chapter eighteen

Refraction at plane surfaces

Laws of Refraction

WHEN a ray of light AO is incident at O on the plane surface of a glass medium, observation shows that some of the light is reflected from the surface along OC in accordance with the laws of reflection, while the rest of the light travels along a new direction, OB, in the glass, Fig. 18.1. On account of the change in direction the light is said to be "refracted" on entering the glass; and the *angle of refraction, r*, is the angle made by the refracted ray OB with the normal at O.

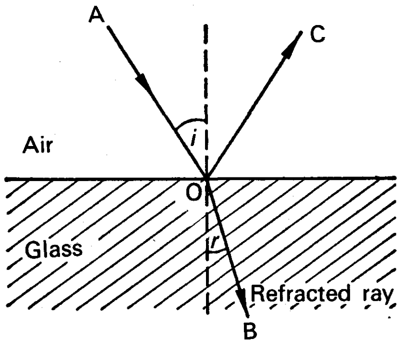


FIG. 18.1. Refraction at plane surface.

Historical records reveal that the astronomer PTOLEMY, who lived about A.D. 140, measured numerous values of the angle of incidence, *i*, and the angle of refraction, *r*, for glass as the angle of incidence was varied. However, he was unable to discover any relation between *i* and *r*. Later scientists were equally unsuccessful, until centuries later SNELL, a Dutch professor, discovered in 1620 that the sines of the angles bear a constant ratio to each other. The *laws of refraction* are:

1. *The incident and refracted rays, and the normal at the point of incidence, all lie in the same plane.*

2. *For two given media,* $\frac{\sin i}{\sin r}$ *is a constant, where i is the angle of incidence and r is the angle of refraction* (Snell's law).

Refractive Index.

The constant ratio $\frac{\sin i}{\sin r}$ is known as the **refractive index** for the two given media; and as the magnitude of the constant depends on the colour of the light, it is usually specified as that obtained for yellow light. If the medium containing the incident ray is denoted by 1, and that containing the refracted ray by 2, the refractive index can be denoted by ${}_1n_2$.

Scientists have drawn up tables of refractive indices when the incident ray is travelling *in vacuo* and is then refracted into the medium concerned, for example, glass or water. The values thus obtained are known as the *absolute* refractive indices of the media; and as a vacuum is always the first medium, the subscripts for the absolute refractive index, symbol n, can be dropped. The magnitude of n for glass is about 1·5, n for water is about 1·33, and n for air at normal pressure is about 1·00028. As the magnitude of the refractive index of a medium is only very slightly altered when the incident light is in air instead of a vacuum, experiments to determine the absolute refractive index n are usually performed with the light incident from air on to the medium; thus we can take ${}_{\text{air}}n_{\text{glass}}$ as equal to ${}_{\text{vacuum}}n_{\text{glass}}$ for most practical purposes.

We have already mentioned that light is refracted because it has different velocities in different media. The Wave Theory of Light, discussed on p. 679, shows that the refractive index ${}_1n_2$ for two given media 1 and 2 is given by

$$ {}_1n_2 = \frac{\textit{velocity of light in medium } 1}{\textit{velocity of light in medium } 2} \quad . \quad . \quad . \quad (1)$$

and this is a *definition* of refractive index which can be used instead of the ratio $\dfrac{\sin i}{\sin r}$. An alternative definition of the absolute refractive index, n, of a medium is thus

$$ n = \frac{\text{velocity of light in a vacuum}}{\text{velocity of light in medium}} \quad . \quad . \quad . \quad (2)$$

In practice the velocity of light in air can replace the velocity in vacuo in this definition.

Relations Between Refractive Indices

(1) Consider a ray of light, AO, refracted from *glass to air* along the direction OB; observation then shows that the refracted ray OB is bent away from the normal, Fig. 18.2. The refractive index from glass to air, ${}_gn_a$, is given by $\sin x/\sin y$, by definition, where x is the angle of incidence in the glass and y is the angle of refraction in the air.

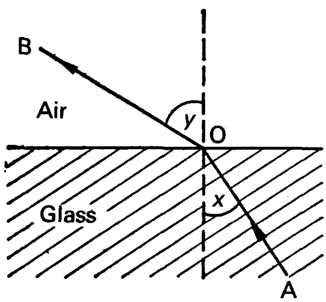


FIG. 18.2. Refraction from glass to air.

From the principle of the reversibility of light (p. 390), it follows that a ray travelling along BO in air is refracted along OA in the glass. The refractive index from air to glass, ${}_an_g$, is then given by $\sin y/\sin x$, by definition. But ${}_gn_a = \dfrac{\sin x}{\sin y}$, from the previous paragraph.

$$\therefore \quad {}_gn_a = \frac{1}{{}_an_g} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If ${}_{a}n_{g}$ is 1·5, then ${}_{g}n_{a} = \frac{1}{1 \cdot 5} = 0 \cdot 67$. Similarly, if the refractive index from air to water is 4/3, the refractive index from water to air is 3/4.

(2) Consider a ray AO incident in air on a plane glass boundary, then refracted from the glass into a water medium, and finally emerging along a direction CD into air. *If the boundaries of the media are parallel, experiment shows that the emergent ray CD is parallel to the incident ray AO*, although there is a relative displacement. Fig. 18.3. Thus the angles made with the normals by AO, CD are equal, and we shall denote them by i_a.

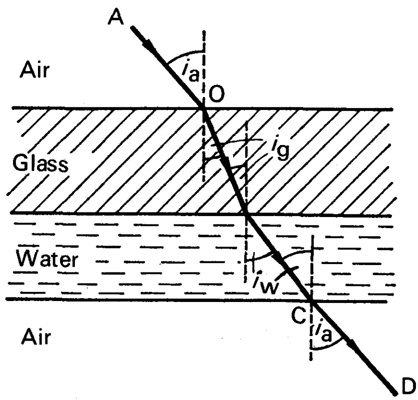


FIG. 18.3. Refraction at parallel plane surfaces.

Suppose i_g, i_w are the angles made with the normals by the respective rays in the glass and water media. Then, by definition, ${}_{g}n_{w} = \frac{\sin i_g}{\sin i_w}$.

But
$$\frac{\sin i_g}{\sin i_w} = \frac{\sin i_g}{\sin i_a} \times \frac{\sin i_a}{\sin i_w},$$

and
$$\frac{\sin i_g}{\sin i_a} = {}_{g}n_{a}, \text{ and } \frac{\sin i_a}{\sin i_w} = {}_{a}n_{w}$$

$$\therefore \ {}_{g}n_{w} = {}_{g}n_{a} \times {}_{a}n_{w} \quad . \quad . \quad . \quad . \quad . \quad (i)$$

Further, as ${}_{g}n_{a} = \frac{1}{{}_{a}n_{g}}$, we can write

$${}_{g}n_{w} = \frac{{}_{a}n_{w}}{{}_{a}n_{g}}.$$

Since ${}_{a}n_{w} = 1 \cdot 33$ and ${}_{a}n_{g} = 1 \cdot 5$, it follows that ${}_{g}n_{w} = \frac{1 \cdot 33}{1 \cdot 5} = 0 \cdot 89$.

From (i) above, it follows that in general

$${}_{1}n_{3} = {}_{1}n_{2} \times {}_{2}n_{3} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The order of the suffixes enables this formula to be easily memorised.

General Relation Between *n* and Sin i

From Fig. 18.3, $\sin i_a/\sin i_g = {}_a n_g$

$$\therefore \sin i_a = {}_a n_g \sin i_g \qquad \text{(i)}$$

Also, $\sin i_w/\sin i_a = {}_w n_a = 1/{}_a n_w$

$$\therefore \sin i_a = {}_a n_w \sin i_w \qquad \text{(ii)}$$

Hence, from (i) and (ii),

$$\sin i_a = {}_a n_g \sin i_g = {}_a n_w \sin i_w$$

If the equations are re-written in terms of the absolute refractive indices of air (n_a), glass (n_g), and water (n_w), we have

$$n_a \sin i_a = n_g \sin i_g = n_w \sin i_w$$

since $n_a = 1$. This relation shows that when a ray is refracted from one medium to another, *the boundaries being parallel,*

$$\mathbf{n \sin i = a\ constant} \qquad (5)$$

where n is the absolute refractive index of a medium and i is the angle made by the ray with the normal in that medium.

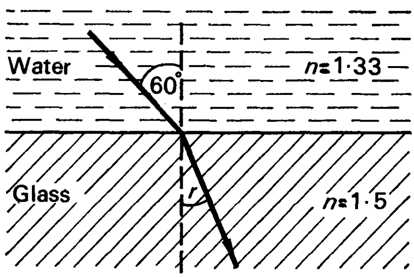


FIG. 18.4. Refraction from water to glass.

This relation applies also to the case of light passing directly from one medium to another. As an illustration of its use, suppose a ray is incident on a water-glass boundary at an angle of 60°, Fig. 18.4. Then, applying "$n \sin i$ is a constant", we have

$$1{\cdot}33 \sin 60° = 1{\cdot}5 \sin r, \qquad \text{(iii)}$$

where r is the angle of refraction in the glass, and 1·33, 1·5 are the respective values of n_w and n_g. Thus $\sin r = 1{\cdot}33 \sin 60°/1{\cdot}5 = 0{\cdot}7679$, from which $r = 50{\cdot}1°$.

Multiple Images in Mirrors

If a candle or other object is held in front of a plane mirror, a series of faint or "ghost" images are observed in addition to one bright image.

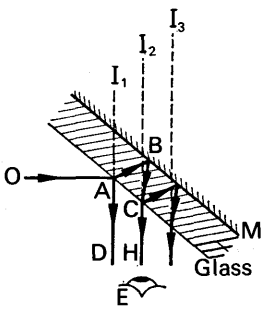


FIG. 18.5. Multiple images.

Suppose O is an object placed in front of a mirror with silvering on the back surface M, as shown in Fig. 18.5. A ray OA from O is then reflected from the front (glass) surface along AD and gives rise to a faint image I_1, while the remainder of the light energy is refracted at A along AB. Reflection then takes place at the silvered surface, and after refraction into the air along CH a bright image is observed at I_2. A small percentage of the light is reflected at C, however, and re-enters the glass again, thus forming a faint image at I_3. Other faint images are formed in the same way. Thus a series of *multiple images* is obtained, the brightest being I_2. The images lie on the normal from O to the mirror, the distances depending on the thickness of the glass and its refractive index and the angle of incidence.

Drawing the Refracted Ray by Geometrical Construction

Since $\sin i/\sin r = n$, the direction of the refracted ray can be calculated when a ray is incident in air at a known angle i on a medium of given refractive index n. The direction of the refracted ray can also be obtained by means of a geometrical construction. Thus suppose AO is a ray incident in air at a given angle i on a medium of refractive index n, Fig. 18.6 (i). With O as centre, two circles, a, b, are drawn whose radii are in the ratio $1:n$, and AO is produced to cut circle a at P. PN is then drawn parallel to the normal at O to intersect circle b at Q. *OQ is then the direction of the refracted ray.*

To prove the construction is correct, we note that angle OPN $= i$, angle OQN $= r$. Thus $\sin i/\sin r$ = ON/OP $\div$ ON/OQ = OQ/OP. But OQ/OP = radius of circle b/radius of circle $a = n$, from our drawing

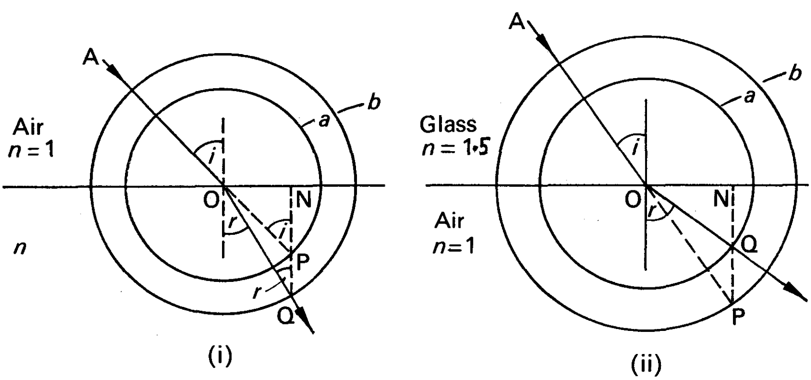


FIG. 18.6. Drawing of refracted rays.

of the circles. Hence sin i/sin $r = n$. Thus OQ must be the refracted ray. Although we have taken the case of the incident ray in air, the same construction will enable the refracted ray to be drawn when the incident ray is in any other medium. The radii of the circles are then in the ratio of the absolute refractive indices.

Fig. 18.6 (ii) illustrates the drawing in the case of a ray AO refracted from a dense medium such as glass ($n = 1{\cdot}5$) into a less dense medium such as air ($n = 1$). Circles a, b are again drawn concentric with O, their radii being in the ratio 1 : n. The incident ray AO, however, is produced to cut the *larger* circle b this time, at P, and a line PN is then drawn parallel to the normal at O to intersect the circle a at Q. OQ is then the direction of the refracted ray. The proof for the construction follows similar lines to that given for Fig. 18.6 (i), and it is left as an exercise for the reader.

The direction of the incident ray AO in Fig. 18.6 (ii) may be such that the line PN does not intersect the circle a. In this case, which is important and is discussed shortly, no refracted ray can be drawn. See *Total Internal Reflection*, p. 430.

Refractive Index of a Liquid by Using a Concave Mirror

We are now in a position to utilise our formulæ in refraction, and we shall first consider a simple method of determining roughly the refractive index, n, of a small quantity of transparent liquid.

If a small drop of the liquid is placed on a concave mirror S, a position H can be located by the no parallax method where the image of a pin held over the mirror coincides in position with the pin itself, Fig. 18.7. The rays from the pin must now be striking the mirror *normally*, in which case the rays are reflected back along the incident path and form an image at the same place as the object. A ray HN close to the axis HP is refracted at N along ND in the liquid, strikes the mirror normally at D, and is reflected back along the path DNH. Thus if DN is produced it passes through the centre of curvature, C, of the concave mirror.

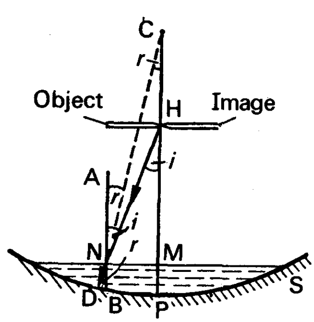


FIG. 18.7. (Depth of liquid exaggerated.)

Let ANB be the normal to the liquid surface at N. Then angle ANH = angle NHM = i, the angle of incidence, and angle BND = angle ANC = angle NCM = r, the angle of refraction in the liquid. From triangles HNM, CNM respectively, $\sin i$ = NM/HN and $\sin r$ = NM/CN. The refractive index, n, of the liquid is thus given by

$$n = \frac{\sin i}{\sin r} = \frac{\text{NM/HN}}{\text{NM/CN}} = \frac{\text{CN}}{\text{HN}}.$$

Now since HN is a ray very close to the principal axis CP, HN = HM and CN = CM, to a very good approximation. Thus $n = \frac{\text{CM}}{\text{HM}}$. Further, if the depth MP of the liquid is very small compared with HM and CM, CM = CP and HM = HP approximately. Hence, approximately,

$$n = \frac{\text{CP}}{\text{HP}}.$$

HP can be measured directly. CP is the radius of curvature of the mirror, which can be obtained by the method shown on page 414. The refractive index, n, of the liquid can thus be calculated.

Apparent Depth

Swimmers in particular are aware that the bottom of a pool of water appears nearer the surface than is actually the case; the phenomenon is due to the refraction of light.

Consider an object O at a distance below the surface of a medium such as water or glass, which has a refractive index n, Fig. 18.8. A ray OM from O perpendicular to the surface passes straight through into the air along MS. A ray ON very close to OM is refracted at N into the air away from the normal, in a direction NT; and an observer viewing O directly overhead sees it in the position I, which is the point of intersection of SM and TN produced. Though we have only considered two rays in the air, a *cone* of rays, with SM as the axis, actually enters the observer's eye.

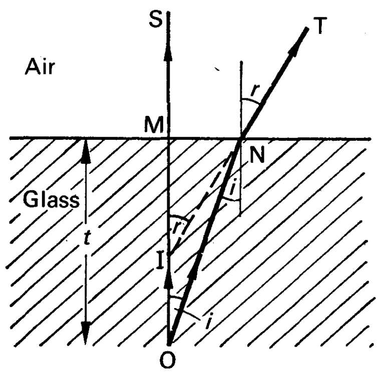


FIG. 18.8. (Inclination of ON to OM exaggerated.)

Suppose the angle of incidence in the glass is i, and the angle of refraction in the air is r. Then, since "$n \sin i$" is a constant (p. 423), we have

$$n \sin i = 1 \times \sin r \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

where n is the refractive index of glass; the refractive index of air is 1.

Since i = angle NOM, and r = MIN, sin i = MN/ON and sin r = MN/IN. From (i), it follows that

$$n\frac{MN}{ON} = \frac{MN}{IN}$$

$$\therefore n = \frac{ON}{IN} \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

Since we are dealing with the case of an observer directly above O, the rays ON, IN are *very* close to the normal OM. Hence to a very good approximation, ON = OM and IN = IM. From (ii),

$$\therefore n = \frac{ON}{IN} = \frac{OM}{IM}.$$

Since the real depth of the object O = OM, and its apparent depth = IM,

$$\therefore \quad n = \frac{\text{real depth}}{\text{apparent depth}} \quad . \quad . \quad . \quad . \quad (6)$$

If the real depth, OM, = t, the apparent depth $= \frac{t}{n}$, from (6). The *displacement*, OI, of the object, which we shall denote by d, is thus given by $t - \frac{t}{n}$, i.e.,

$$d = t\left(1 - \frac{1}{n}\right) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If an object is 6 cm below water of refractive index, $n = 1\frac{1}{3}$, it appears to be displaced upward to an observer in air by an amount, $d = 6\left(1 - \frac{1}{1\frac{1}{3}}\right) = 1\frac{1}{2}$ cm.

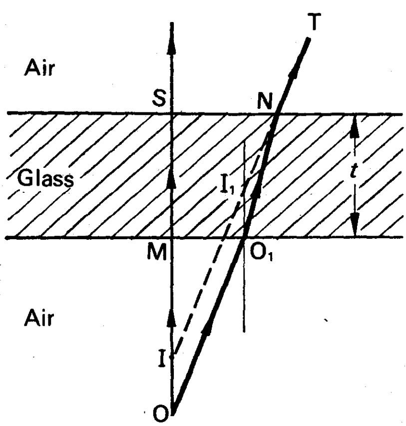


FIG. 18.9. Object below glass block.

Object Below Parallel-sided Glass Block

Consider an object O placed some distance in air below a parallel-sided glass block of thickness t, Fig. 18.9. The ray OMS normal to the surface emerges along MS, while the ray OO_1 close to the normal is refracted along O_1N in the glass and emerges in air along NT in a direction parallel to OO_1 (see p. 422). An observer (not shown) above the glass thus sees the object at I, the point of intersection of TN and SM.

Suppose the normal at O_1 intersects IN at I_1. Then, since O_1I_1 is parallel to OI and IT is parallel to OO_1, OII_1O_1 is a parallelogram. Thus OI = O_1I_1. But OI is the displacement of the object O. Hence O_1I_1 is equal to the displacement. Since the apparent position of an object at O_1 is at I_1 (compare Fig. 18.8), we conclude that *the displacement OI of O is independent of the position of O below the glass*, and is given by

$OI = t\left(1 - \frac{1}{n}\right)$, see p. 427.

Measurement of Refractive Index by Apparent Depth Method

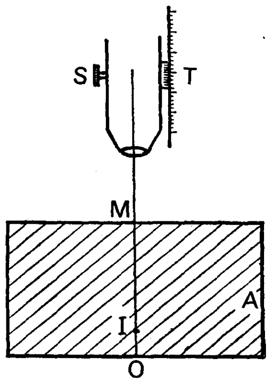


FIG. 18.10. Refractive index by apparent depth.

The formula for the refractive index of a medium in terms of the real and apparent depths is the basis of a very accurate method of measuring refractive index. A *travelling microscope*, S (a microscope which can travel in a vertical direction and which has a fixed graduated scale T beside it) is focused on lycopodium particles at O on a sheet of white paper, and the reading on T is noted, Fig. 18.10. Suppose it is c cm. If the refractive index of glass is required, a glass block A is placed on the paper, and the microscope is raised until the particles are refocused at I. Suppose the reading on T is b cm. Some lycopodium particles are then sprinkled at M on the top of the glass block, and the microscope is raised until they are focused, when the reading on T is noted. Suppose it is a cm.

Then real depth of O = OM = $(a - c)$ cm.

and apparent depth = IM = $(a - b)$ cm.

$$\therefore n = \frac{\text{real depth}}{\text{apparent depth}} = \frac{a - c}{a - b}$$

The high accuracy of this determination of n lies mainly in the fact that the objective of the microscope collects only those rays near to its axis, so that the object O, and its apparent position I, are seen by rays very close to the normal OM. The experiment thus fulfils the theoretical conditions assumed in the proof of the formula $n = \frac{\text{real depth}}{\text{apparent depth}}$ p. 427.

The refractive index of water can also be obtained by an apparent depth method. The block A is replaced by a dish, and the microscope is focused first on the bottom of the dish and then on lycopodium powder sprinkled on the surface of water poured into the dish. The apparent position of the bottom of the dish is also noted, and the refractive index of the water n_w is calculated from the relation

$$n_w = \frac{\text{real depth of water}}{\text{apparent depth of water}}$$

General formula for real and apparent depth. So far we have considered the rays refracted from a medium like glass into air. As a more general case, suppose an object O is in a medium of refractive index n_1 and the rays from it are refracted at M, N into a medium of refractive index n_2, Fig. 18.11. The image of O to an observer in the latter medium is then at I.

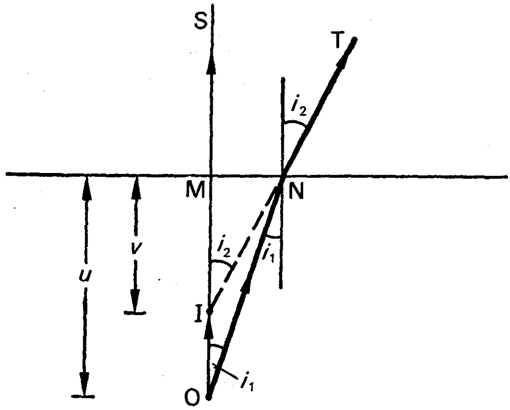


FIG. 18.11. Relation between real and apparent depth.

Suppose i_1 is the angle of incidence at N, and i_2 is the angle of refraction as shown. Then, since "$n \sin i$" is a constant,

$$n_1 \sin i_1 = n_2 \sin i_2.$$

Now angle MON $= i_1$, and angle MIN $= i_2$.

$$\therefore\ n_1 \frac{MN}{ON} = n_2 \frac{MN}{IN}$$

$$\therefore\quad \frac{n_1}{ON} = \frac{n_2}{IN} \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

If we consider rays very close to the normal, then IN $=$ IM $= v$, say, and ON $=$ OM $= u$, say. Substituting in (i),

$$\therefore\quad \frac{n_1}{u} = \frac{n_2}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

This formula can easily be remembered, as the refractive index of a medium is divided by the corresponding distance of the object (or image) *in* that medium.

Total Internal Reflection. Critical Angle

If a ray AO in glass is incident at a small angle α on a glass–air plane boundary, observation shows that part of the incident light is reflected along OE in the glass, while the remainder of the light is refracted away from the normal at an angle β into the air. The reflected ray OE is weak, but the refracted ray OL is bright, Fig. 18.12 (i). This means that most of the incident light energy is transmitted, and a little is reflected.

When the angle of incidence, α, in the glass is increased, the angle of emergence, β, is increased at the same time; and at some angle of incidence c in the glass the refracted ray OL travels along the glass–air boundary, making the angle of refraction of 90°, Fig. 18.12 (ii). The reflected ray OE is still weak in intensity, but as the angle of incidence in the glass is increased slightly the reflected ray suddenly becomes bright, and no refracted ray is then observed, Fig. 18.12 (iii). Since *all* the

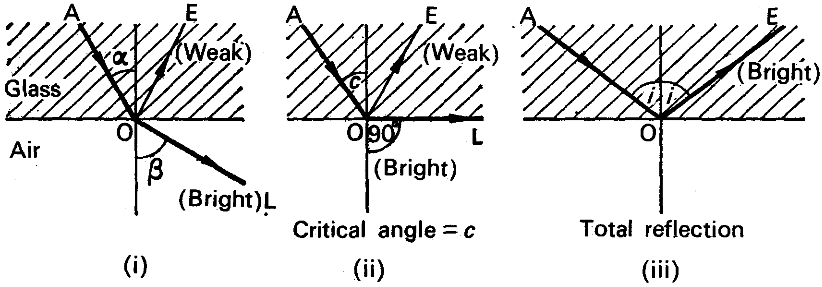


FIG. 18.12. Total internal reflection.

incident light energy is now reflected, **total reflection** is said to take place in the glass at O.

When the angle of refraction in air is 90°, a critical stage is reached at the point of incidence O, and the angle of incidence *in the glass* is accordingly known as the **critical angle** for glass and air, Fig. 18.12 (ii). Since "$n \sin i$" is a constant (p. 423), we have

$$n \sin c = 1 \times \sin 90°,$$

where n is the refractive index of the glass. As $\sin 90° = 1$, then

$$n \sin c = 1,$$

or,
$$\sin c = \frac{1}{n} \quad . \quad . \quad . \quad . \quad (8)$$

Crown glass has a refractive index of about 1·51 for yellow light, and thus the critical angle for glass to air is given by $\sin c = 1/1{\cdot}51 = 0{\cdot}667$. Consequently $c = 41{\cdot}5°$. Thus if the incident angle in the glass is greater than c, for example 45°, total reflection occurs, Fig. 18.12 (iii). The critical angle between two media for blue light is less than for red light, since the refractive index for blue light is greater than that for red light (see p. 458).

The phenomenon of total reflection may occur when light in glass ($n_g = 1{\cdot}51$, say) is incident on a boundary with water ($n_w = 1{\cdot}33$). Applying "$n \sin i$ is a constant" to the critical case, Fig. 18.13, we have

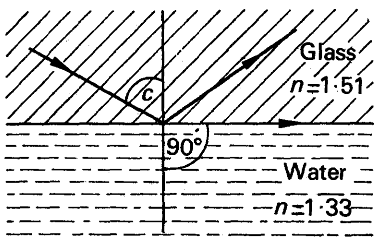


FIG. 18.13. Critical angle for water and glass.

$$n_g \sin c = n_w \sin 90°,$$

where c is the critical angle. As $\sin 90° = 1$

$$n_g \sin c = n_w$$

$$\therefore \sin c = \frac{n_w}{n_g} = \frac{1{\cdot}33}{1{\cdot}51} = 0{\cdot}889$$

$$\therefore c = 63°.$$

Thus if the angle of incidence in the glass exceeds 63°, total internal reflection occurs.

It should be carefully noted that the phenomenon of total internal reflection can occur only when light travels from one medium to another which has a *smaller* refractive index, i.e., which is optically less dense. The phenomenon cannot occur when light travels from one medium to another optically denser, for example from air to glass, or from water to glass, as a refracted ray is then always obtained.

SOME APPLICATIONS OF TOTAL INTERNAL REFLECTION

1. *Reflecting prisms* are pieces of glass of a special shape which are used in prism binoculars and in certain accurate ranging instruments such as submarine periscopes. These prisms, discussed on p. 449, act as reflectors of light by total internal reflection.

2. *The mirage* is a phenomenon due to total reflection. In the desert the air is progressively hotter towards the sand, and hence the density of the air decreases in the direction *bcd*, Fig. 18.14 (i). A downward ray OA from a tree or the sky is thus refracted more and more away from the normal; but at some layer of air *c*, a critical angle is reached, and the ray begins to travel in an upward direction along *cg*. A distant observer P thus sees the object O at I, and hence an image of a palm tree, for example, is seen below the actual position of the tree. As an image of part of the sky is also formed by total reflection round the image of the tree, the whole appearance is similar to that of a pool of water in which the tree is reflected.

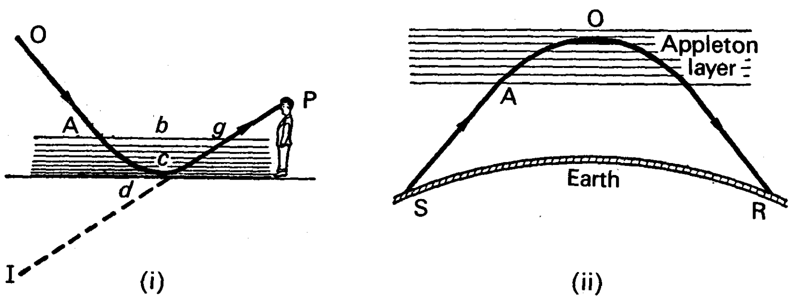


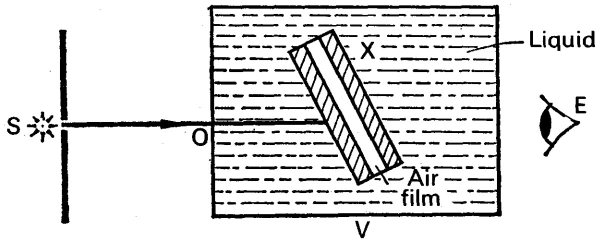
FIG. 18.14. Examples of total reflection.

3. *Total reflection of radio waves.* A radio wave is an example of an *electromagnetic wave* because it comprises electric and magnetic forces. Light waves also are electromagnetic waves (p. 983). Light waves and radio waves are therefore the same in nature, and a close analogy can be made between the refraction of light and the refraction of a radio wave when the latter enters a medium containing electric particles.

In particular, the phenomenon of total reflection occurs when radio waves travel from one place, S, on the earth, for example England, to another place, R, on the other side of the earth, for example America, Fig. 18.14 (ii). A layer of considerable density of electrons exists many miles above the earth (at night this is the *Appleton layer*), and when a radio wave SA from a transmitter is sent skyward it is refracted away from the normal on entering the electron layer. At some height, corresponding to O, a critical angle is reached, and the wave then begins to be refracted downward. After emerging from the electron layer it returns to R on the earth, where its presence can be detected by a radio receiver.

Measurement of Refractive Index of a Liquid by an Air-cell Method

The phenomenon of total internal reflection is utilised in many methods of measuring refractive index. Fig. 18.15 (i) illustrates how the



(i)

FIG. 18.15 (i). Air-cell method.

refractive index of a liquid can be determined. Two thin plane-parallel glass plates, such as microscope slides, are cemented together so as to contain a thin film of air of constant thickness between them, thus forming an air-cell, X. The liquid whose refractive index is required is placed in a glass vessel V having thin plane-parallel sides, and X is placed in the liquid. A bright source of light, S, provides rays which are incident on one side of X in a constant direction SO, and the light through X is observed by a person on the other side at E.

When the light is incident normally on the sides of X, the light passes straight through X to E. When X is rotated slightly about a vertical axis, light is still observed; but as X is rotated farther, the light is suddenly cut off from E, and hence no light now passes through X, Fig. 18.15 (i).

Fig. 18.15 (ii) illustrates the behaviour of the light when this happens. The ray SO is refracted along OB in the glass, but at B *total internal reflection begins*. Suppose i_1 is the angle of incidence in the liquid, i_2 is the angle of incidence in the glass, and n, n_g are the corresponding refractive indices. Since the boundaries of the media are parallel we can apply the relation "$n \sin i$ is a constant", and hence

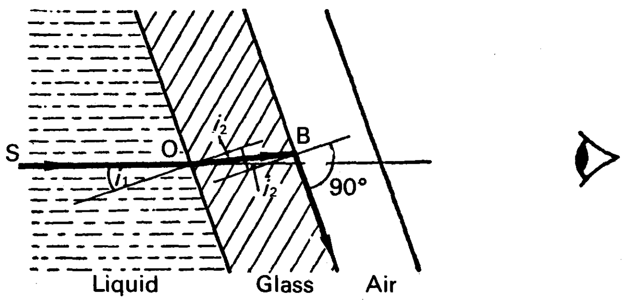
$$n \sin i_1 = n_g \sin i_2 = 1 \times \sin 90°, \quad . \quad . \quad . \quad (i)$$

the last product corresponding to the case of refraction in the air-film.

$$\therefore \quad n \sin i_1 = 1 \times \sin 90° = 1 \times 1 = 1$$

$$\therefore n = \frac{1}{\sin i_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

It should now be carefully noted that i_1 is the angle of incidence in the *liquid* medium, and is thus determined by measuring the rotation of X from its position when normal to SO to the position when the light



(ii)

FIG. 18.15 (ii). Air-cell theory.

is cut off. In practice, it is better to rotate X in opposite directions and determine the angle θ between the *two* positions for the extinction of the light. The angle i_1 is then half the angle θ, and hence $n = 1/\sin\frac{\theta}{2}$.

From equations (i) and (9), it will be noted that i_1 is the critical angle between the liquid and air, and i_2 is the critical angle between the glass and air. We cannot measure i_2, however, as we can i_1, and hence the method provides the refractive index of the *liquid.*

The source of light, S, in the experiment should be a monochromatic source, i.e., it should provide light of one colour, for example yellow light. The extinction of the light is then sharp. If white light is used, the colours in its spectrum are cut off at slightly different angles of incidence, since refractive index depends upon the colour of the light (p. 458). The extinction of the light is then gradual and ill-defined.

Pulfrich Refractometer

A *refractometer* is an instrument which measures refractive index by making use of total internal reflection. PULFRICH designed a refractometer enabling the refractive index of a liquid to be easily obtained, which consists of a block of glass G with a polished and vertical face. On top of G is cemented a circular glass tube V, Fig. 18.16. The liquid L is placed in V, and a convergent beam of monochromatic light is directed so that the liquid–glass interface is illuminated. On observing the light refracted through G by a telescope T, a light and dark field of view are seen.

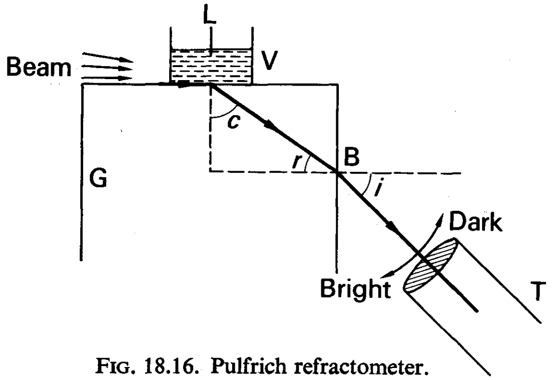


FIG. 18.16. Pulfrich refractometer.

The boundary between the light and dark fields corresponds to the ray which is incident just horizontally on the liquid-glass boundary, as shown in Fig. 18.16. If c is the angle of refraction in the glass, it follows that

$$n \sin 90° = n_g \sin c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$

where n, n_g are the refractive indices of the liquid and glass respectively. For refraction at B,

$$n_g \sin r = \sin i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (ii)$$

Also, $$c + r = 90° \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (iii)$$

From (i), $\sin c = n/n_g$. Now from (iii), $\sin r = \sin (90° - c) = \cos c$. Substituting in (ii), we have $n_g \cos c = \sin i$, or $\cos c = \sin i/n_g$.

But $$\sin^2 c + \cos^2 c = 1$$

$$\therefore \quad \frac{n^2}{n_g^2} + \frac{\sin^2 i}{n_g^2} = 1$$

$$\therefore \quad n^2 + \sin^2 i = n_g^2$$

$$\therefore \quad n = \sqrt{n_g^2 - \sin^2 i}$$

Thus if i is measured and $n_g = 1{\cdot}51$, n can be calculated. In practice, tables are supplied giving the refractive index in terms of i, and another block is used in place of G for liquids of higher n than 1·51.

Abbe refractometer. Abbe designed a refractometer for measuring the refractive index of liquids whose principle is illustrated in Fig. 18.17. Two similar prisms X, Y are placed on a table A, the prism X being hinged at H so that it could be swung away from Y. A drop of the liquid is placed on the surface a, which is matt, and the prisms are placed together so that the liquid is squeezed into a thin film between them. Light from a suitable source is directed towards the prisms by means of a mirror M, where it strikes the surface a and is scattered by the matt surface into the liquid film. The emergent rays are collected in a telescope T directed towards the prisms, and the field of view is divided into a dark and bright portion. The table A is then turned until the dividing line between the dark and bright fields is on the crosswires of the telescope, which is fixed. The reading on the scale S, which is attached to, and moves with, the table, gives the refractive index of the liquid directly, as explained below.

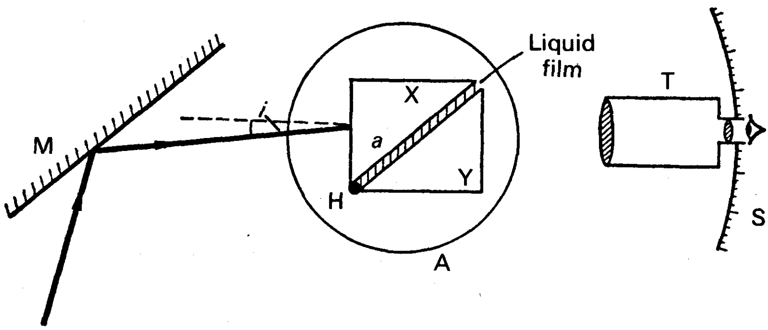


FIG. 18.17. Abbe refractometer principle.

Theory. The dividing line, BQ, between the bright and dark fields corresponds to the case of the ray DA, incident in the liquid L at grazing incidence on the prism Y, Fig. 18.18. The refracted ray AB in the prism then makes the critical angle c with the normal at A, where $n \sin 90° = n_g \sin c$, n and n_g being the respective refractive indices of the liquid and the glass.

$$\therefore \quad n = n_g \sin c \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

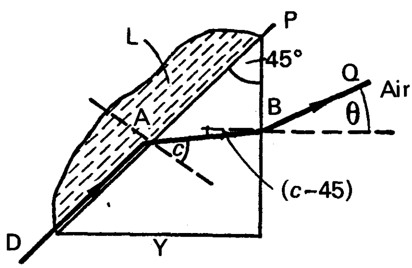


FIG. 18.18. Theory of refractometer.

For simplicity, suppose that Y is a right-angled isosceles prism, so that the angle P of the prism is 45°. The angle of incidence at B in the glass is then $(c - 45°)$, by considering the geometry of triangle PAB, and hence for refraction at B we have

$$n_g \sin (c - 45°) = \sin \theta \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

where θ is the angle with the normal at B made by the emerging ray BQ. By eliminating c from (i) and (ii), we obtain finally

$$n = \sin 45° \, (n_g{}^2 - \sin^2\theta^{\frac{1}{2}}) + \cos 45° \sin \theta$$

$$= \frac{1}{\sqrt{2}} [(n_g{}^2 - \sin^2\theta)^{\frac{1}{2}} + \sin \theta]$$

since $\sin 45° = 1/\sqrt{2} = \cos 45°$. Thus knowing n_g and θ, the refractive index of the liquid, n, can be evaluated. The scale S, Fig. 18.17, which gives θ, can thus be calibrated in terms of n.

EXAMPLES

1. Describe a method, based on grazing incidence or total internal reflection, for finding the refractive index of water for the yellow light emitted by a sodium flame.

The refractive index of carbon bisulphide for red light is 1·634 and the difference between the critical angles for red and blue light at a carbon bisulphide-air interface is 0° 56′. What is the refractive index of carbon bisulphide for blue light? (*N.*)

First part. See air-cell method, p. 432.

Second part. Suppose n_r and c_r are the refractive index and critical angle of carbon bisulphide for red light.

Then $$\sin c_r = \frac{1}{n_r} = \frac{1}{1{\cdot}634} = 0{\cdot}6119$$

$$\therefore \quad c_r = 37^\circ\ 44'$$

The critical angle, c_b, for blue light is less than that for red light.

$$\therefore \quad c_b = 37^\circ\ 44' - 0^\circ\ 56' = 36^\circ\ 48'$$

The refractive index for blue light, n_b, $= \dfrac{1}{\sin c_b}$

$$\therefore \quad n_b = \frac{1}{\sin 36^\circ\ 48'} = 1{\cdot}669$$

2. Find an expression for the distance through which an object appears to be displaced towards the eye when a plate of glass of thickness t and refractive index n is interposed.

A tank contains a slab of glass 8 cm thick and of refractive index 1·6. Above this is a depth of 4·5 cm of a liquid of refractive index 1·5 and upon this floats 6 cm of water ($n = 4/3$). To an observer looking from above, what is the apparent position of a mark on the bottom of the tank? (*O. & C.*)

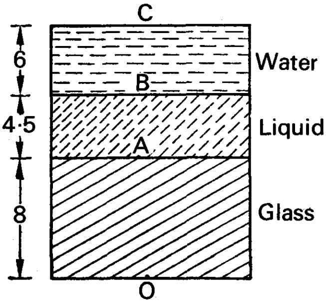


FIG. 18.19.

First part. See text.

Second part. Suppose O is the mark at the bottom of the tank, Fig. 18.19. Then since the boundaries of the media are parallel, the total displacement of O is the sum of the displacements due to each of the media.

For glass, displacement, $d \doteq t\left(1 - \dfrac{1}{n}\right) = 8\left(1 - \dfrac{1}{1{\cdot}6}\right) = 3$ cm

For liguid, $$d = t\left(1 - \frac{1}{n}\right) = 4{\cdot}5\left(1 - \frac{1}{1{\cdot}5}\right) = 1{\cdot}5 \text{ cm}$$

For water, $$d = 6\left(1 - \frac{1}{4/3}\right) = 1{\cdot}5 \text{ cm}$$

$$\therefore \text{ total displacement} = 3 + 1{\cdot}5 + 1{\cdot}5 = 6 \text{ cm}$$

$\therefore$ apparent position of O is 6 cm from bottom.

3. A small object is placed on the principal axis of a concave spherical mirror of radius 20 cm at a distance of 30 cm. By how much will the position and size of the image alter when a parallel-sided slab of glass, of thickness 6 cm and refractive index 1·5, is introduced between the centre of curvature and the object? The parallel sides are perpendicular to the principal axis. Prove any formula used. (*N.*)

Suppose O is the position of the object before the glass is placed in position, Fig. 18.20. The image position is given by $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$,

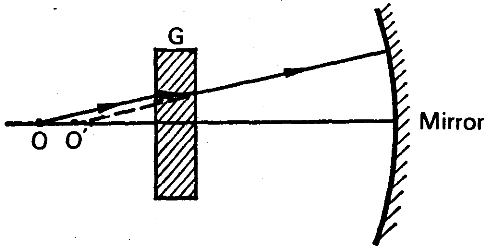


FIG. 18.20.

$$\therefore \quad \frac{1}{v}+\frac{1}{30}=\frac{2}{20}$$

Solving, $v = 15$ cm.

The magnification, $m = \frac{v}{u} = \frac{15}{30} = 0{\cdot}5$.

When the glass slab G of thickness, *t*, 6 cm is inserted, the rays from O appear to come from a point O′ whose displacement from O is $t\left(1-\frac{1}{n}\right)$, where *n* is the glass refractive index. See p. 428. The displacement is thus $6\left(1-\frac{1}{1{\cdot}5}\right) = 2$ cm. The distance of O′ from the mirror is therefore (30 − 2), or 28 cm. Applying the equation $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$, we find $v = +15\frac{5}{9}$ cm. The image position changes by $(15\frac{5}{9} - 15)$ or $\frac{5}{9}$ cm. The magnification becomes $15\frac{5}{9} \div 28$, or 0·52.

EXERCISES 18

1. A ray of light is incident at 60° in air on an air–glass plane surface. Find the angle of refraction in the glass by calculation and by drawing (n for glass $= 1{\cdot}5$).

2. A ray of light is incident in water at an angle of 30° on a water–air plane surface. Find the angle of refraction in the air by calculation and by drawing (n for water $= 4/3$).

3. A ray of light is incident in water at an angle of (i) 30°, (ii) 70° on a water–glass plane surface. Calculate the angle of refraction in the glass in each case (${}_an_g = 1{\cdot}5$, ${}_an_w = 1{\cdot}33$).

4. (*a*) Describe the apparent depth method of finding the refractive index of glass, and prove the formula used. (*b*) What is the apparent position of an object below a rectangular block of glass 6 cm thick if a layer of water 4 cm thick is on top of the glass (refractive index of glass and water = $1\frac{1}{2}$ and $1\frac{1}{3}$ respectively)?

5. Describe and explain a method of measuring approximately the refractive index of a small quantity of liquid.

6. Calculate the critical angle for (i) an air–glass surface, (ii) an air–water surface, (iii) a water–glass surface; draw diagrams in each case illustrating the total reflection of a ray incident on the surface (${}_an_g = 1{\cdot}5$, ${}_an_w = 1{\cdot}33$).

7. Explain what happens in general when a ray of light strikes the surface separating transparent media such as water and glass. Explain the circumstances in which total reflection occurs and show how the critical angle is related to the refractive index.

Describe a method for determining the refractive index of a medium by means of critical reflection. (*L.*)

8. Define *refractive index* of one medium with respect to another and show how it is related to the values of the velocity of light in the two media.

Describe a method of finding the refractive index of water for sodium light, deducing any formula required in the reduction of the observations. (*N.*)

9. Explain carefully why the apparent depth of the water in a tank changes with the position of the observer.

A microscope is focused on a scratch on the bottom of a beaker. Turpentine is poured into the beaker to a depth of 4 cm, and it is found necessary to raise the microscope through a vertical distance of 1·28 cm to bring the scratch again into focus. Find the refractive index of the turpentine. (*C.*)

10. What is meant by *total reflection* and *critical angle*? Describe two methods of measuring the refractive index of a material by determining the critical angle, one of which is suitable for a solid substance and the other for a liquid. (*L.*)

11. (*a*) State the conditions under which total reflection occurs. Show that the phenomenon will occur in the case of light entering normally one face of an isosceles right-angle prism of glass, but not in the case when light enters similarly a similar hollow prism full of water. (*b*) A concave mirror of small aperture and focal length 8 cm lies on a bench and a pin is moved vertically above it. At what point will image and object coincide if the mirror is filled with water of refractive index 4/3? (*N.*)

12. State the laws of refraction, and define *refractive index*.

Describe an accurate method of determining the refractive index of a transparent liquid for sodium light. Give the theory of the method, and derive any formula you require. Discuss the effect of substituting white light for sodium light in your experiment. (*W.*)

13. Describe an experiment for finding the refractive index of a liquid by measuring its apparent depth.

A vessel of depth $2d$ cm is half filled with a liquid of refractive index n_1, and the upper half is occupied by a liquid of refractive index n_2. Show that the apparent depth of the vessel, viewed perpendicularly, is $d\left(\frac{1}{n_1}+\frac{1}{n_2}\right)$. (*L.*)

14. The base of a cube of glass of refractive index n_1 is in contact with the surface of a liquid of refractive index n_2. Light incident on one vertical face of the cube is reflected internally from the base and emerges again from the opposite vertical face in a direction making an angle θ with its normal. Assuming that $n_1 > n_2$, show that the light has just been totally reflected internally if $n_2 = \sqrt{(n_1^2 - \sin^2 \theta)}$.

Describe how the above principle may be used to measure the refractive index of a small quantity of liquid. (*N.*)

15. Explain the meaning of critical angle and total internal reflection. Describe fully (*a*) one natural phenomenon due to total internal reflection, (*b*) one practical application of it. Light from a luminous point on the lower face of a rectangular glass slab, 2·0 cm thick, strikes the upper face and the totally reflected rays outline a circle of 3·2 cm radius on the lower face. What is the refractive index of the glass? (*N.*)

16. Describe one experiment in each case to determine the refractive index for sodium light of (*a*) a sample of glass which could be supplied to any shape and size which you specify, (*b*) a liquid of which only a *very* small quantity is available. Show how the result is calculated in each case. (You are *not* expected to derive standard formulae.)

How would you modify the experiment in (*a*) to find how the refractive index varies with the wavelength of the light used? What general result would you expect? (*O. & C*).

17. Explain the meaning of *critical angle,* and describe how you would measure the critical angle for a water–air boundary.

ABCD is the plan of a glass cube. A horizontal beam of light enters the face AB at grazing incidence. Show that the angle θ which any rays emerging from BC would make with the normal to BC is given by $\sin \theta = \cot a$, where a is the critical angle. What is the greatest value that the refractive index of glass may have if any of the light is to emerge from BC? (*N.*)

18. State the *laws of refraction of light.* Explain how you would measure the refractive index of a transparent liquid available only in *small* quantity, i.e., less than 0·5 cm³.

A ray of light is refracted through a sphere, whose material has a refractive index n, in such a way that it passes through the extremities of two radii which make an angle a with each other. Prove that if γ is the deviation of the ray caused by its passage through the sphere

$$\cos \tfrac{1}{2} (a - \gamma) = n \cos \tfrac{1}{2} a. \qquad (L.)$$

19. Explain what is meant by the terms *critical angle* and *total reflection.* Describe an accurate method of determining the critical angle for a liquid, indicating how you would calculate the refractive index from your measurements.

A man stands at the edge of the deep end of a swimming bath, the floor of which is covered with square tiles. If the water is clear and undisturbed, explain carefully how the floor of the bath appears to him. (*O. & C.*)

20. Summarize the various effects that may occur when a parallel beam of light strikes a plane interface between two transparent media.

Explain why, when looking at the windows of a railway carriage from inside, one sees by day the country outside and by night the reflection of the inside of the carriage.

An observer looks normally through a thick window of thickness d and refractive index n at an object at a distance e behind the farther surface. Where does the object appear to be, and how can this apparent position be found experimentally? (*O. & C.*)