure spectrum.

If only one lens, L, is available, the prism P *must* be placed in the minimum deviation position for yellow light in order to obtain a fairly pure spectrum, Fig. 20.3. The prism is then also approximately in the minimum deviation position for the various colours in the incident convergent beam, and hence the rays of one colour are approximately deviated by the same amount by the prism, thus forming an image of the slit S at roughly the same place.

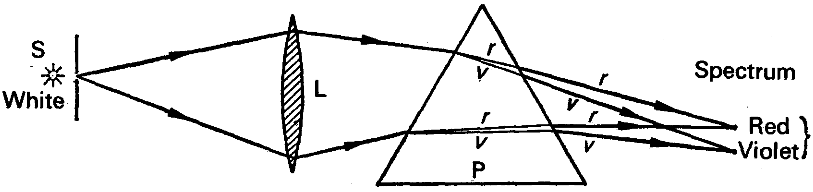

FIG. 20.3. Fairly pure spectrum.

**Infra-red and ultra-violet rays.** In 1800 HERSCHEL discovered the existence of *infra-red rays,* invisible rays beyond the red end of the spectrum. Fundamentally, they are of the same nature as rays in the visible spectrum but having longer wavelengths than the red, and produce a sensation of heat (see p. 344). Their existence may be demonstrated in the laboratory by means of an arc light in place of S in Fig. 20.3, a rocksalt lens at L and a rocksalt prism at P. A phototransistor, such as Mullard OCP71, connected to an amplifier and galvanometer is very sensitive to infra-red light. When this detector is moved into the dark part beyond the red end of the spectrum, a deflection is obtained in the galvanometer. Since they are not scattered by fine particles as much as the rays in the visible spectrum, infra-red rays can penetrate fog and mist. Clear pictures have been taken in mist by using infra-red filters and photographic plates.

About 1801 RITTER discovered the existence of invisible rays beyond the violet end of the visible spectra. *Ultra-violet rays,* as they are known, affect photographic plates and cause certain minerals to fluoresce. They can also eject electrons from metal plates (see *Photoelectric effect,* p. 1077). Ultra-violet rays can be detected in the laboratory by using an arc light in the place of S in Fig. 20.3, a quartz lens at L, and a quartz prism at P. A sensitive detector is a photoelectric cell connected to a galvanometer and battery. When the cell is moved beyond the violet into the dark part of the spectrum a deflection is observed in the galvanometer.

## Deviation Produced by Small-angle Prism for Small Angles of Incidence

Before discussing in detail the colour effect produced when white light is incident on a prism, we must derive an expression for the deviation produced by a *small-angle prism.*

Consider a ray PM of monochromatic light incident almost normally on the face TM of a prism of small angle $A$, so that the angle of incidence, $i_1$, is small, Fig. 20.4. Then $\sin i_1/\sin r_1 = n$, where $r_1$ is the angle of refraction in the prism, and $n$ is the refractive index for the colour of the light. As $r_1$ is less than $i_1$, $r_1$ also is a small angle. Now the sine of a small angle is practically equal to the angle measured in radians. Thus $i_1/r_1 = n$, or

$$i_1 = nr_1 \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

From the geometry of Fig. 20.4, the angle of incidence $r_2$ on the face TN of the prism is given by $r_2 = A - r_1$; and since $A$ and $r_1$ are both

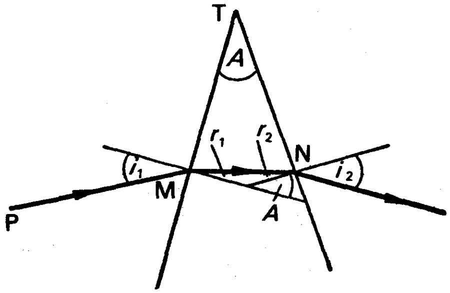

FIG. 20.4. Deviation through small-angle prism.

small, it follows that $r_2$ is a small angle. The angle of emergence $i_2$ is thus also small, and since $\sin i_2/\sin r_2 = n$ we may state that $i_2/r_2 = n$, or

$$i_2 = nr_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

The deviation, $d$, of the ray on passing through the prism is given by $d = (i_1 - r_1) + (i_2 - r_2)$. Substituting for $i_1$ and $i_2$ from (i) and (ii),

$$\therefore \quad d = nr_1 - r_1 + nr_2 - r_2 = n(r_1 + r_2) - (r_1 + r_2)$$

$$\therefore \quad d = (n - 1)(r_1 + r_2)$$

But $\quad r_1 + r_2 = A$

$$\therefore \quad \mathbf{d = (n - 1) A} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This is the magnitude of the deviation produced by a *small*-angle prism for *small* angles of incidence. If $A$ is expressed in radians, then $d$ is in radians; if $A$ is expressed in degrees, then $d$ is in degrees. If $A = 6°$ and $n = 1{\cdot}6$ for yellow light, the deviation $d$ of that colour for small angles of incidence is given by $d = (1{\cdot}6 - 1)\,6° = 3{\cdot}6°$. It will be noted that the deviation is independent of the magnitude of the small angle of incidence on the prism.

## Dispersion by Small-angle Prism

We have already seen from Newton's experiment that the colours in a beam of white light are separated by a glass prism into red, orange, yellow, green, blue, indigo, violet, so that the emergent light is no longer white but coloured. The separation of the colours by the prism is known generally as the phenomenon of *dispersion*, and the *angular dispersion* between the red and blue emergent rays, for example, is defined as the *angle* between these two rays. Thus, in Fig. 20.5, $\theta$ is the angular dispersion between the red and blue rays. Of course, the angular dispersion is also equal to the *difference in deviation* of the two colours produced by the prism; and since we have already derived the expression $d = (n - 1) A$ for the deviation of monochromatic light by a small-angle prism we can obtain the angular dispersion between any two colours.

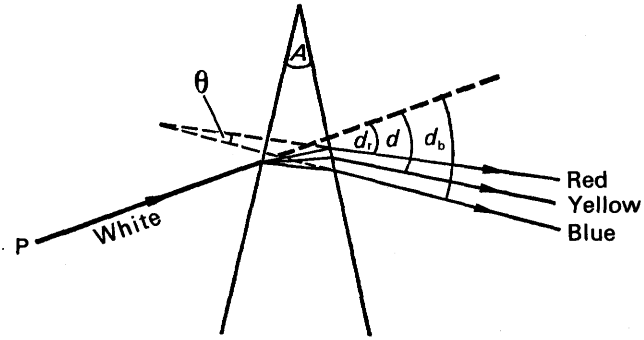

FIG. 20.5. Dispersion.

Suppose $d_b$, $d_r$ are the respective deviations of the blue and red light when a ray of white light is incident at a small angle on a prism of small angle $A$, Fig. 20.5. Then, if $n_b$, $n_r$ are the refractive indices of the prism material for blue and red light respectively,

$$d_b = (n_b - 1)\,A,$$

and

$$d_r = (n_r - 1)\,A.$$

$$\therefore \text{ angular dispersion, } d_b - d_r = (n_b - 1)\,A - (n_r - 1)\,A$$

$$\therefore \quad d_b - d_r = (n_b - \mu_r)\,A \quad . \quad . \quad (2)$$

For a particular crown glass, $n_b = 1{\cdot}521$, $n_r = 1{\cdot}510$. Thus if $A = 8°$, the angular dispersion between the blue and red colours =

$$d_b - d_r = (n_b - n_r)\,A = (1{\cdot}521 - 1{\cdot}510)\,8° = 0{\cdot}09°$$

The *mean deviation* of the white light by the prism is commonly chosen as the deviation of the *yellow* light, since this is the colour approximately in the middle of the spectrum; the mean refractive index of a material is also specified as that for yellow light. Now the deviation, $d$, of monochromatic light is given by $d = (n - 1)\,A$, from equation (1), and unless otherwise stated, the magnitudes of $d$ and $n$ will be understood to be those for yellow light when these symbols contain no suffixes. If $n_b = 1{\cdot}521$ and $n_r = 1{\cdot}510$, then *approximately* the refractive index, $n$, for yellow light is the average of $n_b$ and $n_r$, or $\frac{1}{2}(1{\cdot}521 + 1{\cdot}510)$; thus $n = 1{\cdot}515$. Hence if the prism has an angle of 8°, the mean deviation, $d$, $= (n - 1)\,A = (1{\cdot}515 - 1)\,8° = 4{\cdot}1°$.

## Dispersive Power

The *dispersive power*, $\omega$, of the material of a small-angle prism for blue and red rays may be defined as the ratio

$$\omega = \frac{\textit{angular dispersion between blue and red rays}}{\textit{mean deviation}} \quad . \quad (3)$$

The dispersive power depends on the material of the prism. As an illustration, suppose that a prism of angle 8° is made of glass of a type X, say, and another prism of angle 8° is made of glass of a type Y.

| | $n_b$ | $n_r$ | $n$ |
|---|---|---|---|
| Crown glass, X | 1·521 | 1·510 | 1·515 |
| Flint glass, Y | 1·665 | 1·645 | 1·655 |

Further, suppose the refractive indices of the two materials for blue red, and yellow light are those shown in the above table.

For a small angle of incidence on the prism of glass X, the angular dispersion

$$= d_b - d_r = (n_b - 1)A - (n_r - 1)A$$

$$= (n_b - n_r)A = (1{\cdot}521 - 1{\cdot}510)\,8° = 0{\cdot}09° \quad . \qquad \text{(i)}$$

The mean deviation, $d = (n - 1)A = (1{\cdot}515 - 1)\,8° = 4{\cdot}1°$ . (ii)

$$\therefore \text{ dispersive power, } \omega, = \frac{0{\cdot}09}{4{\cdot}1} = 0{\cdot}021 \quad . \quad . \quad . \quad . \qquad \text{(iii)}$$

Similarly, for the prism of glass Y,

$$\text{angular dispersion} = (n_b - n_r)A = (1{\cdot}665 - 1{\cdot}645)\,8 = 0{\cdot}16°$$

and $\quad$ mean deviation $= (n - 1)A = 1{\cdot}655 - 1)\,8 = 5{\cdot}24°$

$$\therefore \text{ dispersive power} = \frac{0{\cdot}16}{5{\cdot}24} = 0{\cdot}03 \quad . \quad . \quad . \quad . \qquad \text{(iv)}$$

From (iii) and (iv), it follows that the dispersive power of glass Y is about 1·5 times as great as that of glass X.

## General Formula for Dispersive Power

We can now derive a general formula for dispersive power, ω, which is independent of angles. From equation 3, it follows that

$$\omega = \frac{d_b - d_r}{d}$$

as $d_b$, $d_r$, $d$ denote the deviations of blue, red, and yellow light respectively.

But $\quad d_b - d_r = (n_b - 1)A - (n_r - 1)A = (n_b - n_r)A$

and $\quad d = (n - 1)A.$

$$\therefore \quad \omega = \frac{d_b - d_r}{d} = \frac{(n_b - n_r)A}{(n - 1)A}$$

$$\therefore \quad \omega = \frac{n_b - n_r}{n - 1} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

From this formula, it can be seen that (i) ω depends only on the material of the prism and is independent of its angle, (ii) ω is a number and has therefore no units. In contrast to "dispersive power", it should be noted that "dispersion" is an angle, and that its magnitude depends on the angle $A$ of the prism and the two colours concerned, for $d_b - d_r = \text{dispersion} = (n_b - n_r)A$.

## Achromatic Prisms

We have seen that a prism separates the colours in white light. If a prism is required to deviate white light without dispersing it into colours, *two* prisms of different material must be used to eliminate the dispersion, as shown in Fig. 20.6. The prism P is made of crown glass, and causes dispersion between the red and blue in the incident white light. The prism Q is inverted with respect to P, and with a suitable choice of its angle $A'$ (discussed fully later), the red and blue rays incident on it can be made to emerge in *parallel* directions. If the rays are viewed the eye-lens brings them to a focus at the same place on the retina, and hence the colour effect due to red and blue rays is eliminated. The dispersion of the other colours in white light still remains, but most of the colour effect is eliminated as the red and blue rays are the "outside" (extreme) rays in the spectrum of white light.

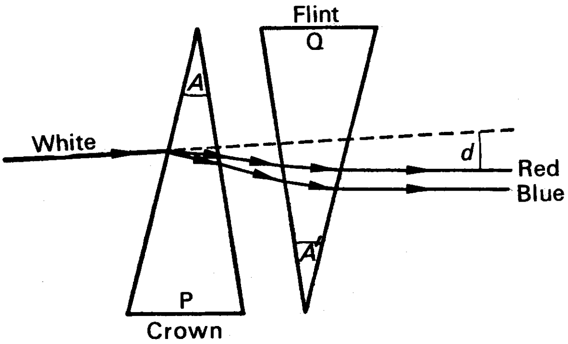

FIG. 20.6. Achromatic prisms.

Prisms which eliminate dispersion between two colours, blue and red say, are said to be *achromatic* prisms for those colours. Suppose $n_b$, $n_r$ are the refractive indices of crown glass for blue and red light, and $A$ is the angle of the crown glass prism P. Then, from p. 458,

$$\text{dispersion} = (n_b - n_r)\,A \quad . \quad . \quad . \quad . \quad \text{(i)}$$

If $n'_b$, $n'_r$ are the refractive indices of flint glass for blue and red light and $A'$ is the angle of the flint glass prism Q, then similarly,

$$\text{dispersion} = (n'_b - n'_r)\,A' \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

Now prism P produces its dispersion in a "downward" direction since a prism bends rays towards its base, Fig. 20.6, and prism Q produces its dispersion in an "upward" direction. For achromatic prisms, therefore, the dispersions produced by P and Q must be equal.

$$\therefore (n_b - n_r)\,A = (n'_b - n'_r)\,A' \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Suppose P has an angle of 6°. Then, using the refractive indices for $n_b, n_r, n'_b, n'_r$ in the table on p. 458, it follows from (5) that the angle A′ is given by

$$(1{\cdot}521 - 1{\cdot}510)\,6° = (1{\cdot}665 - 1{\cdot}645)\,A'$$

Thus $$A' = \frac{0{\cdot}011}{0{\cdot}02} \times 6° = 3{\cdot}3°$$

## Deviation Produced by Achromatic Prisms

Although the colour effects between the red and blue rays are eliminated by the use of achromatic prisms, it should be carefully noted that the incident light beam, as a whole, has been deviated. This angle of deviation, $d$, is shown in Fig. 20.6, and is the angle between the incident and emergent beams. The deviation of the mean or yellow light by prism P is given by $(n - 1)\,A$, and is in a "downward" direction.

Since the deviation of the yellow light by the prism Q is in an opposite direction, and is given by $(n' - 1)A'$, the net deviation, $d$, is given by

$$d = (n - 1)A - (n' - 1)A'.$$

Using the angles 6° and 3·3° obtained above, with $n = 1{\cdot}515$ and $n' = 1{\cdot}655$,

$$d = (1{\cdot}515 - 1)\,6^\circ - (1{\cdot}655 - 1)\,3{\cdot}3^\circ = 0{\cdot}93^\circ.$$

## Direct-vision Spectroscope

The direct-vision spectroscope is a simple instrument used for examining the different colours in the spectrum obtained from a glowing gas in a flame or in a discharge tube. It contains several crown and flint prisms cemented together, and contained in a straight tube having lenses which constitute an eye-piece. The tube is pointed at the source of light examined, when various colours are seen on account of the dispersion produced by the prisms, Fig. 20.7.

In practice, the direct-vision spectroscope contains several crown and flint glass prisms, but for convenience suppose we consider two such prisms, as in Fig. 20.7. For "direct vision", the *net deviation of the mean* (yellow) *ray* produced by the prisms must be zero. Thus the mean deviation caused by the crown glass prism in one direction must be equal to that caused by the flint glass prism in the opposite direction. Hence, with the notation already used, we must have

$$(n - 1)A = (n' - 1)A'.$$

Suppose $A = 6^\circ$, $n = 1{\cdot}515$, $n' = 1{\cdot}655$. Then $A'$ is given by

$$(1{\cdot}515 - 1)\,6^\circ = (1{\cdot}655 - 1)\,A'$$

$$\therefore \qquad A' = \frac{0{\cdot}515}{0{\cdot}655} \times 6^\circ = 4{\cdot}7^\circ$$

The *net dispersion* of the blue and red rays is given by

$$\begin{aligned} &(n_b - n_r)\,A - (n_b' - n_r')\,A' \\ &= (1{\cdot}521 - 1{\cdot}510)\,6^\circ - (1{\cdot}665 - 1{\cdot}645)\,4{\cdot}7^\circ \\ &= 0{\cdot}066 - 0{\cdot}094 = -0{\cdot}028^\circ. \end{aligned}$$

The minus indicates that the net dispersion is produced in a "blue-upward" direction, as the dispersion of the flint glass prism is greater than that of the crown glass prism.

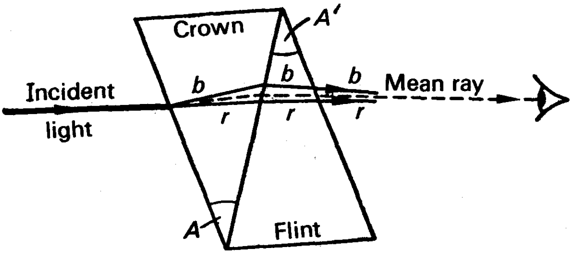

FIG. 20.7. Dispersion; but no deviation of mean ray.

# SPECTRA

## The Importance of the Study of Spectra

The study of the wavelengths of the radiation from a hot body comes under the general heading of *Spectra.* The number of spectra of elements and compounds which have been recorded runs easily into millions, and it is worth while stating at the outset the main reasons for the interest in the phenomenon.

It is now considered that an atom consists of a nucleus of positive electricity surrounded by electrons moving in various orbits, and that a particular electron in an orbit has a definite amount of energy. In certain circumstances the electron may jump from this orbit to another, where it has a smaller amount of energy. When this occurs radiation is emitted, and the energy in the radiation is equal to the difference in energy of the atom between its initial and final states. The displacement of an electron from one orbit to another occurs when a substance is raised to a high temperature, in which case the atoms present collide with each other very violently. Light of a definite wavelength will then be emitted, and will be characteristic of the electron energy changes in the atom. There is usually more than one wavelength in the light from a hot body (iron has more than 4,000 different wavelengths in its spectrum), and each wavelength corresponds to a change in energy between two orbits. A study of spectra should therefore reveal much important information concerning the structure and properties of atoms.

Every element has a unique spectrum. Consequently a study of the spectrum of a substance enables its composition to be readily determined. *Spectroscopy* is the name given to the exact analysis of mixtures or compounds by a study of their spectra, and the science has developed to such an extent that the presence in a substance of less than a millionth of a milligram of sodium can be detected.

**Types of emission spectra.** There are three different types of spectra, which are easily recognised. They are known as (*a*) *line spectra,* (*b*) *band spectra,* (*c*) *continuous spectra.*

(*a*) *Line spectra.* When the light emitted by the atoms of a glowing substance (such as vaporised sodium or helium gas) is examined by a prism and spectrometer, lines of various wavelengths are obtained. These lines, it should be noted, are images of the narrow slit of the spectrometer on which the light is incident. The spectra of hydrogen, Fig. 20.8, and helium are line spectra, and it is generally true that line spectra are obtained from *atoms.*

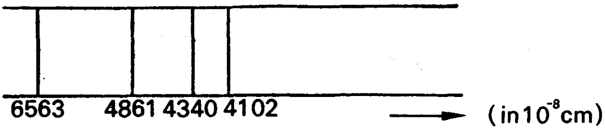

FIG. 20.8. Visible line spectra of hydrogen.

(*b*) *Band spectra.* Band spectra are obtained from *molecules,* and consist of a series of bands each sharp at one end but "fading" at the other end, Fig.

20.9. The term "fluting" is often used to describe the way in which the bands are spaced. Careful examination reveals that the bands are made up of numerous fine lines very close to each other. Two examples of band spectra are those usually obtained from nitrogen and oxygen.

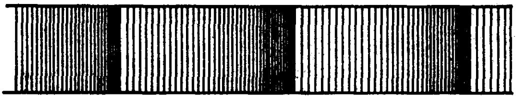

FIG. 20.9. Diagrammatic representation of band spectra.

(*c*) *Continuous spectra.* The spectrum of the sun is an example of a continuous spectrum, and, in general, the latter are obtained from *solids and liquids*. In these states of matter the atoms and molecules are close together, and electron orbital changes in a particular atom are influenced by neighbouring atoms to such an extent that radiations of all different wavelengths are emitted. In a gas the atoms are comparatively far apart, and each atom is uninfluenced by any other. The gas therefore emits radiations of wavelengths which result from orbital changes in the atom due solely to the high temperature of the gas, and a line spectrum is obtained. When the temperature of a gas is decreased and pressure applied so that the liquid state is approached, the line spectrum of the gas is observed to broaden out considerably.

**Production of spectra.** In order to produce its spectrum the substance under examination must be heated to a high temperature. There are four main methods of *excitation*, as the process is called, and spectra are classified under the method of their production.

(*a*) *Flame spectra.* The temperature of a Bunsen flame is high enough to vaporise certain solids. Thus if a piece of platinum wire is dipped into a sodium salt and then placed in the flame, a vivid yellow colour is obtained which is characteristic of the element sodium. This method of excitation can only be used for a limited number of metals, the main class being the alkali and alkaline earth metals such as sodium, potassium, lithium, calcium, and barium. The line spectra produced in each case consist of lines of different colours, but some lines have a greater intensity than others. Thus sodium is characterised by two prominent yellow lines barely distinguishable in a small spectroscope, and lithium by a prominent green line.

(*b*) *Spark spectra.* If metal electrodes are connected to the secondary of an induction coil and placed a few millimetres apart, a spark can be obtained which bridges the gap. It was discovered that a much more intense and violent spark could be obtained by placing a capacitor in parallel with the gap. This spark is known as a *condensed* spark. The solid under investigation forms one of the electrodes, and is vaporised at the high temperature obtained.

(*c*) *Arc spectra.* This is the method most used in industry. If two metal rods connected to a d.c. voltage supply are placed in contact with each other and then drawn a few millimetres apart, a continuous spark, known as an arc, is obtained across the gap. The arc is a source of very high temperature, and therefore vaporises substances very readily. In practice the two rods are placed in a vertical position, and a small amount of the substance investigated is placed on the lower rod.

(*d*) *Discharge-tube spectra.* If a gas is contained at low pressure inside a tube having two aluminium electrodes and a high a.c. or d.c. voltage is applied to the gas, a "discharge" occurs between the electrodes and the gas becomes luminous. This is the most convenient method of examining the spectra of gases. The luminous neon gas in a discharge tube has a reddish colour, while mercury vapour is greenish-blue.

## Absorption Spectra. Kirchhoff's Law

The spectra just discussed are classified as *emission spectra*. There is another class of spectra known as *absorption spectra*, which we shall now briefly consider.

If light from a source having a continuous spectrum is examined after it has passed through a sodium flame, the spectrum is found to be crossed by a dark line; this dark line is in the position corresponding to the bright line emission spectrum obtained with the sodium flame alone. The continuous spectrum with the dark line is naturally characteristic of the absorbing substance, in this case sodium, and it is known as an *absorption spectrum*. An absorption spectrum is obtained when red glass is placed in front of sunlight, as it allows only a narrow band of red rays to be transmitted.

KIRCHHOFF's investigations on absorption spectra in 1855 led him to formulate a simple law concerning the emission and absorption of light by a substance. This states: *A substance which emits light of a certain wavelength at a given temperature can also absorb light of the same wavelength at that temperature.* In other words, a good emitter of a certain wavelength is also a good absorber of that wavelength. From Kirchhoff's law it follows that if the radiation from a hot source emitting a continuous spectrum is passed through a vapour, the absorption spectrum obtained is deficient in those wavelengths which the vapour would emit if it were raised to the same high temperature. Thus if a sodium flame is observed through a spectrometer in a darkened room, a bright yellow line is seen; if a strong white arc light, richer in yellow light than the sodium flame, is placed behind the flame, a dark line is observed in the place of the yellow line. The sodium absorbs yellow light from the white light, and re-radiates it in all directions. Consequently there is less yellow light in front of the sodium flame than if it were removed, and a dark line is thus observed.

## Fraunhofer Lines

In 1814 FRAUNHOFER noticed that the sun's spectrum was crossed by many hundreds of dark lines. These *Fraunhofer lines*, as they are called, were mapped out by him on a chart of wavelengths, and the more prominent were labelled by the letters of the alphabet. Thus the dark line in the blue part of the spectrum was known as the *F* line, the dark line in the yellow part as the *D* line, and the dark line in the red part as the *C* line.

The Fraunhofer lines indicate the presence in the sun's atmosphere of certain elements in a vaporised form. The vapours are cooler than the central hot portion of the sun, and they absorb their own characteristic wavelengths from the sun's continuous spectrum. Now every element has a characteristic spectrum of wavelengths. Accordingly, it became possible to identify the elements round the sun from a study of the wavelengths of the Fraunhofer (dark) lines in the sun's spectrum, and it was then found that hydrogen and helium were present. This was how helium was first discovered. The *D* line is the yellow sodium line.

The incandescent gases round the sun can be seen as flames many miles high during a total eclipse of the sun, when the central portion of the sun is cut off from the observer. If the spectrum of the sun is observed just before an eclipse takes place, a continuous spectrum with Fraunhofer lines is obtained, as already stated. At the instant when the eclipse becomes total, however, bright emission lines are seen in exactly the same position as those previously occupied by the Fraunhofer lines, and they correspond to the emission spectra of the vapours alone. This is an illustration of Kirchhoff's law, p. 464.

## Measurement of Wavelengths by Spectrometer

As we shall discuss later (p. 690) the light waves produced by different colours are characterised by different *wavelengths*. Besides measuring refractive index, the spectrometer can be adapted for measuring unknown wavelengths, corresponding to the lines in the spectrum of a glowing gas in a discharge tube, for example.

A prism is first placed on the spectrometer table in the minimum deviation position for yellow (sodium) light, thus providing a reference position for the prism in relation to incident light from the collimator. The source of yellow light is now replaced by a helium discharge tube, which contains helium at a very low pressure, glowing as a result of the high voltage placed across the tube. Several bright lines of various colours can now be observed through the telescope (they are differently coloured images of the slit), and the *deviation*, $\theta$, of each of the lines is obtained by rotating the telescope until the image is on the cross-wires, and then noting the corresponding reading on the circular graduated scale. Since the wavelengths, $\lambda$, of the various lines in the helium spectrum are known very accurately from tables, a graph can now be plotted between $\theta$ and $\lambda$. The helium discharge tube can then be replaced by a hydrogen or mercury discharge tube, and the deviations due to other lines of known wavelength obtained. In this way a *calibration curve* for the spectrometer can be obtained, Fig. 20.10.

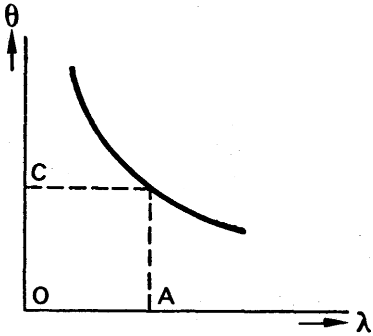

FIG. 20.10. Measurement of wavelength by spectrometer.

The wavelength due to a line *Q* in the spectrum of an unknown glowing gas can now be easily derived. With the prism still in the minimum deviation position for yellow light, the deviation, $\theta$, of Q is measured. If this angle corresponds to C in Fig. 20.10, the wavelength $\lambda$ is OA.

## EXAMPLES

**1.** Show that when a ray of light passes nearly normally through a prism of small angle $\alpha$ and refractive index $n$, the deviation $\delta$ is given by $\delta = (n-1)\alpha$. A parallel beam of light falls normally upon the first face of a prism of small angle. The portion of the beam which is refracted at the second surface is deviated through an angle of 1° 35′, and the portion which is reflected at the second surface and emerges again at the first surface makes an angle of 8° 9′ with the incident beam. Calculate the angle of the prism and the refractive index of the glass. (*C.*)

First part. See text.

Second part. Let $\theta$ = angle of prism, $n$ = the refractive index, and RH the ray incident normally on the face AN, striking the second face at K, Fig. 20.11.

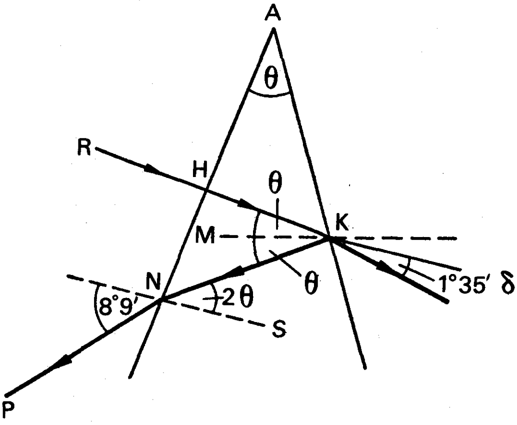

FIG. 20.11. Example.

Then the angle of incidence at K = $\theta$, and angle HKN = $2\theta$. By drawing the normal NS at N, which is parallel to HK, it can be seen that angle KNS = $2\theta$. The angle of emergence from the prism = 8° 9′ since the incident beam was normal to AN.

The angle of deviation, $\delta$, of the beam by the prism is given by

$$\delta = (n-1)\,\theta$$

$$\therefore \quad 1^\circ\,35' = (n-1)\,\theta \qquad \text{(i)}$$

$$\text{For refraction at N, } n = \frac{\sin 8^\circ\,9'}{\sin 2\theta}$$

Since the angles concerned are small,

$$n = \frac{8^\circ\,9'}{2\theta} \qquad \text{(ii)}$$

where $\theta$ is in degrees.

From (ii), $\theta = \dfrac{8^\circ\,9'}{2n}$; substituting in (i),

$$\therefore\quad 1^\circ\,35' = (n-1)\frac{8^\circ\,9'}{2n}$$

$$\therefore\quad \frac{n-1}{2n} = \frac{95}{489}$$

$$\therefore\quad 489n - 489 = 190n$$

$$\therefore\quad n = 1{\cdot}63$$

$$\therefore\quad \theta = \frac{8^\circ\,29'}{2n} = \frac{8^\circ\,29'}{3{\cdot}26} = 2^\circ\,30'$$

**2.** Define dispersive power. The following table gives the refractive indices of crown and flint glass for three lines of the spectrum.

|  | C | D | F |
|---|---|---|---|
| Crown | 1·514 | 1·517 | 1·523 |
| Flint | 1·644 | 1·650 | 1·664 |

Calculate the refracting angle of a flint glass prism which, when combined with a crown glass prism of refracting angle 5°, produces a combination that does not deviate the light corresponding to the D line. What separation of the rays corresponding to the C and F lines will such a compound prism produce? (*L.*)

For definition, see text.

The D line corresponds to the mean, or yellow, ray, the F and C lines to the blue and red rays respectively. Let $n'$, $n$ = the refractive indices for crown and flint glass respectively, $A'$, $A$ = the corresponding angles of the prisms.

For no deviation $(n'_D - 1)A' - (n_D - 1)A = 0$,

$$\therefore\quad (1{\cdot}517 - 1)\,5^\circ - (1{\cdot}650 - 1)A = 0$$

$$\therefore\quad A = \frac{0{\cdot}517}{0{\cdot}650} \times 5 = 3{\cdot}99^\circ$$

The separation of the F and C lines

$$= (n_F - n_C)A - (n'_F - n'_C)A'$$
$$= (1{\cdot}664 - 1{\cdot}644)\,3{\cdot}99^\circ - (1{\cdot}523 - 1{\cdot}514)\,5^\circ$$
$$= 0{\cdot}0798^\circ - 0{\cdot}045^\circ = 0{\cdot}0348^\circ$$

**3.** Prove that for a prism of small angle $A$ the deviation of a ray of light is $(n-1)A$, provided that the angle of incidence also is small. A crown glass prism of refracting angle 6° is to be achromatised for red and blue light with a flint glass prism. Using the data below and the formula above find (*a*) the angle of the flint glass prism, (*b*) the mean deviation.

|  | *Crown glass* | *Flint glass* |
|---|---|---|
| $n$ red | 1·513 | 1·645 |
| $n$ blue | 1·523 | 1·665 |

(*N.*)

First part. See text.

Second part. Let $A$ = the angle of the flint prism, $n'$, $n$ = the refractive indices of the crown and flint glass respectively. For achromatism, $(n'_b - n'_r)\,6^\circ = (n_b - n_r)\,A$

$$\therefore \quad (1{\cdot}523 - 1{\cdot}513)\,6^\circ = 1{\cdot}665 - 1{\cdot}645)\,A$$

$$\therefore \quad A = \frac{0{\cdot}010}{0{\cdot}020} \times 6^\circ = 3^\circ$$

The mean refractive index, $n'$, for crown glass $= \dfrac{1{\cdot}523 + 1{\cdot}513}{2} = 1{\cdot}518$

and mean refractive index $n$, for flint glass $= \dfrac{1{\cdot}665 + 1{\cdot}645}{2} = 1{\cdot}655$

$$\therefore \text{ deviation of mean ray} = (n' - 1)\,6^\circ - (n - 1)\,3^\circ$$
$$= (1{\cdot}518 - 1)\,6^\circ - (1{\cdot}655 - 1)\,3^\circ = 1{\cdot}043^\circ$$

## EXERCISES 20

1. Write down the formula for the deviation of a ray of light through a prism of small angle $A$ which has a refractive index $n$ for the colour concerned. Using the following table, calculate the deviation of (i) red light, (ii) blue light, (iii) yellow light through a flint glass prism of refracting angle 4°, and through a crown glass prism of refracting angle 6°.

|  | *Crown glass* | *Flint glass* |
|---|---|---|
| $n$ red | 1·512 | 1·646 |
| $n$ blue | 1·524 | 1·666 |

2. Using the above data, calculate the *dispersive powers* of crown glass and flint glass.

3. Explain how it is possible with two prisms to produce dispersion without mean deviation. A prism of crown glass with refracting angle of 5° and mean refractive index 1·51 is combined with one flint glass of refractive index 1·65 to produce no mean deviation. Find the angle of the flint glass prism. The difference in the refractive indices of the red and blue rays in crown glass is 0·0085 and in flint glass 0·0162. Find the inclination between the red and blue rays which emerge from the composite prism. (*L.*)

4. Draw a ray diagram showing the passage of light of two different wavelengths through a prism spectrometer. Why is it that such a spectrometer is almost invariably used with (*a*) a very narrow entrance slit, (*b*) parallel light passing through the prism, (*c*) the prism set at, or near, minimum deviation?

A spectrometer is used with a *small angle* prism made from glass which has a refractive index of 1·649 for the blue mercury line and 1·631 for the green mercury line. The collimator lens and the objective of the spectrometer both have a focal length of 30 cm. If the angle of the prism is 0·1 radian what is the spacing of the centres of the blue and green mercury lines in the focal plane of the objective, and what maximum slit width may be used without the lines overlapping? The effect of diffraction need not be considered. (*O. & C.*)

**5.** A glass prism of refracting angle 6·0° and of material of refractive index 1·50 is held with its refracting angle downwards alongside another prism of angle 4·0° which has its refracting angle pointing upwards. A narrow parallel beam of yellow light is incident nearly normally on the first prism, passes through both prisms, and is observed to emerge parallel to its original direction. Calculate the refractive index of the material of the second prism. If white light were used and the glasses of the two prisms were very different in their power to disperse light, describe very briefly what would be seen on a white screen placed at right angles to the emergent light. (*C.*)

**6.** A ray of monochromatic light is incident at an angle $i$ on one face of a prism of refracting angle $A$ of glass of refractive index $n$ and is transmitted. The deviation of the ray is $D$.

Considering only rays incident on the side of the normal away from the refracting angle, sketch graphs on the same set of axes showing how $D$ varies with $i$ when (*a*) $A$ is about 60°, (*b*) $A$ is very small.

From first principles derive an expression for $D$ when $i$ and $A$ are both very small angles. (*N.*)

**7.** Distinguish between emission spectra and absorption spectra. Describe the spectrum of the light emitted by (i) the sun, (ii) a car headlamp fitted with yellow glass, (iii) a sodium vapour street lamp.

What are the approximate wavelength limits of the visible spectrum? How would you demonstrate the existence of radiations whose wavelengths lie just outside these limits? (*O. & C.*)

**8.** State what is meant by *dispersion* and describe, with diagrams, the principle of (i) an achromatic and (ii) a direct-vision prism.

Derive an expression for the refractive index of the glass of a *narrow* angle prism in terms of the angle of minimum deviation and the angle of the prism. If the refractive index of the glass of refracting angle 8° is 1·532 and 1·514 for blue and red light respectively, determine the angular dispersion produced by the prism. (*L.*)

**9.** Describe the processes which lead to the formation of numerous dark lines (Fraunhofer lines) in the solar spectrum. Explain why the positions of these lines in the spectrum differ very slightly when the light is received from opposite ends of an equatorial diameter of the sun. (*N.*)

**10.** Describe with the aid of diagrams what is meant by *dispersion* and *deviation* by a glass prism. Derive a formula for the deviation $D$ produced by a glass prism of small refracting angle $A$ for small angles of incidence. Sketch the graph showing how the deviation varies with angle of incidence for a beam of light striking such a prism, and on the same axes indicate what would happen with a prism of much larger refracting angle but of material of the same index of refraction. (*C.*)

**11.** Describe the optical system of a simple prism spectrometer. Illustrate your answer with a diagram showing the paths through the spectrometer of the pencils of rays which form the red and blue ends of the spectrum of a source of white light. (Assume in your diagram that the lenses are achromatic.)

The prism of a spectrometer has a refracting angle of 60° and is made of glass whose refractive indices for red and violet are respectively 1·514 and 1·530. A white source is used and the instrument is set to give minimum deviation for red light. Determine (*a*) the angle of incidence of the light on the prism, (*b*) the angle of emergence of the violet light, (*c*) the angular width of the spectrum. (*N.*)

**12.** Calculate the angle of a crown glass prism which makes an achromatic combination for red and blue light with a flint glass prism of refracting angle 4°. What is the mean deviation of the light by this combination? Use the data given in question 1.

**13.** Describe and give a diagram of the optical system of a spectrometer. What procedure would you adopt when using the instrument to measure the refractive index of the glass of a prism for sodium light? What additional observations would be necessary in order to determine the dispersive power of the glass?

The refractive index of the glass of a prism for red light is 1·514 and for blue light 1·523. Calculate the difference in the velocities of the red and blue light in the prism if the velocity of light *in vacuo* is $3 \times 10^5$ kilometres per second. (*N.*)

**14.** Explain, with diagrams, how a 'pure' spectrum is produced by means of a spectrometer. What source of light may be used and what readings must be taken in order to find the dispersive power of the material of which the prism is made? (*L.*)

**15.** (*a*) Explain, giving a carefully drawn, labelled diagram, the function of the various parts of a spectrometer. How is it adjusted for normal laboratory use? (*b*) Distinguish between a continuous spectrum, an absorption spectrum, a band spectrum, and a line spectrum. State briefly how you would obtain each type with a spectrometer. (*W.*)

**16.** Describe a prism spectrometer and the adjustment of it necessary for the precise observation of the spectrum of light by a gaseous source.

Compare and contrast briefly the spectrum of sunlight and of light emitted by hydrogen at low pressure contained in a tube through which an electric discharge is passing. (*L.*)