chapter seventeen
Reflection at curved mirrors

CURVED mirrors are reputed to have been used thousands of years ago. Today motor-cars and other vehicles are equipped with driving mirrors which are curved, searchlights have curved mirrors inside them, and the largest telescope in the world utilises a huge curved mirror (p. 544).

Convex and Concave Mirrors. Definitions

In the theory of Light we are mainly concerned with curved mirrors which are parts of *spherical* surfaces. In Fig. 17.1 (*a*), the mirror APB is part of a sphere whose centre C is in front of the reflecting surface; in Fig. 17.1 (*b*), the mirror KPL is part of a sphere whose centre C is behind its reflecting surface. To a person in front of it APB curves inwards and is known as a **concave** mirror, while KPL bulges outwards and is known as a **convex** mirror.

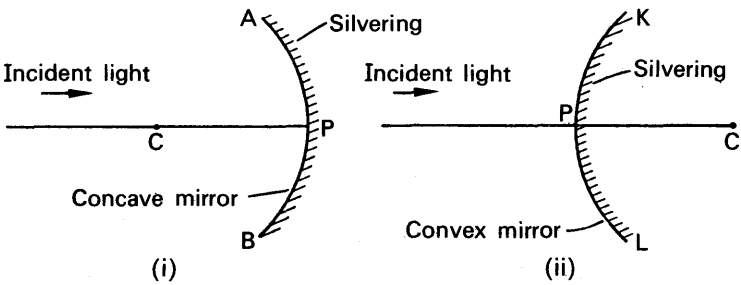

FIG. 17.1. Concave (converging) and convex (diverging) mirrors.

The mid-point, P, of the mirror is called its *pole*; C, the centre of the sphere of which the mirror is part, is known as the *centre of curvature*; and AB is called the *aperture* of the mirror. The line PC is known as the *principal axis*, and plays an important part in the drawing of images in the mirrors; lines parallel to PC are called *secondary axes*.

Narrow and Wide Beams. The Caustic

When a very narrow beam of rays, parallel to the principal axis and close to it, is incident on a concave mirror, experiment shows that all the reflected rays converge to a point F on the principal axis, which is therefore known as the *principal focus* of the mirror, Fig. 17.2 (i). On this account a concave mirror is better described as a "converging" mirror. An image of the sun, whose rays on the earth are parallel, can hence be received on a screen at F, and thus a concave mirror has a *real* focus.

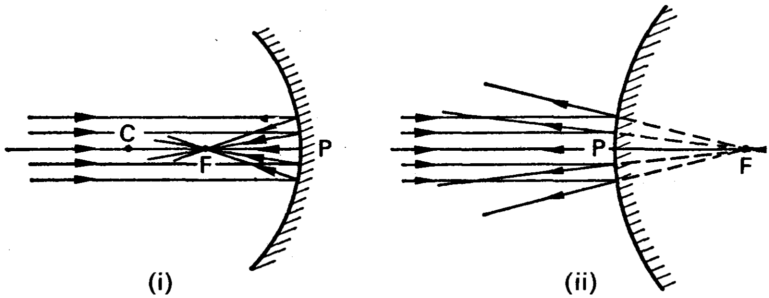

FIG. 17.2. Foci of concave and convex mirrors.

If a narrow beam of parallel rays is incident on a convex mirror, experiment shows that the reflected rays form a divergent beam which appear to come from a point F *behind* the mirror, Fig. 17.2 (ii). A convex mirror has thus a *virtual* focus, and the image of the sun cannot be received on a screen using this type of mirror. To express its action on a parallel beam of light, a convex mirror is often called a "diverging" mirror.

When a *wide* beam of light, parallel to the principal axis, is incident on a concave spherical mirror, experiment shows that reflected rays do not pass through a single point, as was the case with a narrow beam. The reflected rays appear to touch a surface known as a *caustic surface*, S, which has an apex, or cusp, at F, the principal focus, Fig. 17.3. Similarly, if a wide beam of parallel light is incident on a convex mirror, the reflected rays do not appear to diverge from a single point, as was the case with a narrow beam.

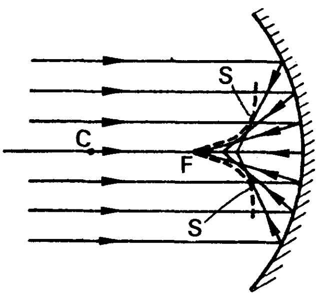

FIG. 17.3. Caustic surface.

Parabolic Mirrors

If a small lamp is placed at the focus, F, of a concave mirror, it follows from the principle of the reversibility of light (p. 390) that rays striking the mirror round a small area about the pole are reflected parallel. See Fig. 17.2 (i). But those rays from the lamp which strike the mirror at points well away from P will be reflected in different directions, because a *wide* parallel beam is not brought to a focus at F, as shown in Fig. 17.3. The beam of light reflected from the mirror thus diminishes

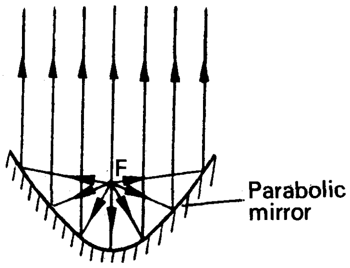

FIG. 17.4. Parabolic mirror.

in intensity as its distance from the mirror increases, and a concave spherical mirror is hence useless as a searchlight mirror.

A mirror whose section is the shape of a parabola (the path of a cricket-ball thrown forward into the air) is used in searchlights. A parabolic mirror has the property of reflecting the wide beam of light from a lamp at its focus F as a perfectly parallel beam, in which case the intensity of the reflected beam is practically undiminished as the distance from the mirror increases, Fig. 17.4.

Focal Length (f) and Radius of Curvature (r)

From now onwards we shall be concerned with curved spherical mirrors of small aperture, so that a parallel incident beam will pass through the focus after reflection. The diagrams which follow are exaggerated for purposes of clarity.

The distance PC from the pole to the centre of curvature is known as the *radius of curvature* (r) of a mirror; the distance PF from the pole to the focus is known as the *focal length* (f) of the mirror. As we shall now prove, there is a simple relation between f and r.

Consider a ray AX parallel to the principal axis of either a concave or a convex mirror, Fig. 17.5 (i), (ii). The normal to the mirror at X is

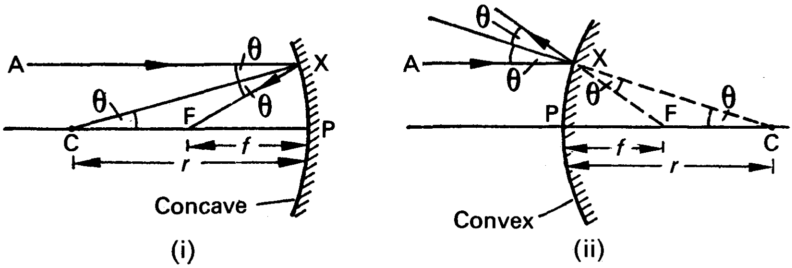

FIG. 17.5. Relation between f and r.

CX, because the radius of the spherical surface is perpendicular to the surface, and hence the reflected ray makes an angle, θ, with CX equal to the incident angle θ. Taking the case of the concave mirror, angle AXC = angle XCP, alternate angles, Fig. 17.5 (i). Thus triangle FXC is isosceles, and FX = FC. As X is a point very close to P we assume to a very good approximation that FX = FP.

$$\therefore \quad FP = FC, \text{ or } FP = \tfrac{1}{2}\,CP.$$

$$\therefore \quad f = \frac{r}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This relation between f and r is the same for the case of the convex mirror, Fig. 17.5 (ii), as the reader can easily verify.

Images in Concave Mirrors

Concave mirrors produce images of different sizes; sometimes they are inverted and real, and on other occasions they are erect (the same way up as the object) and virtual. As we shall see, the nature of the image formed depends on the distance of the object from the mirror.

Consider an object of finite size OH placed at O perpendicular to the principal axis of the mirror, Fig. 17.6 (i). The image, R, of the top point

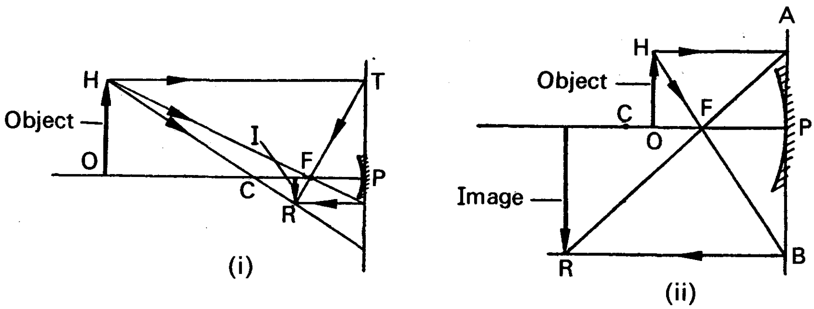

FIG. 17.6. Images in concave mirrors.

H can be located by the intersection of two reflected rays coming initially from H, and the rays usually chosen are two of the following: (1) The ray HT parallel to the principal axis, which is reflected to pass through the focus, F, (2) the ray HC passing through the centre of curvature, C, which is reflected back along its own path because it is a normal to the mirror, (3) the ray HF passing through the focus, F, which is reflected parallel to the principal axis. Since the mirror has a small aperture, and we are considering a narrow beam of light, the mirror must be represented in accurate image drawings by a *straight* line. Thus PT in Fig. 17.6 (i) represents a perfect mirror.

When the object is a very long distance away (at infinity), the image is small and is formed inverted at the focus (p. 402). As the object approaches the centre of curvature, C, the image remains real and inverted, and is formed in front of the object, Fig. 17.6 (i). When the object is between C and F, the image is real, inverted, and larger than the object; it is now further from the mirror than the object, Fig. 17.6 (ii).

As the object approaches the focus, the image recedes further from the mirror, and when the object is at the focus, the image is at infinity. When the object is nearer to the mirror than the focus the image IR becomes *erect* and *virtual*, as shown in Fig. 17.7 (i). In this case the image is *magnified*, and the concave mirror can thus be used as a shaving mirror.

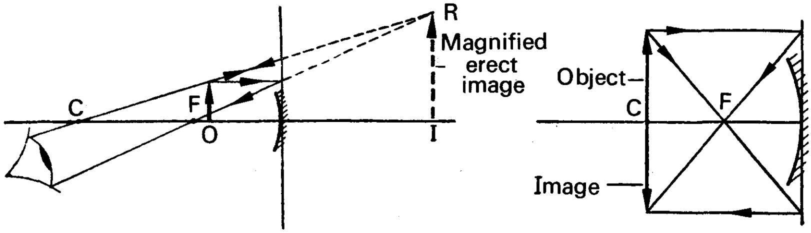

FIG. 17.7. Images in concave mirrors.

A special case occurs when the object is at the centre of curvature, C. The image is then real, inverted, and the same size of the object, and it is also situated at C, Fig. 17.7 (ii). This case provides a simple method of locating the centre of curvature of a concave mirror (p. 413).

Images in Convex Mirrors

Experiment shows that the image of an object in a convex mirror is erect, virtual, and diminished in size, no matter where the object is situated. Suppose an object OH is placed in front of a convex mirror, Fig. 17.8 (i). A ray HM parallel to the principal axis is reflected as if it

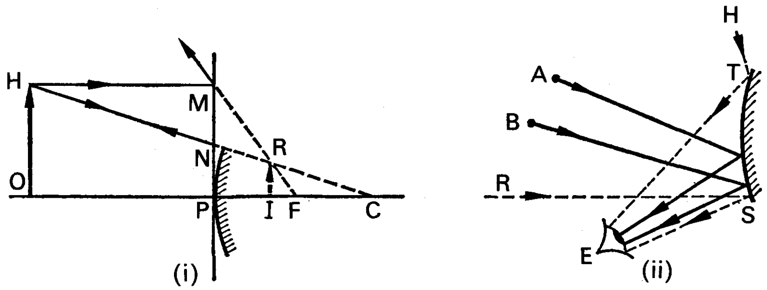

FIG. 17.8. Images in convex mirrors.

appeared to come from the virtual focus, F, and a ray HN incident towards the centre of curvature, C, is reflected back along its path. The two reflected rays intersect *behind* the mirror at R, and IR is a virtual and erect image.

Objects well outside the principal axis of a convex mirror, such as A, B in Fig. 17.8 (ii), can be seen by an observer at E, whose *field of view* is that between HT and RS, where T, S are the edges of the mirror. Thus in addition to providing an erect image the convex mirror has a wide field of view, and is hence used as a driving mirror.

Formulæ for Mirrors. Sign Convention

Many of the advances in the uses of curved mirrors and lenses have resulted from the use of *optical formulæ*,and we have now to consider the relation which holds between the object and image distances in mirrors and their focal length. In order to obtain a formula which holds for both concave and convex mirrors, a *sign rule* or *convention* must be obeyed, and we shall adopt the following:

A real object or image distance is a positive distance.

A virtual object or image distance is a negative distance.

In brief, "real is positive, virtual is negative". The focal length of a concave mirror is thus a positive distance; the focal length of a convex mirror is a negative distance.

Concave Mirror

Consider a point object O on the principal axis of a concave mirror. A ray OX from O is reflected in the direction XI making an equal angle θ with the normal CX; a ray OP from O, incident at P, is reflected back along PO, since CP is the normal at P. The point of intersection, I, of the two rays is the image of O. Fig. 17.9.

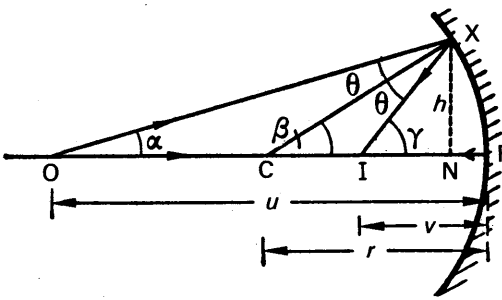

FIG. 17.9. Mirror formula.

Suppose α, β, γ are the angles made by OX, CX, IX respectively with the axis. Since we are considering a mirror of small aperture these angles are small in practice, Fig. 17.9 being exaggerated. As β is the exterior angle of triangle CXO, we have $\beta = \alpha + \theta$.

$$\therefore \theta = \beta - \alpha \quad . \quad . \quad . \quad . \quad \text{(i)}$$

Since γ is the exterior angle of triangle IXC, we have $\gamma = \beta + \theta$.

$$\therefore \theta = \gamma - \beta \quad . \quad . \quad \text{(ii)}$$

From (i) and (ii), it follows that

$$\beta - \alpha = \gamma - \beta$$

$$\therefore \alpha + \gamma = 2\beta \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

We can now substitute for α, β, γ in terms of h, the height of X above the axis, and the distances OP, CP, IP. In so doing (*a*) we assume N is practically coincident with P, as X is very close to P in practice, (*b*) the appropriate sign, $+$ or $-$, must precede all the numerical values of the distances concerned. Also, as $\alpha = \tan\alpha$ in radians when α is very small, we have

$$\alpha = \frac{XN}{ON} = \frac{XN}{+OP} = \frac{h}{+OP},$$

where OP is the distance of the real object O from the mirror in centimetres, say, and XN $= h$. Similarly,

$$\beta = \frac{XN}{CN} = \frac{XN}{+CP} = \frac{h}{+CP},$$

as CP, the radius of curvature of the concave mirror, is real.

Also,
$$\gamma = \frac{XN}{IN} = \frac{XN}{+IP} = \frac{h}{+IP},$$

where IP is the distance of the real image I from the mirror. Substituting for α, β, γ in (iii),

$$\frac{h}{(+IP)} + \frac{h}{(+OP)} = 2\frac{h}{(+CP)}.$$

Dividing by h,
$$\frac{1}{IP} + \frac{1}{OP} = \frac{2}{CP}.$$

If we let v represent the image distance from the mirror, u the object distance from the mirror, and r the radius of curvature, we have

$$\frac{1}{\mathbf{v}} + \frac{1}{\mathbf{u}} = \frac{2}{\mathbf{r}} \qquad \qquad (2)$$

Further, since $f = r/2$, then $\frac{2}{r} = \frac{1}{f}$.

$$\therefore \frac{1}{\mathbf{v}} + \frac{1}{\mathbf{u}} = \frac{1}{\mathbf{f}} \qquad \qquad (3)$$

The relations (2), (3) are general formulae for curved spherical mirrors; and when they are used the appropriate sign for v, u, f, or r must always precede the corresponding numerical value.

Convex Mirror

We now obtain a relation for object distance (u), image distance (v), and focal length (f) of a convex mirror. In this case the incident rays OX, OP are reflected as if they appear to come from the point I behind the mirror, which is therefore a virtual image, and hence the image distance IP is *negative*, Fig. 17.10. Further, CP is negative, as the centre of curvature of X, a convex mirror, is behind the mirror.

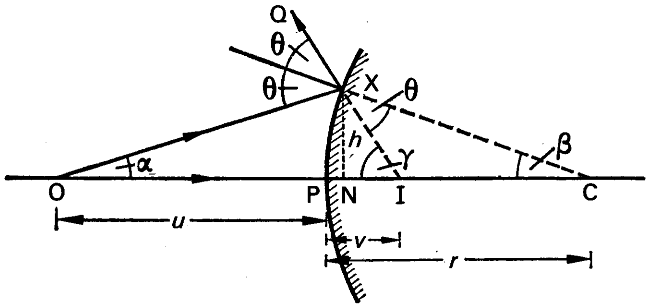

FIG. 17.10. Mirror formula.

Since θ is the exterior angle of triangle COX, $\theta = \alpha + \beta$. As γ is the exterior angle of triangle CIX, $\gamma = \theta + \beta$, or $\theta = \gamma - \beta$.

$$\therefore \gamma - \beta = \alpha + \beta$$

$$\therefore \gamma - \alpha = 2\beta \quad . \quad . \quad . \quad . \quad \text{(i)}$$

Now $\gamma = \dfrac{h}{\mathrm{IN}} = \dfrac{h}{(-\mathrm{IP})}$, as I is virtual; $\alpha = \dfrac{h}{\mathrm{ON}} = \dfrac{h}{(+\mathrm{OP})}$, as O is real,

$\beta = \dfrac{h}{\mathrm{NC}} = \dfrac{h}{(-\mathrm{PC})}$, as C is virtual. Substituting in (i),

$$\therefore \quad \frac{h}{(-\mathrm{IP})} - \frac{h}{(+\mathrm{OP})} = \frac{2h}{(-\mathrm{CP})}$$

$$\therefore \quad \frac{1}{\mathrm{IP}} + \frac{1}{\mathrm{OP}} = \frac{2}{\mathrm{CP}}$$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{2}{r}$$

and

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

Thus, using the sign convention, the same formula holds for concave and convex mirrors.

Formula for Magnification

The lateral magnification, *m*, produced by a mirror is defined by

$$m = \frac{\textit{height of image}}{\textit{height of object}}.$$

Suppose IR is the image of an object OH in a concave or convex mirror. Fig. 17.11 (i), (ii). Then a ray HP from the top point H of the

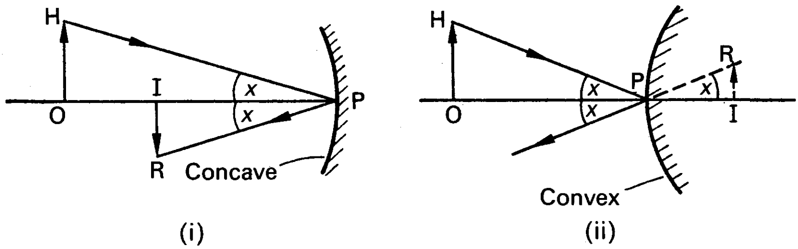

FIG. 17.11. Magnification formula.

object passes through the top point R of the image after reflection from the mirror. Now the normal to the mirror at P is the principal axis, OP. Thus angle OPH = angle IPR, from the law of reflection.

$$\therefore \quad \tan \text{OPH} = \tan \text{IPR}$$

$$\text{i.e.,} \quad \frac{\text{OH}}{\text{OP}} = \frac{\text{IR}}{\text{IP}}$$

$$\therefore \quad \frac{\text{IR}}{\text{OH}} = \frac{\text{IP}}{\text{OP}}$$

But IP = image distance = v, and OP = object distance = u

$$\therefore \quad \frac{\text{IR}}{\text{OH}} = \frac{v}{u}$$

$$\therefore \quad \mathbf{m} = \frac{\mathbf{v}}{\mathbf{u}} \quad . \quad . \quad . \quad . \quad (4)$$

Thus if the image distance is half the object distance, the image is half the length of the object.

Since
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore \frac{v}{v} + \frac{v}{u} = \frac{v}{f},$$

multiplying throughout by v.

$$1 + \frac{v}{u} = \frac{v}{f}$$

$$\therefore \ 1 + m = \frac{v}{f}$$

$$\therefore \quad m = \frac{v}{f} - 1$$

Some Applications of Mirror Formulæ

The following examples will assist the reader to understand how to apply the formulæ $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ and $m=\frac{v}{u}$ correctly:

1. An object is placed 10 cm in front of a concave mirror of focal length 15 cm. Find the image position and the magnification.

Since the mirror is concave, $f = +15$ cm. The object is real, and hence $u = +10$ cm. Substituting in $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$\frac{1}{v}+\frac{1}{(+10)}=\frac{1}{(+15)}$$

$$\therefore \frac{1}{v}=\frac{1}{15}-\frac{1}{10}=-\frac{1}{30}$$

$$\therefore v=-30$$

Since v is negative in sign the image is *virtual*, and it is 30 cm from the mirror. See Fig. 17.7 (i). The magnification, $m=\frac{v}{u}=\frac{30}{10}=3$, so that the image is three times as high as the object.

2. The image of an object in a convex mirror is 4 cm from the mirror. If the mirror has a radius of curvature of 24 cm, find the object position and the magnification.

The image in a convex mirror is always virtual (p. 406). Hence $v = -4$ cm. The focal length of the mirror $= \frac{1}{2}r = 12$ cm; and since the mirror is convex, $f = -12$ cm. Substituting in $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{(-4)}+\frac{1}{u}=\frac{1}{(-12)}$$

$$\therefore \frac{1}{u}=-\frac{1}{12}+\frac{1}{4}=\frac{1}{6}$$

$$\therefore u=6$$

Since u is positive in sign the object is real, and it is 6 cm from the mirror. The magnification, $m=\frac{v}{u}=\frac{4}{6}=\frac{2}{3}$, and hence the image is two-thirds as high as the object. See Fig. 17.8 (i).

3. An erect image, three times the size of the object, is obtained with a concave mirror of radius of curvature 36 cm. What is the position of the object?

If x cm is the numerical value of the distance of the object from the mirror, the image distance must be $3x$ cm, since the magnification $m = \frac{\text{image distance}}{\text{object distance}}$ $= 3$. Now an *erect* image is obtained with a concave mirror only when the image is *virtual* (p. 406).

$$\therefore \quad \text{image distance, } v = -3x$$

$$\text{Also, object distance, } u = +x$$

$$\text{and focal length, } f = \tfrac{1}{2}r = +18 \text{ cm.}$$

Substituting in $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$,

$$\therefore \frac{1}{(-3x)} + \frac{1}{(+x)} = \frac{1}{(+18)}$$

$$\therefore -\frac{1}{3x} + \frac{1}{x} = \frac{1}{18}$$

$$\therefore \frac{2}{3x} = \frac{1}{18}$$

$$\therefore \quad x = 12$$

Thus the object is 12 cm from the mirror.

Virtual Object and Convex Mirror

We have already seen that a convex mirror produces a virtual image of an object in front of it, which is a real object. A convex mirror may sometimes produce a real image of a *virtual* object.

As an illustration, consider an incident beam of light bounded by AB, DE, converging to a point O *behind* the mirror, Fig. 17.12. O is

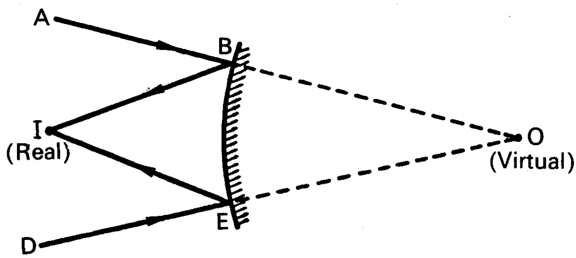

FIG. 17.12. Real image in convex mirror.

regarded as a virtual object, and if its distance from the mirror is 10 cm, then the object distance, $u = -10$. Suppose the convex mirror has a focal length of 15 cm, i.e., $f = -15$.

Since $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

$$\frac{1}{v}+\frac{1}{(-10)}=\frac{1}{(-15)}$$

$$\therefore \quad \frac{1}{v}=-\frac{1}{15}+\frac{1}{10}=+\frac{1}{30}$$

$$\therefore \quad v=+30$$

The point image, I, is thus 30 cm from the mirror, and is *real.* The beam reflected from the mirror is hence a convergent beam, Fig. 17.12; a similar case with a plane mirror is shown in Fig. 16.9 (ii).

Object at Centre of Curvature of Concave Mirror

Suppose an object is placed at the centre of curvature of a concave mirror. Then $u = +r$, where r is the numerical value of the radius of curvature. Substituting in $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$ to find the image distance v,

$$\frac{1}{v}+\frac{1}{(+r)}=\frac{2}{(+r)}$$

$$\therefore \quad \frac{1}{v}=\frac{2}{r}-\frac{1}{r}=\frac{1}{r}$$

$$\therefore \quad v=r$$

The image is therefore also formed at the centre of curvature. The magnification in this case is given by $m=\frac{v}{u}=\frac{r}{r}=1$, and hence the object and image are the same size. This case is illustrated in Fig. 17.7 (ii), to which the reader should now refer.

SOME METHODS OF DETERMINING FOCAL LENGTH AND RADIUS OF CURVATURE OF MIRRORS

Concave Mirror

Method 1. A pin O is placed above a concave mirror M so that an inverted image of the pin can be seen, Fig. 17.13. If the pin is moved up and down with its point on the axis of the mirror, and an observer E moves his eye perpendicularly to the pin at the same time, a position of O is reached when the image I remains perfectly in line with O as E moves; i.e., there is no parallax (no relative displacement) between

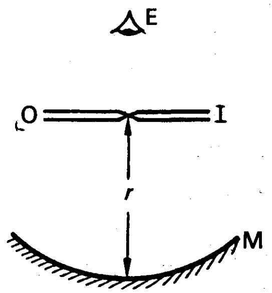

FIG. 17.13. Centre of curvature of concave mirror.

pin and image. The pin is now at exactly the same place as its image I, Fig. 17.13. Since an object and image coincide in position at the centre of curvature of a concave mirror (p. 413), the distance from the point of the pin to the mirror is equal to its radius of curvature, r. The focal length, f, which is $\frac{r}{2}$, is then easily obtained.

If an illuminated object is available instead of a pin, the object is moved to or from the mirror until a clear image is obtained beside the object. The distance of the object from the mirror is then equal to r. In general the method of no parallax, using a pin, gives a higher degree of accuracy in locating an image.

Method 2. By using the method of no parallax, or employing an illuminated object, several, say six, values of the image distance, v, can be obtained with the concave mirror, corresponding to six different values of the object distance u. Substituting for u, v in the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, six values of f can be calculated, and the average value taken.

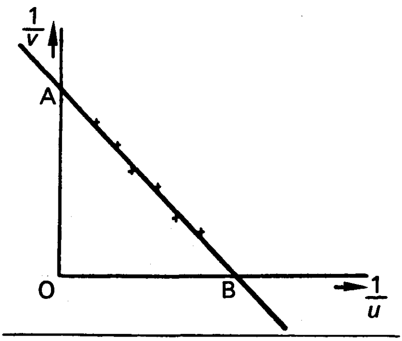

FIG. 17.14. Graph of $\frac{1}{v}$ against $\frac{1}{u}$.

A better method of procedure, however, is to plot the magnitudes of $\frac{1}{v}$ against $\frac{1}{u}$; a straight line BA can be drawn through the points thus obtained, Fig. 17.14. Now $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$. Hence, when $\frac{1}{v}=0$, $\frac{1}{u}=\frac{1}{f}$.

But
$$\text{OB} = \frac{1}{u} \text{ when } \frac{1}{v} = 0$$
$$\therefore \text{OB} = \frac{1}{f}, \quad \text{i.e., } f = \frac{1}{\text{OB}}.$$

Thus the focal length can be determined from the reciprocal value of the intercept OB on the axis of $\frac{1}{u}$.

From $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, $\frac{1}{v}=\frac{1}{f}$ when $\frac{1}{u}=0$. Thus OA $=\frac{1}{f}$, Fig. 17.14, and hence $f=\frac{1}{\text{OA}}$. It can thus be seen that (i) f can also be calculated from the reciprocal of the intercept OA on the axis of $\frac{1}{v}$, (ii) OAB is an isosceles triangle if the same scale is employed for $\frac{1}{v}$ and $\frac{1}{u}$.

Convex Mirror

Method 1. By using a convex lens, L, a real image of an object O can be formed at a point C on the other side of L, Fig. 17.15. The convex mirror, MN, is then placed between L and C with its reflecting face facing the lens, so that a convergent beam of rays is incident on the mirror. When the latter is moved along the axis OC a position will be reached when the beam is incident *normally* on the mirror, in which case the rays are reflected back along the incident path. *A real inverted image is then formed at O.*

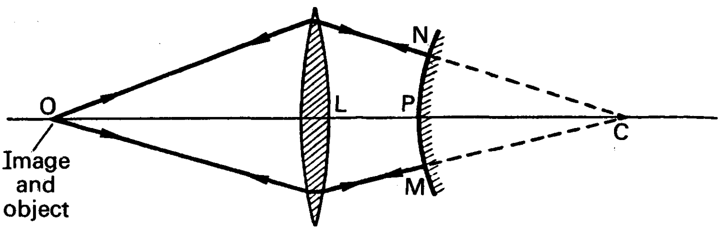

FIG. 17.15. Convex mirror measurement.

Since the rays incident on the mirror, for example at N or M, are normal to the mirror surface, they will, if produced, pass through the centre of curvature, C, of the mirror. Thus the distance PC = *r*, the radius of curvature. Since PC = LC − LP, this distance can be obtained from measurement of LP and LC, the latter being the image distance from the lens when the mirror is taken away.

Method 2. A more difficult method than the above consists of positioning a pin O in front of the convex mirror, when a virtual image, I, is formed, Fig. 17.16. A small plane mirror M is then moved between O and P until the image I′ of the lower part of O *in M* coincides in position with the upper part of the image of O in the convex mirror. The distances OP, MP are then measured.

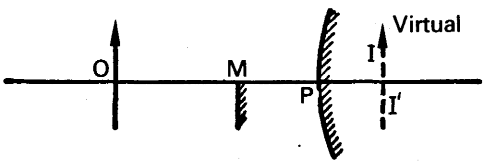

FIG. 17.16. Convex mirror measurement.

Since M is a plane mirror, the image I′ of O in it is such that OM = MI′. Thus PI = MI′ − MP = OM − MP, and hence PI can be calculated. But PI = v, the image distance of O in the convex mirror, and OP = u, the object distance. Substituting for the virtual distance v, and u, in $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, the focal length of the convex mirror can be found.

EXAMPLES

1. Show that a concave spherical mirror can produce a focused image of an object when certain conditions are observed, and prove the usual relation between the object and image distances. A linear object, 10 cm long, lies along the axis of a concave mirror whose radius of curvature is 30 cm, the near end of the object lying 18 cm from the mirror. Find the magnification of the image. (*W.*)

First part. The condition for a focused image is that the light from the object must be incident as a narrow beam round the pole of the mirror. This implies that the object must be small (p. 402). The usual relation between the object and image distances is proved on p. 407.

Second part. Suppose Q is the near end of the object, which is 18 cm from the mirror. The distance of the image of Q is given by

$$\frac{1}{v} + \frac{1}{(+18)} = \frac{1}{(+15)}$$

since $f = \frac{r}{2} = \frac{30}{2} = 15$ cm

$$\therefore \frac{1}{v} = \frac{1}{15} - \frac{1}{18}$$

from which $v = 90$ cm

The other end P of the object is (10 + 18) cm from the mirror, or 28 cm The image of P is given by

$$\frac{1}{v} + \frac{1}{(+28)} = \frac{1}{(+15)}$$

$$\therefore \frac{1}{v} = \frac{1}{15} - \frac{1}{28}$$

from which $v = 32{\cdot}3$ cm

$$\therefore \text{length of image} = 90 - 32{\cdot}3 = 57{\cdot}7 \text{ cm}$$

$$\therefore \text{magnification of image} = \frac{57{\cdot}7}{10} = 5{\cdot}8$$

2. PBCA is the axis of a concave spherical mirror, A being a point object, B its image, C the centre of curvature of the mirror and P the pole. Find a relation between PA, PB, and PC, supposing the aperture of the mirror to be small. A concave mirror forms, on a screen, a real image of twice the linear dimensions of the object. Object and screen are then moved until the image is three times the size of the object. If the shift of the screen is 25 cm determine the shift of the object and the focal length of the mirror. (*N.*)

First part. See text.

Second part. Suppose v is the distance of the screen from the mirror when the image is twice the length of the object. Since the magnification, m, is given by

$$m = \frac{v}{f} - 1 \quad . \quad . \quad . \quad . \quad \text{(i)}$$

where f is the focal length of the mirror (see p. 410),

$$\therefore 2 = \frac{v}{f} - 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

When $m = 3$, the image distance is $(v + 25)$ cm. Substituting in (i),

$$\therefore 3 = \frac{v + 25}{f} - 1 \quad . \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

Subtracting (ii) from (iii), we have

$$1 = \frac{25}{f}$$

$$\therefore f = 25 \text{ cm.}$$

From (ii), $2 = \frac{v}{25} - 1$, or $v = 75$ cm. The object distance, u, is thus given by $\frac{1}{75} + \frac{1}{u} = \frac{1}{25}$, from which $u = 37\frac{1}{2}$ cm. From (iii), $v = 100$ cm. The object distance, u, is then given by $\frac{1}{100} + \frac{1}{u} = \frac{1}{25}$, from which $u = 33\frac{1}{3}$ cm. Thus the shift of the object $= 37\frac{1}{2} - 33\frac{1}{3} = 4\frac{1}{6}$ cm.

EXERCISES 17

1. An object is placed (i) 10 cm, (ii) 4 cm from a concave mirror of radius curvature 12 cm. Calculate the image position in each case, and the respective magnifications.

2. Repeat Q. 1 by accurate drawings to scale. (*Note.*—The mirror must be represented by a straight line.)

3. An object is placed 15 cm from a convex mirror of focal length 10 cm. Calculate the image distance and the magnification produced. Draw an accurate diagram to scale, and verify your drawing from the calculated results.

4. Explain with the aid of diagrams why a curved mirror can be used (i) as a driving mirror, (ii) in a searchlight, (iii) as a shaving mirror. Why is a special form of mirror required in the searchlight?

5. Describe and explain a method of finding the focal length of (*a*) a concave mirror, (*b*) a convex mirror.

6. A pole 4 m long is laid along the principal axis of a convex mirror of focal length 1 m The end of the pole nearer the mirror is 2 m from it. Find the length of the image of the pole.

7. Deduce a formula connecting u, v and r, the distances of object, image and centre of curvature from a spherical mirror.

A mirror forms an erect image 30 cm from the object and twice its height. Where must the mirror be situated? What is its radius of curvature? Assuming the object to be real, determine whether the mirror is convex or concave. (*L.*)

8. Establish the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ for a concave mirror.

In an experiment with a concave mirror the magnification m of the image is measured for a series of values of v, and a curve is plotted between m and v. What curve would you expect to obtain, and how would you use it to deduce the focal length of the mirror? (*C.*)

9. Derive an approximate relation connecting the distances of an object and its image from the surface of a convex spherical mirror.

A small object is placed at right angles to the axis of a concave mirror so as to form (*a*) a real, (*b*) a virtual image, twice as long as the object. If the radius of curvature of the mirror is R what is the distance between the two images? (*L.*)

10. Deduce a formula connecting the distances of object and image from a spherical mirror. What are the advantages of a concave mirror over a lens for use in an astronomical telescope?

A driving mirror consists of a cylindrical mirror of radius 10 cm and length (over the curved surface) of 10 cm. If the eye of the driver be assumed at a great distance from the mirror, find the angle of view. (*O. & C.*)

11. Find the relation connecting the focal length of a convex spherical mirror with the distances from the mirror of a small object and the image formed by the mirror.

A convex mirror, radius of curvature 30 cm, forms a real image 20 cm from its surface. Explain how this is possible and find whether the image is erect or inverted. (*L.*)

12. What conditions must be satisfied for an optical system to form an image of an object? Show how these conditions are satisfied for a convex spherical mirror when a small object is placed on its axis and derive a relationship showing how the position of the image depends on the position of the object and the radius of curvature of the mirror.

A millimetre scale is placed at right angles to the axis of a convex mirror of radius of curvature 12 cm. This scale is 18 cm away from the pole of the mirror. Find the position of the image of the scale. What is the size of the divisions of the image? What is the ratio of the angle subtended by the image to that subtended by the object at a point on the axis 25 cm away from the object on the side remote from the mirror? (*O. & C.*)

13. Describe an experiment to determine the radius of curvature of a convex mirror by an optical method. Illustrate your answer with a ray diagram and explain how the result is derived from the observations.

A small convex mirror is placed 60 cm from the pole and on the axis of a large concave mirror, radius of curvature 200 cm. The position of the convex mirror is such that a real image of a distant object is formed in the plane of a hole drilled through the concave mirror at its pole. Calculate (*a*) the radius of curvature of the convex mirror, (*b*) the height of the real image if the distant object subtends an angle of 0·50° at the pole of the concave mirror. Draw a ray diagram to illustrate the action of the convex mirror in producing the image of a non-axial point of the object and suggest a practical application of this arrangement of mirrors. (*N.*)