chapter sixteen

Reflection at plane surfaces

HIGHLY-POLISHED metal surfaces reflect about 80 to 90 per cent of the light incident on them; *mirrors* in everyday use are therefore usually made by depositing silver on the back of glass. In special cases the front of the glass is coated with the metal; for example, the largest reflector in the world is a curved mirror nearly 5 metres across, the front of which is coated with aluminium (p. 544). Glass by itself will also reflect light, but the percentage reflected is small compared with the case of a silvered surface; it is about 5 per cent for an air-glass surface.

Laws of Reflection

If a ray of light, AO, is incident on a plane mirror XY at O, the angle AON made with the *normal* ON to the mirror is called the "angle of incidence", *i*, Fig. 16.1. The angle BON made by the reflected ray OB with the normal is called the "angle of reflection", *r;* and experiments with a ray-box and a plane mirror, for example, show that:

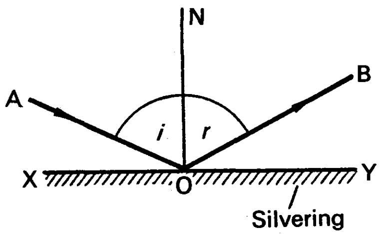

FIG. 16.1. Plane mirror.

(1) *The reflected ray, the incident ray, and the normal to the mirror at the point of incidence all lie in the same plane.*

(2) *The angle of incidence = the angle of reflection.*

These are called the two *laws of reflection*, and they were known to PLATO, who lived about 400 B.C.

Regular and Diffuse Reflection

In the case of a plane mirror or glass surface, it follows from the laws of reflection that a ray incident at a given angle on the surface is reflected in a definite direction. Thus a parallel beam of light incident on a plane mirror in the direction AO is reflected as a parallel beam in the direction OB, and this is known as a case of *regular reflection*, Fig. 16.2 (i). On the other hand, if a parallel beam of light is incident on a sheet of paper in a direction AO, the light is reflected in all different directions from the paper: this is an example of *diffuse reflection*, Fig. 16.2 (ii). Objects in everyday life, such as flowers, books, people, are seen by light diffusely reflected from them. The explanation of the diffusion of light is that the surface of paper, for example, is not perfectly smooth like a mirrored surface; the "roughness" in a paper surface can be seen with a micro-

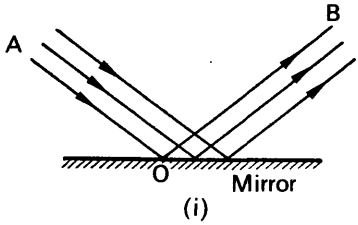

FIG. 16.2 (i). Regular reflection.

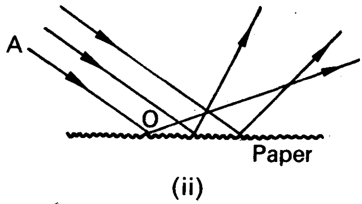

FIG. 16.2 (ii). Diffuse reflection.

scope. At each point on the paper the laws of reflection are obeyed, but the angle of incidence varies, unlike the case of a mirror.

Deviation of Light at Plane Mirror Surface

Besides other purposes, plane mirrors are used in the sextant (p. 395), in simple periscopes, and in signalling at sea. These instruments utilise the property of a plane mirror to deviate light from one direction to another.

Consider a ray AO incident at O on a plane mirror XY, Fig. 16.3 (i). The angle AOX made by AO with XY is known as the *glancing angle*, *g*, with the mirror; and since the angle of reflection is equal to the angle of incidence, the glancing angle BOY made by the reflected ray OB with the mirror is also equal to *g*.

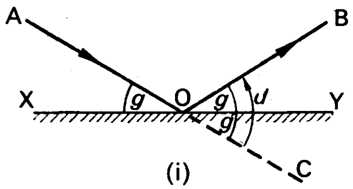

FIG. 16.3 (i).
Deviation of light at at plane mirror.

The light has been deviated from a direction AO to a direction OB. Thus if OC is the extension of AO, the *angle of deviation*, *d*, is angle COB. Since angle COY = vertically opposite angle XOA, it follows that

$$d = 2g \qquad (1);$$

so that, in general, *the angle of deviation of a ray by a plane surface is twice the glancing angle.*

Deviation of Reflected Ray by Rotated Mirror

Consider a ray AO incident at O on a plane mirror M_1, a being the glancing angle with M_1, Fig. 16.3 (ii). If OB is the reflected ray, then, as shown above, the angle of deviation COB = $2g = 2a$.

Suppose the mirror is rotated through an angle θ to a position M_2, the direction of the incident ray AO being *constant*. The ray is now reflected from M_2, in a direction OP, and the glancing angle with M_2 is $(\alpha + \theta)$. Hence the new angle of deviation COP $= 2g = 2(\alpha + \theta)$. The reflected ray has thus been rotated through an angle BOP when the mirror rotated through an angle θ; and since

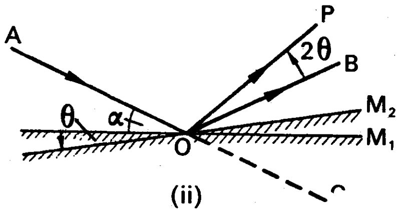

FIG. 16.3 (ii). Rotation of reflected ray.

$$\angle BOP = \angle COP - \angle COB,$$

then

$$\angle BOP = 2(\alpha + \theta) - 2\alpha = 2\theta.$$

Thus, *if the direction of an incident ray is constant, the angle of rotation oj the reflected ray is twice the angle of rotation of the mirror.* If the mirror rotates through 4°, the reflected ray turns through 8°, the direction of the incident ray being kept unaltered.

Optical Lever in Mirror Galvanometer

In a number of instruments a beam of light is used as a "pointer", which thus has a negligible weight and is sensitive to deflections of the moving system inside the instrument. In a mirror galvanometer, used for measuring very small electric currents, a small mirror M_1 is rigidly attached to a system which rotates when a current flows in it, and a beam of light from a fixed lamp L shines on the mirror, Fig. 16.4. If the light is incident normally on the mirror at A, the beam is reflected directly back, and a spot of light is obtained at O on a graduated scale S

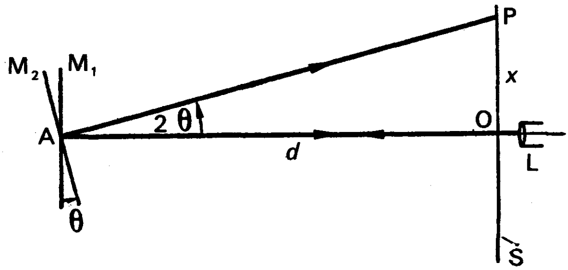

FIG. 16.4. Optical lever principle.

placed just above L. Suppose that the moving system, to which the mirror is attached, undergoes a rotation θ. The mirror is then rotated through this angle to a position M_2, and the spot of light is deflected through a distance x, say to a position P on the scale.

Since the direction OA of the incident light is constant, the rotation of the reflected ray is twice the angle of rotation of the mirror (p. 393). Thus angle OAP $= 2\theta$. Now $\tan 2\theta = x/d$, where d is the distance OA. Thus 2θ can be calculated from a knowledge of x and d, and hence θ

is obtained. If 2θ is small, then $\tan 2\theta$ is approximately equal to 2θ in radians, and in this case θ is equal to $x/2d$ radians.

In conjunction with a mirror, a beam of light used as a "pointer" is known as an "optical lever". Besides a negligible weight, it has the advantage of magnifying by two the rotation of the system to which the mirror is attached, as the angle of rotation of the reflected light is twice the angle of rotation of the mirror. An optical lever can be used for measuring small increases of length due to the expansion or contraction of a solid.

Deviation by Successive Reflections at Two Inclined Mirrors

Before we can deal with the principle of the sextant, the deviation of light by successive reflection at two inclined mirrors must be discussed.

Consider two mirrors, XO, XB, inclined at an angle θ, and suppose AO is a ray incident on the mirror XO at a glancing angle α, Fig. 16.5 (i). The reflected ray OB then also makes a glancing angle α with OX, and from our result on p. 392, the angle of deviation produced by XO in a clockwise direction (angle LOB) $= 2\alpha$.

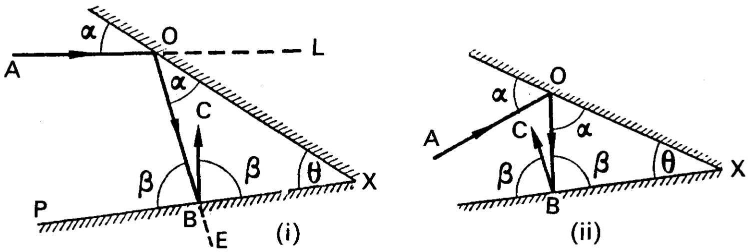

FIG. 16.5. Successive reflection at two plane mirrors.

Suppose OB is incident at a glancing angle β on the second mirror XB. Then, if the reflected ray is BC, the angle of deviation produced by this mirror (angle EBC) $= 2\beta$, in an anti-clockwise direction. Thus the net deviation D of the incident ray AO produced by both mirrors $= 2\beta - 2\alpha$, in an anti-clockwise direction.

Now from triangle OBX,

$$\text{angle PBO} = \text{angle BOX} + \text{angle BXO},$$

i.e.,

$$\beta = \alpha + \theta$$

Thus $\theta = \beta - \alpha$, and hence

$$D = 2\beta - 2\alpha = 2\theta.$$

But θ is a *constant* when the two mirrors are inclined at a given angle. *Thus, no matter what angle the incident ray makes with the first mirror, the deviation D after two successive reflections is constant and equal to twice the angle between the mirrors.*

Fig. 16.5 (ii) illustrates the case when the ray BC reflected at the second mirror travels in an opposite direction to the incident ray AO, unlike the case in Fig. 16.5 (i). In Fig. 16.5 (ii), the net deviation, D, after two successive reflections in a clockwise direction is $2\alpha + 2\beta$. But $\alpha + \beta = 180° - \theta$. Hence $D = 2\alpha + 2\beta = 360° - 2\theta$. Thus the deviation, D, in an anti-clockwise direction is 2θ, the same result as obtained above.

Principle of the Sextant

The *sextant* is an instrument used in navigation for measuring the angle of elevation of the sun or stars. It consists essentially of a fixed glass B, silvered on a vertical half, and a silvered mirror O which can be rotated about a horizontal axis. A small fixed telescope T is directed towards B, Fig. 16.6.

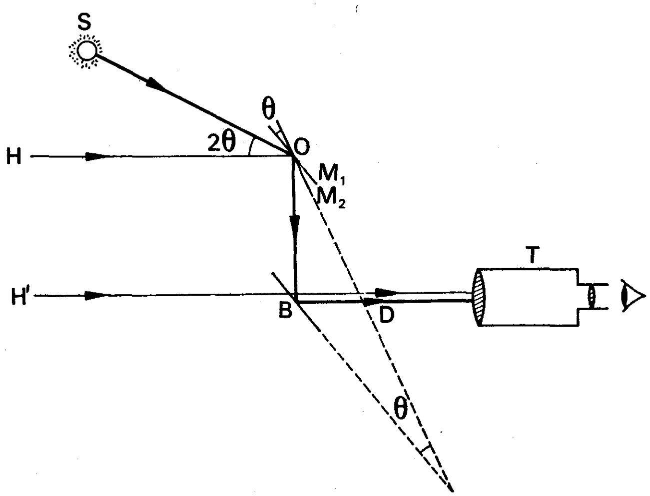

FIG. 16.6. Sextant principle.

Suppose that the angle of elevation of the sun, S, is required. Looking through T, the mirror O is turned until the view H′ of the horizon seen directly through the unsilvered half of B, and also the view of it, H, seen by successive reflection at O and the silvered half of B, are coincident. The mirror O is then parallel to B in the position M_1, and the ray HO is reflected along OB and BD to enter the telescope T. The mirror O is now rotated to a position M_2 until the image of the sun S, seen by successive reflections at O and B, is on the horizon H′, and the angle of rotation, θ, of the mirror is noted, Fig. 16.6.

The ray SO from the sun is now reflected in turn from O and B so that it travels along BD, the direction of the horizon, and the angle of deviation of the ray is thus angle SOH. But the angle between the mirrors M_2 and B is θ. Thus, from our result for successive reflections at two inclined mirrors, angle SOH $= 2\theta$. Now the angle of elevation of the sun, S, is angle SOH. Hence *the angle of elevation is twice the angle* *of rotation of the mirror O*, and can thus be easily measured from a scale (not shown) which measures the rotation of O.

Since the angle of deviation after two successive reflections is independent of the angle of incidence on the first mirror (p. 394), the image of the sun S through T will continue to be seen on the horizon once O is adjusted, no matter how the ship pitches or rolls. This is an advantage of the sextant.

Images in Plane Mirrors

So far we have discussed the deviation of light by a plane mirror. We have now to consider the *images* in plane mirrors.

Suppose that a **point object** A is placed in front of a mirror M, Fig. 16.7. A ray AO from A, incident on M, is reflected along OB in such a way that angle AON = angle BON, where ON is the normal at O to the mirror. A ray AD incident normally on the mirror at D is reflected back along DA. Thus the rays reflected from M appear to come from a point I *behind* the mirror, where I is the point of intersection of BO and AD produced. As we shall prove shortly, any ray from A, such as AP, is also reflected as if it comes from I, and hence an observer at E sees the image of A at I.

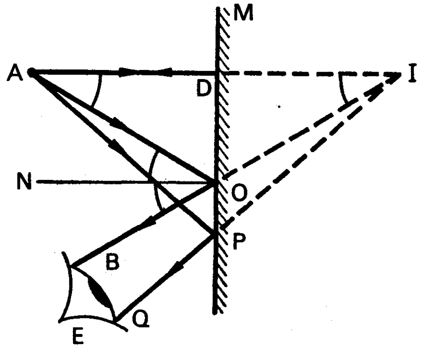

FIG. 16.7. Image in plane mirror.

Since angle AON = alternate angle DAO, and angle BON = corresponding angle DIO, it follows that angle DAO = angle DIO. Thus in the triangles ODA, ODI, angle DAO = angle DIO, OD is common, and angle ADO = 90° = angle IDO. The two triangles are hence congruent, and therefore AD = ID. For a given position of the object, A and D are fixed points. Consequently, since AD = ID, the point I is a fixed point; and hence *any* ray reflected from the mirror must pass through I, as was stated above.

We have just shown that *the object and image in a plane mirror are at equal perpendicular distances from the mirror.* It should also be noted that AO = OI in Fig. 16.7, and hence the object and image are at equal distances from any point on the mirror.

Image of Finite-sized Object. Perversion

If a right-handed batsman observes his stance in a plane mirror, he appears to be left-handed. Again, the words on a piece of blotting-paper become legible when the paper is viewed in a mirror. This

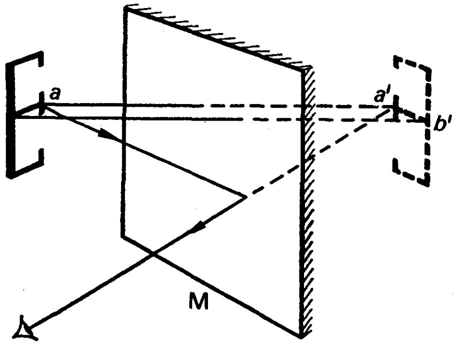

FIG. 16.8. Perverted (laterally inverted) image.

phenomenon can be explained by considering an E-shaped object placed in front of a mirror M, Fig. 16.8. The image of a point *a* on the object is at *a'* at an equal distance behind the mirror, and the image of a point *b* on the left of *a* is at *b'*, which is on the *right* of *a'*. The left-hand side of the image thus corresponds to the right-hand side of the object, and vice-versa, and the object is said to be *perverted*, or *laterally inverted* to an observer.

Virtual and Real Images

As was shown on p. 396, an object O in front of a mirror has an image I behind the mirror. The rays reflected from the mirror do not actually pass through I, but only *appear* to do so, and the image cannot be received on a screen because the image is behind the mirror, Fig. 16.9 (i). This type of image is therefore called an unreal or **virtual** image.

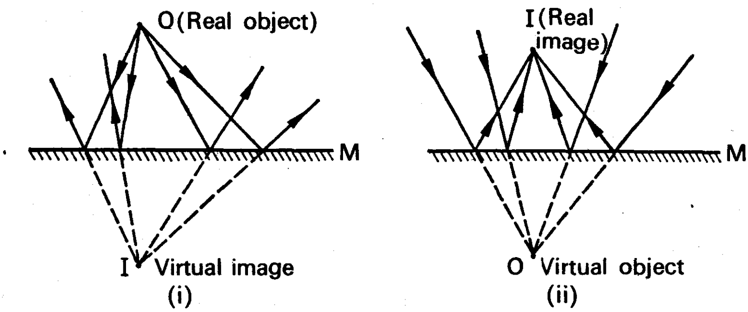

FIG. 16.9. Virtual and real image in plane mirror.

It must not be thought, however, that only virtual images are obtained with a plane mirror. If a *convergent* beam is incident on a plane mirror M, the reflected rays pass through a point I *in front of* M, Fig. 16.9 (ii). If the incident beam converges to the point O, the latter is termed a "virtual" object; I is called a **real** image because it can be received on a screen. Fig. 16.9 (i) and (ii) should now be compared. In the former, a real object (divergent beam) gives rise to a virtual image; in the latter, a virtual object (convergent beam) gives rise to a real image. In each case the image and object are at equal distances from the mirror.

Location of Images by No Parallax Method

A virtual image can be located by the *method of no parallax*, which we shall now describe.

Suppose O is a pin placed in front of a plane mirror M, giving rise to a virtual image I behind M, Fig. 16.10. A pin P behind the mirror is then

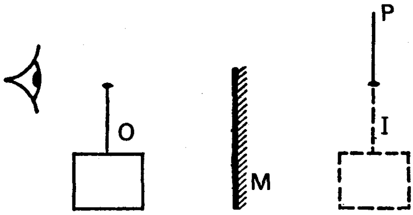

FIG. 16.10. Location of image by no parallax method.

moved towards or away from M, each time moving the head from side to side so as to detect any relative motion between I and P. When the latter appear to move together they are coincident in position, and hence P is at the place of the image I, which is thus located. When P and I do not coincide, they appear to move relative to one another when the observer's head is moved; this relative movement is called "parallax". It is useful to note that the nearer object moves in the opposite direction to the observer.

The method of no parallax can be used, as we shall see later, to locate the positions of real images, as well as virtual images, obtained with lenses and curved mirrors.

Images in Inclined Mirrors

A *kaleidoscope*, produced as a toy under the name of "mirrorscope", consists of two inclined pieces of plane glass with some coloured tinsel between them. On looking into the kaleidoscope a beautiful series of coloured images of the tinsel can often be seen, and the instrument is used by designers to obtain ideas on colouring fashions.

Suppose OA, OB are two mirrors inclined at an angle θ, and P is a point object between them, Fig. 16.11 (i). The image of P in the mirror OB is B_1, and $OP = OB_1$ (see p. 396). B_1 then forms an image B_2 in the mirror OA, with $OB_2 = OB_1$, B_2 forms an image B_3 in OB, and so on. All the images thus lie on a circle of centre O and radius OP. Another set of images, A_1, A_2, A_3 . . ., have their origin in the image A_1 formed by P in the mirror OA. When the observer looks into the mirror OB he sees

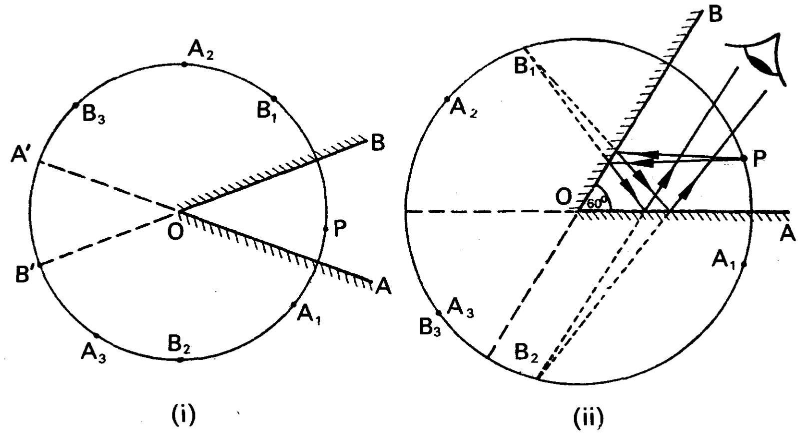

FIG. 16.11. Images in inclined mirrors.

the series of images B_1, A_2, B_3 . . .; when he looks into the mirror OA he sees the series of images A_1, B_2, A_3 . . . A finite series of images is seen in either mirror, the last image (not shown) being the one formed on the arc A′B′, because it is then *behind* the silvering of the next mirror.

When the mirrors are inclined at an angle of 60°, the final images of P, A_3, B_3, of each series coincide, Fig. 16.11 (ii). The total number of images is now 5, as the reader can verify. Fig. 16.11 (ii) illustrates the cone of light received by the pupil of the eye when the image B_2 is observed, reflection occurring successively at the mirrors. The drawing is started by joining B_2 to the boundary of the eye, then using B_1, and finally using P.

EXAMPLE

State the laws of reflection of light and describe how you would verify these laws.

A man 2 m tall, whose eye level is 1·84 m above the ground, looks at his image in a vertical mirror. What is the minimum vertical length of the mirror if the man is to be able to see the whole of himself? Indicate its position accurately in a diagram.

First part. See text. A ray-box can be used to verify the laws.

Second part. Suppose the man is represented by HF, where H is his head and F is his feet; suppose that E represents his eyes, Fig. 16.12. Since the man

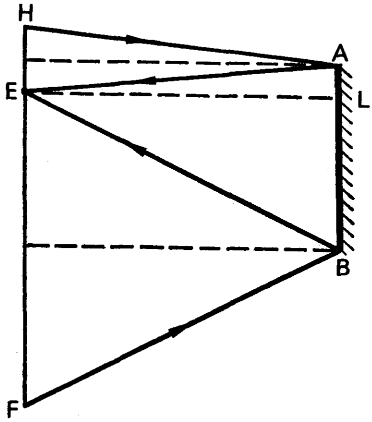

FIG. 16.12. Example.

sees his head H, a ray HA from H to the top A of the mirror is reflected to E. Thus A lies on the perpendicular bisector of HE, and hence AL $= \frac{1}{2}$ HE $= 0{\cdot}08$ m, where L is the point on the mirror at the same level as E. Since the man sees his feet F, a ray FB from F to the bottom B of the mirror is also reflected to E. Thus the perpendicular bisector of EF passes through B, and hence BL $= \frac{1}{2}$ FE $= \frac{1}{2} \times 1{\cdot}84$ m $= 0{\cdot}92$ m.

$\therefore$ length of mirror $=$ AL $+$ LB $= 0{\cdot}08$ m $+ 0{\cdot}92$ m $= 1$ m.

EXERCISES 16

1. Prove the relation between the angle of rotation of a mirror and the angle of deflection of a reflected ray, when the direction of the incident ray is constant.

2. Two plane mirrors are inclined at an angle of 35°. A ray of light is incident on one mirror at 60°, and undergoes two successive reflections at the mirrors. Show by accurate drawing that the angle of deviation produced is 70°.

Repeat with an angle of incidence of 45°, instead of 60°, and state the law concerning the angle of deviation.

3. Two plane mirrors are inclined to each other at a fixed angle. If a ray travelling in a plane perpendicular to both mirrors is reflected first from one and then from the other, show that the angle through which it is deflected does not depend on the angle at which it strikes the first mirror.

Describe and explain the action of *either* a sextant *or* a rear reflector on a bicycle. (*L.*)

4. State the laws of reflection of light. Two plane mirrors are parallel and face each other. They are *a* cm apart and a small luminous object is placed *b* cm from one of them. Find the distance from the object of an image produced by four reflections. Deduce the corresponding distance for an image produced by 2*n* reflections. (*L.*)

5. Two vertical plane mirrors A and B are inclined to one another at an angle *a*. A ray of light, travelling horizontally, is reflected first from A and then from B. Find the resultant deviation and show it is independent of the original direction of the ray. Describe an optical instrument that depends on the above proposition. (*N.*)

6. State the laws of reflection for a parallel beam of light incident upon a plane mirror.

Indicate clearly by means of diagrams (*a*) how the position and size of the image of an extended object may be determined by geometrical construction, in the case of reflection in a plane; (*b*) how the positions of the images of a small lamp, placed unsymmetrically between parallel reflecting planes, may be graphically determined. (*W.*)

7. Describe the construction of the *sextant* and the *periscope*. Illustrate your answer by clear diagrams and indicate the optical principles involved. (*L.*)