chapter nineteen

Refraction through prisms

IN Light, a *prism* is a transparent object usually made of glass which has two plane surfaces, XDEY, XDFZ, inclined to each other, Fig. 19.1

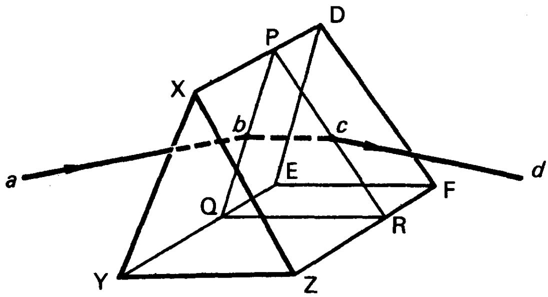

FIG. 19.1. Prism.

Prisms are used in many optical instruments, for example prism binoculars, and they are also utilised for separating the colours of the light emitted by glowing objects, which affords an accurate knowledge of their chemical composition. A prism of glass enables the refractive index of this material to be measured very accurately.

The angle between the inclined plane surfaces XDFZ, XDEY is known as the *angle of the prism*, or the *refracting angle*, the line of intersection XD of the planes is known as the *refracting edge*, and any plane in the prism perpendicular to XD, such as PQR, is known as a *principal section* of the prism. A ray of light *ab*, incident on the prism at *b* in a direction perpendicular to XD, is refracted towards the normal along *bc* when it enters the prism, and is refracted away from the normal along *cd* when it emerges into the air. From the law of refraction (p. 420), the rays *ab*, *bc*, *cd* all lie in the same plane, which is PQR in this case. If the incident ray is directed towards the refracting angle, as in Fig. 19.1, the light is always deviated by the prism towards its base.

Refraction Through a Prism

Consider a ray HM incident in air on a prism of refracting angle A, and suppose the ray lies in the principal section PQR, Fig. 19.2. Then, if i_1, r_1 and i_2, r_2 are the angles of incidence and refraction at M, N as shown, and n is the prism refractive index,

$$\sin i_1 = n \sin r_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

$$\sin i_2 = n \sin r_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

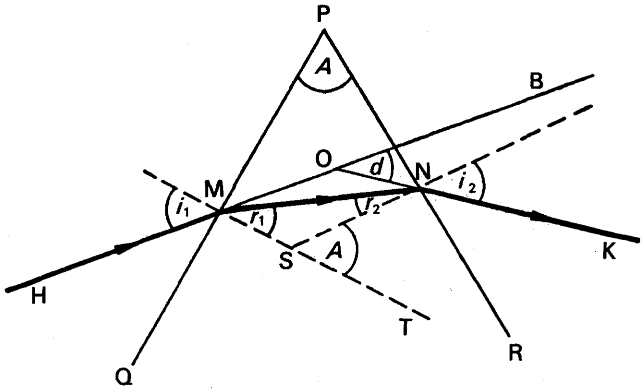

FIG. 19.2. Refraction through prism.

Further, as MS and NS are normals to PM and PN respectively, angle MPN + angle MSN = 180°, considering the quadrilateral PMSN. But angle NST + angle MSN = 180°.

$$\therefore \quad \text{angle NST} = \text{angle MPN} = A$$

$$\therefore \ A = r_1 + r_2 \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

as angle NST is the exterior angle of triangle MSN.

In the following sections, we shall see that the *angle of deviation, d,* of the light, caused by the prism, is utilised considerably. The angle of deviation at M = angle OMN = $i_1 - r_1$; the angle of deviation at N = angle MNO = $i_2 - r_2$. Since the deviations at M, N are in the same direction, the total deviation, d (angle BOK), is given by

$$d = (i_1 - r_1) + (i_2 - r_2) \quad . \quad . \quad . \quad . \quad \text{(iv)}$$

Equations (i) – (iv) are the general relations which hold for refraction through a prism. In deriving them, it should be noted that the geometrical form of the prism base plays no part.

Minimum Deviation

The angle of deviation, d, of the incident ray HM is the angle BOK in Fig. 19.2. The variation of d with the angle of incidence, i, can be obtained experimentally by placing the prism on paper on a drawing board and using a ray AO from a ray-box (or two pins) as the incident ray, Fig. 19.3 (i). When the direction AO is kept constant and the drawing board is turned so that the ray is always incident at O on the prism, the angle of incidence i is varied; the corresponding emergent rays CE, HK, LM, NP can be traced on the paper. Experiment shows that as the angle of incidence i is increased from zero, the deviation d begins to decrease continuously to some value D, and then increases to a maximum as i is increased further to 90°. A *minimum deviation,* corresponding to the emergent ray NP, is thus obtained. A graph of d plotted against i has the appearance of the curve X, which has a minimum value at R, Fig. 19.3 (ii).

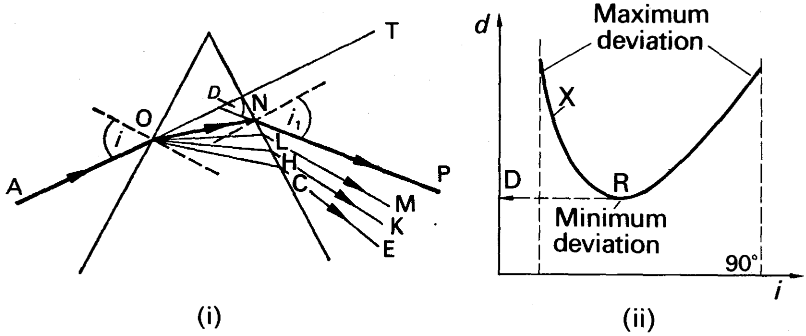

FIG. 19.3. Minimum deviation.

Experiment and theory show that *the minimum deviation, D, of the light occurs when the ray passes symmetrically through the prism.* Suppose this corresponds to the case of the ray AONP in Fig. 19.3 (i). Then the corresponding incident angle, i, is equal to the angle of emergence, i_1, into the air at N for this special case. See also Fig. 19.5 and Fig. 19.9.

A proof of symmetrical passage of ray at minimum deviation. Experiment shows that minimum deviation is obtained at *one* particular angle of incidence. On this assumption it is possible to prove by a *reductio ad absurdum* method that the angle of incidence is equal to the angle of emergence in this case. Thus

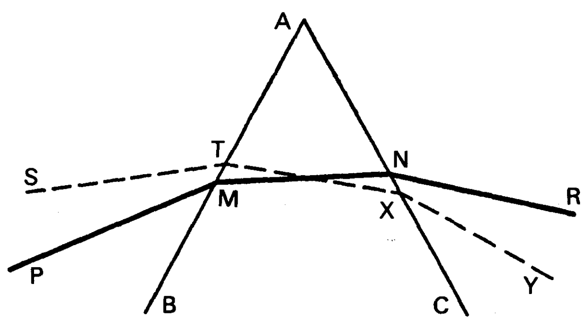

FIG. 19.4. Minimum deviation proof.

suppose that minimum deviation is obtained with a ray PMNR when these angles are *not* equal, so that angle PMB is not equal to angle RNC, Fig. 19.4. It then follows that a ray YX, incident on AC at an angle CXY equal to angle PMB, will emerge along TS, where angle BTS = angle CNR; and from the principle of the reversibility of light, a ray incident along ST on the prism emerges along XY. We therefore have *two* cases of minimum deviation, corresponding to two different angles of incidence. But, from experiment, this is impossible. Consequently our initial assumption must be wrong, and hence the angle of emergence *does* equal the angle of incidence. Thus the ray passes symmetrically through the prism in the minimum deviation case.

Relation Between A, D, and n

A very convenient formula for refractive index, n, can be obtained in the minimum deviation case. The ray PQRS then passes symmetrically through the prism, and the angles made with the normal in the air and in the glass at Q, R respectively are equal, Fig. 19.5. Suppose the angles are denoted by i, r, as shown. Then, as explained on p. 442,

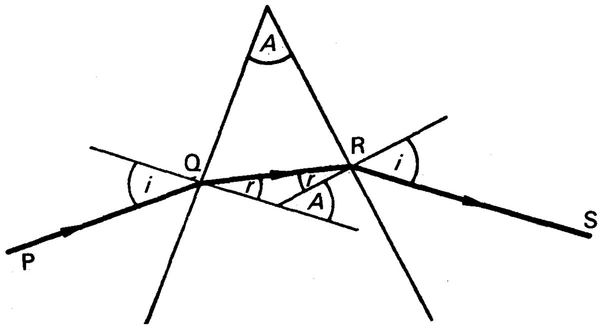

FIG. 19.5. Formula for n of prism.

$$i - r + i - r = D \quad \text{(i)}$$

and

$$r + r = A \quad \text{(ii)}$$

From (ii), $$r = \frac{A}{2}$$

Substituting for r in (i), $$2i = A + D$$

$$\therefore i = \frac{A + D}{2}$$

$$\therefore n = \frac{\sin i}{\sin r} = \frac{\sin \frac{A + D}{2}}{\sin \frac{A}{2}} \quad (1)$$

The Spectrometer

The spectrometer is an optical instrument which is mainly used to study the light from different sources. As we shall see later, it can be used to measure accurately the refractive index of glass in the form of a prism. The instrument consists essentially of a *collimator*, C, a *telescope*, T, and a *table*, R, on which a prism B can be placed. The lenses in C, T are achromatic lenses (p. 515). The collimator is fixed, but the table and the telescope can be rotated round a circular scale graduated in half-degrees (not shown) which has a common vertical axis with the table, Fig. 19.6. A vernier is also provided for this scale. The *source of light*, S, used in the experiment is placed in front of a narrow slit at one end of the collimator, so that the prism is illuminated by light from S.

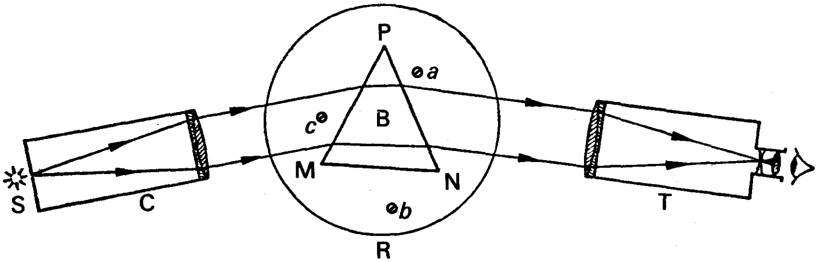

FIG. 19.6. Spectrometer.

Before the spectrometer can be used, however, three adjustments must be made: (1) The collimator C must be adjusted so that parallel light emerges from it; (2) the telescope T must be adjusted so that parallel rays entering it are brought to a focus at cross-wires near its eye-piece; (3) the refracting edge of the prism must be parallel to the axis of rotation of the telescope, i.e., the table must be "levelled".

Adjustments of Spectrometer

The telescope adjustment is made by first moving its eye-piece until the cross-wires are distinctly seen, and then sighting the telescope on to a *distant* object through an open window. The length of the telescope is now altered by a screw arrangement until the object is clearly seen at the same place as the cross-wires, so that parallel rays now entering the telescope are brought to a focus at the cross-wires.

The collimator adjustment. With the prism removed from the table, the telescope is now turned to face the collimator, C, and the slit in C is illuminated by a sodium flame which provides yellow light. The edges of the slit are usually blurred, showing that the light emerging from the lens of C is not a parallel beam. The position of the slit is now adjusted by moving the tube in C, to which the slit is attached, until the edges of the latter are sharp.

"Levelling" the table. If the rectangular slit is not in the centre of the field of view when the prism is placed on the table, the refracting edge of the prism is not parallel to the axis of rotation of the telescope. The table must then be adjusted, or "levelled", by means of the screws *a*, *b*, *c* beneath it. One method of procedure consists of placing the prism on the table with one face MN approximately perpendicular to the line joining two screws *a*, *b*, as shown in Fig. 19.6. The table is turned until MN is illuminated by the light from C, and the telescope T is then moved to receive the light reflected from MN. The screw *b* is then adjusted until the slit appears in the centre of the field of view. With C and T fixed, the table is now rotated until the slit is seen by reflection at the face NP of the prism, and the screw *c* is then adjusted until the slit is again in the middle of the field of view. The screw *c* moves MN in its own plane, and hence the movement of *c* will not upset the adjustment of MN in the perpendicular plane.

Measurement of the Angle, A, of a Prism

The angle of a prism can be measured very accurately by a spectrometer. The refracting edge, P, of the prism is turned so as to face the collimator lens, which then illuminates the two surfaces containing

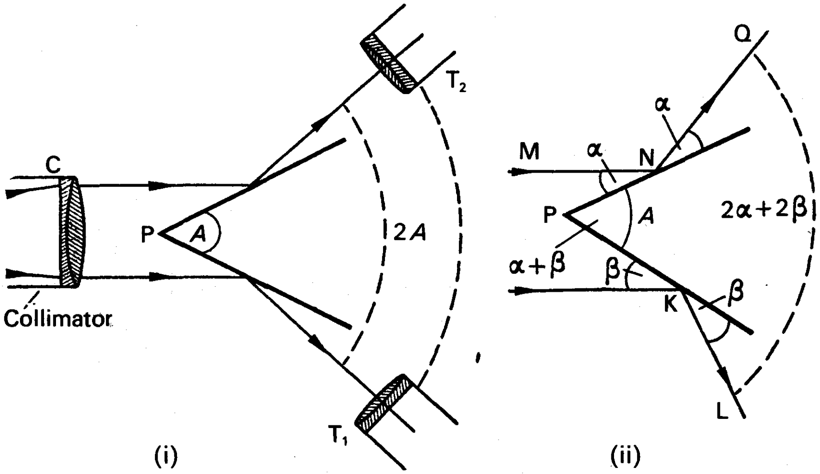

FIG. 19.7. Measurement of angle of prism.

the refracting angle A with parallel light, Fig. 19.7 (i). An image of the collimator slit is hence observed with the telescope in positions T_1, T_2, corresponding to reflection of light at the respective faces of the prism. It is shown below that the angle of rotation of the telescope from T_1 to T_2 is equal to $2A$, and hence the angle of the prism, A, can be obtained.

Proof. Suppose the incident ray MN makes a glancing angle α with one face of the prism, and a parallel ray at K makes a glancing angle β with the other face, Fig. 19.7 (ii). The reflected ray NQ then makes a glancing angle α with the prism surface, and hence the deviation of MN is 2α (see p. 392). Similarly, the deviation by reflection at K is 2β. Thus the reflected rays QN, LK are inclined at an angle equal to $2\alpha + 2\beta$, corresponding to the angle of rotation of the telescope from T_1 to T_2. But the angle, A, of the prism $= \alpha + \beta$, as can be seen by drawing a line through P parallel to MN, and using alternate angles. Hence the rotation of the telescope $= 2\alpha + 2\beta = 2A$.

Measurement of the Minimum Deviation, *D*

In order to measure the minimum deviation, D, caused by refraction through the prism, the latter is placed with its refracting angle A pointing *away* from the collimator, as shown in Fig. 19.8 (i). The telescope is then turned until an image of the slit is obtained on the cross-wires, corresponding to the position T_1. The table is now slowly rotated so that the angle of incidence on the left side of the prism decreases, and the image of the slit is kept on the cross-wires by moving the telescope at the same time. The image of the slit, and the telescope, then slowly approach the fixed line XY. But at one position, corresponding to T_2, the image of the slit begins to move *away* from XY. If the table is now turned in the opposite direction the image of the slit again moves back when the telescope reaches the position T_2. The angle between the emergent ray CH and the line XY is hence the smallest angle of deviation caused by the prism, and is thus equal to D.

The minimum deviation is obtained by finding the angle between the positions of the telescope (i) at T_2, (ii) at T; the prism is removed in the latter case so as to view the slit directly. Alternatively, the experiment to find the minimum deviation is repeated with the refracting angle pointing

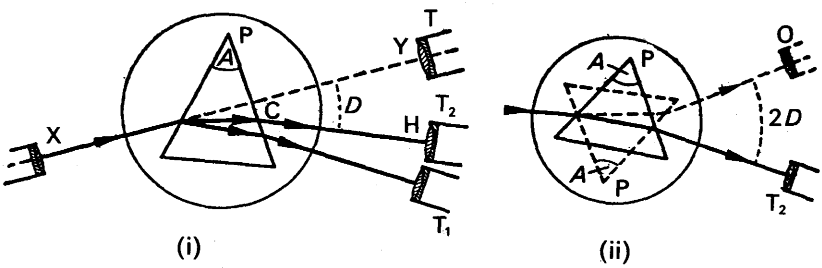

FIG. 19.8. Measurement of minimum deviation.

the opposite way, the prism being represented by dotted lines in this case, Fig. 19.8 (ii). If the position of the telescope for minimum deviation is now O, it can be seen that the angle between the position O and the other minimum deviation position T_2 is $2D$. The value of D is thus easily calculated.

The Refractive Index of the Prism Material

The refractive index, n, of the material of the prism can be easily calculated once A and D have been determined, since, from p. 444.

$$n = \sin\frac{A+D}{2} \Big/ \sin\frac{A}{2}.$$

In an experiment of this nature, the angle, A, of a glass prism was found to be 59° 52′, and the minimum deviation, D, was 40° 30′. Thus

$$n = \sin\frac{59^\circ\ 52' + 40^\circ\ 30'}{2} \Big/ \sin\frac{59^\circ\ 52'}{2}$$

$$= \sin 50^\circ\ 11' / \sin 29^\circ\ 56'$$

$$= 1{\cdot}539$$

The spectrometer prism method of measuring refractive index is capable of providing an accuracy of one part in a thousand. The refractive index of a liquid can also be found by this method, using a hollow glass prism made from thin parallel-sided glass strips.

Grazing Incidence for a Prism

We shall now leave any further considerations of minimum deviation, and shall consider briefly other special cases of refraction through a prism.

When the surface PQ of a prism is illuminated by a source of yellow light placed near Q, the field of view seen through the other surface PR is divided into a bright and dark portion, Fig. 19.9. If NM is the

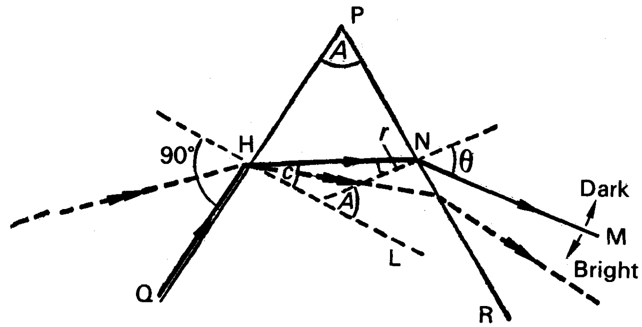

FIG. 19.9. Grazing incidence.

emergent ray corresponding to the incident ray QH which just grazes the prism surface, the dark portion lies above NM and the bright portion exists below NM. The boundary of the light and dark portions is thus NM.

Since the angle of incidence of QH is 90°, angle NHL is equal to c, the critical angle for the glass of the prism. From Fig. 19.9, it follows that $A = c + r$, and hence

$$r = A - c \quad . \quad . \quad . \quad . \quad \text{(i)}$$

Further, for refraction at N,

$$\sin\theta = n \sin r.$$

$$\therefore \sin\theta = n\sin(A - c) = n(\sin A\cos c - \cos A \sin c) \quad . \quad \text{(ii)}$$

But $\sin c = \dfrac{1}{n}$, i.e., $\cos c = \sqrt{1 - \sin^2 c} = \sqrt{1 - \dfrac{1}{n^2}} = \dfrac{1}{n}\sqrt{n^2 - 1}$.

Substituting in (ii) and simplifying, we obtain finally

$$n = \sqrt{1 + \left(\frac{\cos A + \sin\theta}{\sin A}\right)^2}.$$

Thus if A and θ are measured, the refractive index of the prism material can be calculated.

It should be noted that *maximum* deviation by a prism is obtained (i) at grazing incidence, $i = 90°$, (ii) at an angle of incidence θ (Fig. 19.9), corresponding to grazing emergence.

Grazing Incidence and Grazing Emergence

If a ray BM is at grazing incidence on the face of a prism, and the angle A of the prism is increased, a calculation shows that the refracted ray MN in the glass will make a bigger and bigger angle of incidence on the other face PR, Fig. 19.10. This is left as an exercise for the reader. At

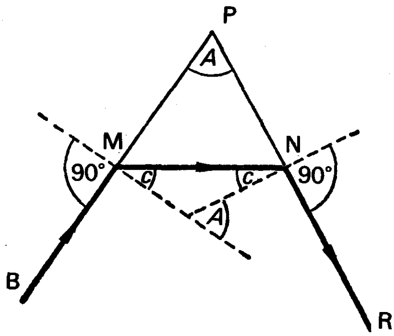

FIG. 19.10. Maximum angle of prism.

a certain value of A, MN will make the critical angle, c, with the normal at N, and the emergent ray NR will then graze the surface PR, as shown in Fig. 19.10. As A is increased further, the rays in the glass strike PR at angles of incidence greater than c, and hence no emergent rays are obtained. Thus Fig. 19.10 illustrates the largest angle of a prism for which emergent rays are obtained, and this is known as the *limiting angle* of the prism. It can be seen from the simple geometry of Fig. 19.10 that $A = c + c$ in this special case, and hence *the limiting angle of a prism is twice the critical angle*. For crown glass of $n = 1{\cdot}51$ the critical angle c is 41° 30′, and hence transmission of light through a prism of crown glass is impossible if the angle of the prism exceeds 83°.

Total Reflecting Prisms

When a plane mirror silvered on the back is used as a reflector, multiple images are obtained (p. 424). This disadvantage is overcome by using right-angled isosceles prisms as reflectors of light in optical instruments such as submarine periscopes (see p. 540).

Consider a ray OQ incident normally on the face AC of such a prism. Fig. 19.11 (i). The ray is undeviated, and is therefore incident at P in the

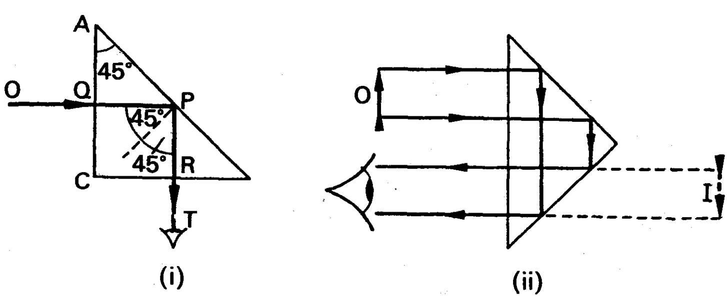

FIG. 19.11. Images in prisms.

glass at an angle of 45° to the normal at P. If the prism is made of crown glass its critical angle is 41° 30′. Hence the incident angle, 45°, in the glass is greater than the critical angle, and consequently the light is *totally* reflected in the glass at P. A bright beam of light thus emerges from the prism along RT, and since the angle of reflection at P is equal to the incident angle, RT is perpendicular to OQ. The prism thus deviates the light through 90°. If the prism is positioned as shown in Fig. 19.11 (ii), an inverted bright virtual image I of the object O is seen by total reflection at the two surfaces of the prism.

There is no loss of brightness when total internal reflection occurs at a surface, whereas the loss may be as much as 10 per cent or more in reflection at a silver surface.

EXAMPLES

1. Describe a good method of measuring the refractive index of a substance such as glass and give the theory of the method. A glass prism of angle 72° and index of refraction 1·66 is immersed in a liquid of refractive index 1·33. What is the angle of minimum deviation for a parallel beam of light passing through the prism? (*L.*)

First part. Spectrometer can be used, p. 444.

Second part.

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\frac{A}{2}}$$

where n is the *relative refractive index* of glass with respect to the liquid.

But

$$n = \frac{1\cdot66}{1\cdot33}$$

$$\therefore \quad \frac{1\cdot66}{1\cdot33} = \frac{\sin\left(\frac{72^\circ+D}{2}\right)}{\sin\frac{72^\circ}{2}} = \frac{\sin\left(\frac{72^\circ+D}{2}\right)}{\sin 36^\circ}$$

$$\therefore \quad \sin\left(\frac{72^\circ+D}{2}\right) = \frac{1\cdot66}{1\cdot33}\sin 36^\circ = 0\cdot7335$$

$$\therefore \quad \frac{72^\circ+D}{2} = 47^\circ\ 11'$$

$$\therefore \quad D = 22^\circ\ 22'$$

2. How would you measure the angle of minimum deviation of a prism? (*a*) Show that the ray of light which enters the first face of a prism at grazing incidence is least likely to suffer total internal reflection at the other face. (*b*) Find the least value of the refracting angle of a prism made of glass of refractive index 7/4 so that no rays incident on one of the faces containing this angle can emerge from the other. (*N.*)

First part. See text.

Second part. (*a*) Suppose PM is a ray which enters the first face of the prism at grazing incidence, i.e., at an angle of incidence of 90°, Fig. 19.12. The refracted ray MQ then makes an angle of refraction c, where c is the critical angle. Suppose QN is the normal at Q on the other face of the prism. Then since angle BNQ $= A$, where A is the angle of the prism, the angle of refraction MQN at Q $= A - c$, from the exterior angle property of triangle MQN. Similarly, if RM is a ray at an angle of incidence i at the first face less than 90°, the angle of refraction at S at the second face $= A - r$, where r is the angle of refraction BMS.

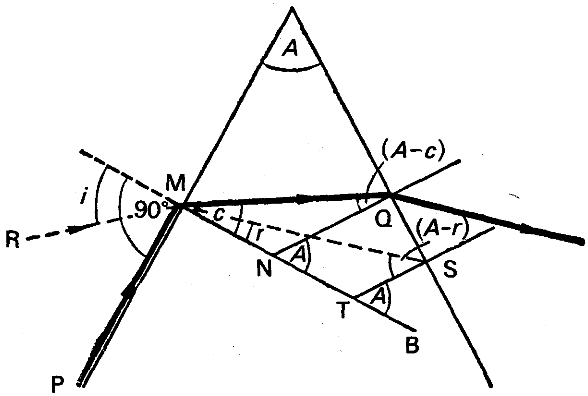

FIG. 19.12. Example

Now c is the *maximum* angle of refraction in the prism.

Hence MQ makes the minimum angle of incidence on the second face, and thus is least likely to suffer total internal reflection.

(*b*) The least value of the refracting angle of the prism corresponds to a ray at grazing incidence and grazing emergence, as shown in Fig. 19.10.

Thus $$\text{minimum angle} = 2c$$
where c is the critical angle (p. 449).

But $$\sin c = \frac{1}{n} = \frac{4}{7} = 0{\cdot}5714$$

$$\therefore \quad c = 34° \, 51'$$

$$\therefore \quad \text{minimum angle} = 2c = 69° \, 42'$$

EXERCISES 19

1. A ray of light is refracted through a prism of angle 70°. If the angle of refraction in the glass at the first face is 28°, what is the angle of incidence in the glass at the second face?

2. (i) The angle of a glass prism is 60°, and the minimum deviation of light through the prism is 39°. Calculate the refractive index of the glass. (ii) The refractive index of a glass prism is 1·66, and the angle of the prism is 60°. Find the minimum deviation.

3. By means of a labelled diagram show the paths of rays from a monochromatic source to the eye through a correctly adjusted prism spectrometer.

Obtain an expression relating the deviation of the beam by the prism to the refracting angle and the angles of incidence and emergence.

A certain prism is found to produce a minimum deviation of 51° 0′, while it produces a deviation of 62° 48′ for two values of the angle of incidence, namely 40° 6′ and 82° 42′ respectively. Determine the refracting angle of the prism, the angle of incidence at minimum deviation and the refractive index of the material of the prism. (*L*).

4. A ray of light passing symmetrically through a glass prism of refracting angle A is deviated through an angle D. Derive an expression for the refractive index of the glass.

A prism of refracting angle about 60° is mounted on a spectrometer table and all the preliminary adjustments are made to the instrument. Describe and explain how you would then proceed to measure the angles A and D.

PQR represents a right-angled isosceles prism of glass of refractive index 1·50. A ray of light enters the prism through the hypotenuse *QR* at an angle of incidence i, and is reflected at the critical angle from *PQ* to *PR*. Calculate and draw a diagram showing the path of the ray through the prism. (Only rays in the plane of *PQR* need be considered.) (*N.*)

5. Give a labelled diagram showing the essential optical parts of a prism spectrometer. Describe the method of adjusting a spectrometer and using it to measure the angle of a prism.

A is the vertex of a triangular glass prism, the angle at A being 30°. A ray of light *OP* is incident at P on one of the faces enclosing the angle A, in a direction such that the angle $OPA = 40°$. Show that, if the refractive index of the glass is 1·50, the ray cannot emerge from the second face. (*L.*)

6. Define refractive index and derive an expression relating the relative refractive index n_{AB} for light travelling out of medium A into medium B with the velocities of light v_A and v_B respectively in those media.

Draw a diagram showing how a parallel beam of monochromatic light is deviated by its passage through a triangular glass prism. Given that the angle of deviation is a minimum when the angles of incidence and emergence are equal show that the refractive index n of the glass is related to the refracting angle a of the prism and the minimum deviation δ by the equation

$$n = \sin \tfrac{1}{2}(a + \delta)/\sin \tfrac{1}{2}a.$$

Describe how you would apply this result to measure the dispersive power of the glass of a given triangular prism. You may assume the availability of sources of light of standard wavelengths. (*O. & C.*)

7. Explain how you would adjust the telescope of a spectrometer before making measurements.

Draw and label a diagram of the optical parts of a prism spectrometer after the adjustments have been completed. Indicate the position of the crosswires and show the paths through the instrument of two rays from a monochromatic source when the setting for minimum deviation has been obtained.

The refracting angle of a prism is 62·0° and the refractive index of the glass for yellow light is 1·65°. What is the smallest possible angle of incidence of a ray of this yellow light which is transmitted without total internal reflection? Explain what happens if white light is used instead, and the angle of incidence is varied in the neighbourhood of this minimum. (*N.*)

8. Explain the meaning of the term *critical angle*. Describe and give the theory of a critical angle method for determining the refractive index of water.

A right-angled prism ABC has angle BAC = angle $ACB = 45°$, and is made of glass of refractive index 1·60. A ray of light is incident upon the hypotenuse face AC so that after refraction it strikes face AB and emerges at minimum deviation. What is the angle of incidence upon AC?

What is the smallest angle of incidence upon AC for which the ray can still emerge at AB? If the angle of incidence upon AC is made zero, what will be the whole deviation of the ray? (*L.*)

9. Draw a labelled diagram of a spectrometer set up for studying the deviation of light through a triangular prism. Describe how you would adjust the instrument and use it to find the refractive index of the prism material.

Indicate briefly how you would show that the radiation from an arc lamp is not confined to the visible spectrum. (*L.*)

10. How would you investigate the way in which the deviation of a ray of light by a triangular glass prism varies with the angle of incidence on the first face of the prism? What result would you expect to obtain?

The deviation of a ray of light incident on the first face of a 60° glass prism at an angle of 45° is 40°. Find the angle which the emergent ray makes with the normal to the second face of the prism and determine, preferably by graphical construction, the refractive index of the glass of the prism. (*L.*)

11. A prism has angles of 45°, 45°, and 90° and all three faces polished. Trace the path of a ray entering one of the smaller faces in a direction parallel to the larger face and perpendicular to the prism edges. Assume 1·5 for the refractive index.

If you had two such prisms how would you determine by a simple pin or ray-box method the refractive index of a liquid available only in small quantity? (*L.*)

12. Under what circumstances does total internal reflection occur? Show that a ray of light incident in a principal section of an equilateral glass prism of refractive index 1·5, can only be transmitted after two refractions at adjacent faces if the angle of incidence on the prism exceeds a certain value. Find this limiting angle of incidence. (*W.*)

13. Draw a graph showing, in a general way, how the deviation of a ray of light when passed through a triangular prism depends on the angle of incidence.

You are required to measure the refractive index of glass in the form of a prism by means of a spectrometer provided with a vertical slit. Explain how you would level the spectrometer table and derive the formula from which you would calculate the refractive index. (You are not required to explain any other adjustment of the apparatus nor to explain how you would find the refracting angle of the prism.) (*L.*)