chapter eleven

Thermal expansion

In this chapter we shall discuss the thermal expansion of solids and liquids.

SOLIDS

Linear Expansion

Most solids increase in length when they are warmed. Fig. 11.1 shows a simple apparatus with which we can measure the linear expansion of a metal tube A. We first measure the length of the tube, l_1, at room temperature, θ_1; then we screw the spherometer S against the end of the tube and take its reading, S_1. We next heat the tube by passing steam through it. At intervals we re-adjust the spherometer; when its reading becomes constant, the temperature of the rod is steady. We measure the temperature θ_2 on the thermometer B, and take the new reading of the spherometer, S_2. The expansion of the tube is

$$e = S_2 - S_1.$$

The increase in length, λ, of unit length of the material for one degree temperature rise is then given by

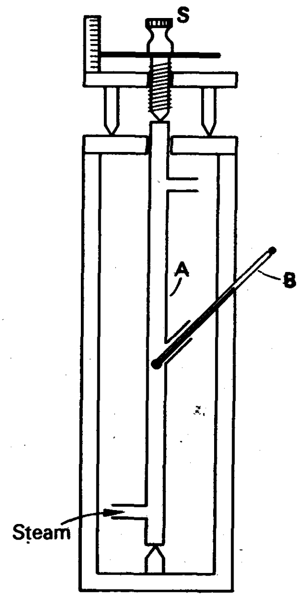

FIG. 11.1. Linear expansivity.

$$\lambda = \frac{\text{expansion}}{\text{original length} \times \text{temperature rise}} = \frac{e}{l_1(\theta_2 - \theta_1)}.$$

The quantity λ is called the *mean linear expansivity* of the metal, over the range θ_1 to θ_2. If this range is not too great—say less than 100°C—the quantity λ may, to a first approximation, be taken as constant.

The linear expansivity of a solid, like the pressure and volume coefficients of a gas, has the unit deg C^{-1} or 'K^{-1}' in SI units; its dimensions are

$$[\lambda] = \frac{[\text{length}]}{[\text{length}] \times [\text{temp.}]} = [\text{temp.}]^{-1}.$$

From the definition of λ, we can estimate the new length of a rod, l_2, at a temperature θ_2 from the equation

$$l_2 = l_1\{1+\lambda(\theta_2-\theta_1)\}, \quad . \quad . \quad . \quad (1)$$

where l_1 is the length of the rod at the temperature θ_1, and λ is the mean value over a range which includes θ_2 and θ_1.

For accurate work, however, the length of a solid at a temperature θ must be represented by an equation of the form

$$l = l_0(1+a\theta+b\theta^2+c\theta^3+\ldots), \quad . \quad . \quad . \quad (2)$$

where l_0 is the length at 0°C, and *a*, *b*, *c* are constants. The constant *a* is of the same order of magnitude as the mean coefficient λ; the other constants are smaller.

MEAN LINEAR EXPANSIVITY

(Near room temperature)

Substance	λ, K^{-1} $\times 10^{-6}$	Substance	λ, K^{-1} $\times 10^{-6}$
Copper	17	Bakelite	22
Iron	12	Brick	9·5
Brass	19	Glass (soda) . .	8·5
Nickel	13	Quartz (fused) (0–30°C) .	0·42
Platinum . . .	9	Pine—across grain .	*c.* 0·34
Invar (36% nickel-steel) .	*c.* 0·1	Pyrex	3

When a solid is subjected to small changes of temperature, about a mean value θ, its linear expansivity λ_θ in the neighbourhood of θ may be defined by the equation

$$\lambda_\theta = \frac{1}{l}\frac{dl}{d\theta},$$

where l is the length of the bar at the temperature θ. The following table shows how the coefficient varies with temperature.

VALUES OF λ_θ COPPER

θ	−87	0	100	400	600	°C
λ_θ	14·1	16·1	16·9	19·3	20·9	$\times 10^{-6} K^{-1}$

Accurate Measurement of Expansion

An instrument for accurately measuring the length of a bar, at a controlled temperature, is called a *comparator* (Fig. 11.2). It consists of two microscopes, M_1, M_2, rigidly attached to steel or concrete pillars P_1, P_2. Between the pillars are rails R_1, R_2, carrying water-baths such as B. One of these baths contains the bar under test, X, which has scratches near its ends; the scratches are nominally a metre apart. Another water bath contains a substandard metre. The eye-pieces of the microscopes are fitted with cross-webs carried on micrometer screws, m_1, m_2.

First the substandard metre is run under the microscopes, and the temperature of its bath is adjusted to that at which the bar was calibrated (usually about 18°C).

When the temperature of the bar is steady, the eyepiece webs are adjusted to intersect the scratches on its ends (Fig. 11.2 (b)), and their micrometers are read. The distance between the cross-webs is then 1 metre.

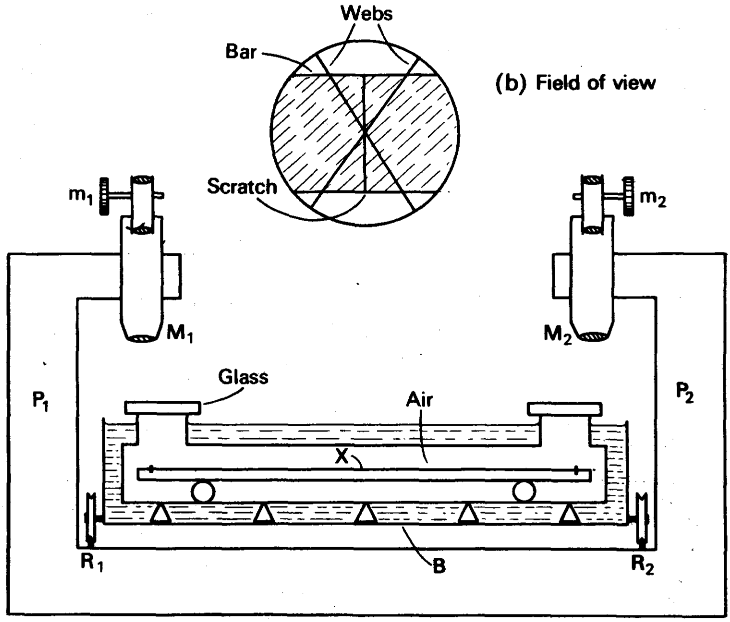

FIG. 11.2. The comparator.

The substandard is now removed, and the unknown bar put in its place; the temperature of the bar is brought to 0°C by filling its bath with ice-water. When the temperature of the bar is steady, the eyepiece webs are re-adjusted to intersect the scratches on its ends, and their micrometers are read. If the right-hand web has been shifted x mm to the right, and the left-hand y mm, also to the right, then the length of the bar at 0°C is

$$l_0 = 1 \text{ metre} + (x - y) \text{ mm}.$$

The bath is now warmed to say 10°C, and the length of the bar again measured. In this way the length can be measured at small intervals of temperature, and the mean linear expansivity, or the coefficients a, b, c in equation (2), can be determined.

Expansion at High Temperatures

Fig. 11.3 illustrates the principle of a method for measuring the expansion of a solid at high temperatures. A is a tube of fused silica,

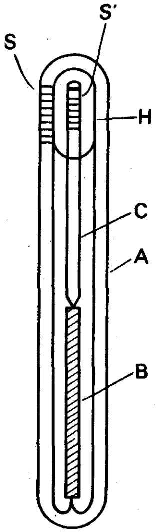

FIG. 11.3. Apparatus for measuring expansion at high temperatures.

having a scale S engraved on the edge of an opening, H, in its side. B is the specimen, and C is a rod of fused silica with a second scale S′ engraved on it.

The thermal expansion of fused silica is much less than that of most solids, over a given temperature range, and has been accurately measured by a method depending on optical interference. When the apparatus shown is placed in a high-temperature bath, the rod C rises by an amount equal to the difference in expansion of the specimen, and an equal length of fused silica. Its rise is measured by observing the displacement of the scale S′ relative to S through a microscope.

Force set up when Expansion is Resisted

Consider a metal rod between two supports P, which we suppose are immovable (Fig. 11.4). Let l_1 be the distance between the supports, and θ_1 the temperature at which the rod just fits between them. If the rod is heated to θ_2, it will try to expand to a greater length l_2, but will not be able to do so.

The value of l_2 would be

$$l_2 = l_1\{1+\lambda(\theta_2-\theta_1)\},$$

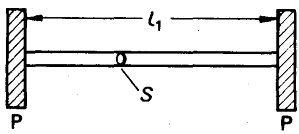

FIG. 11.4. Force in bar.

and the expansion would be

$$e = l_2-l_1 = l_1\lambda(\theta_2-\theta_1),$$

where λ is the mean linear expansivity of the rod.

The force which opposes the expansion is the force which would compress a bar of natural length l_2 by the amount e. Its magnitude F depends on the cross-section of the bar, A, and the Young's modulus of its material, E:

$$\frac{F}{A} = E\frac{e}{l_2}.$$

To a very good approximation we may replace l_2 by l_1, because their difference is small compared with either of them. Thus

$$\frac{F}{A} = E\frac{e}{l_1} = E\frac{\lambda l_1(\theta_2-\theta_1)}{l_1}$$

$$= E\lambda(\theta_2-\theta_1),$$

$$\therefore F = EA\lambda(\theta_2-\theta_1).$$

For steel, $\lambda = 12\times10^{-6}$ K^{-1} and $E = 2\times10^{11}$ newton per m^2. If the temperature difference, $\theta_2-\theta_1$, is 100°C, then, for a cross-sectional area S of 4 cm^2 or 4×10^{-4} m^2,

$$F = 2\times10^{11}\times12\times10^{-6}\times4\times10^{-4}\times100 \text{ newton}$$
$$= 9{\cdot}6\times10^{4} \text{ newton}.$$

On converting to kgf we find F is nearly 10000 kgf.

Expansion of a Measuring Scale

A scale, such as a metre rule, expands with rise in temperature; its readings, therefore, are correct at one temperature, θ_1 say. When the temperature of the scale is greater than θ_1, the distance between any two of its divisions increases, and its reading is therefore too low (Fig. 11.5); when the scale is below θ_1, its reading is too high. Let us suppose that, at θ_1, the distance between any two points P, Q, on the scale is l_1 cm. At θ_2 it is

$$l_2 = l_1\{1+\lambda(\theta_2-\theta_1)\},$$

where λ is the mean linear expansivity of the material of this scale. According to the divisions on the scale, however, the distance between P and Q will still be l_1 cm. Thus

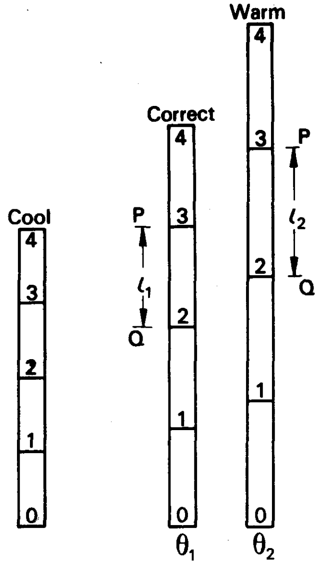

FIG. 11.5. Expansion of a measuring scale.

$$\text{true distance at } \theta_2 = \text{scale value} \times \{1+\lambda(\theta_2-\theta_1)\} \quad . \qquad (3)$$

If a sheet of material with a hole in it is warmed, it expands, and the hole expands with it. In Fig. 11.6, A represents a hole in a plate, and A′ represents a plug, of the same material, that fits the hole. If A and A′ are at the same temperature, then A′ will fit A, whatever the value of that temperature; for we can always imagine A′ to have just been cut out, without loss of material. It follows that the expansion of the hole A, in every direction, is the same as the expansion of the *solid* plug A′.

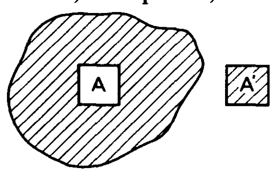

FIG. 11.6. Expansion of a hole.

Differential Expansion

The difference in the expansions of different materials is used in practical arrangements discussed shortly. Fig. 11.7 (A) shows two rods AB, AB′, of different metals, rigidly connected at A. If l_1, l_1', are their lengths at a temperature θ_1, their difference is

$$d_1 = \text{BB}' = l_1 - l_1'$$

If λ and λ' are the mean linear expansivities of the materials of the rods, then the lengths of the rods at θ_2 are

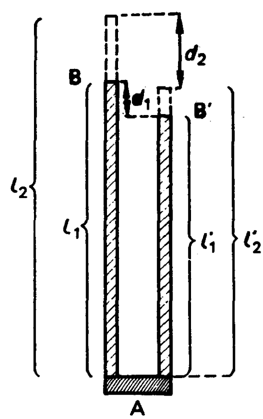

FIG. 11.7A. Differential expansion.

$$l_2 = l_1\{1+\lambda(\theta_2-\theta_1)\},$$
$$l'_2 = l'_1\{1+\lambda'(\theta_2-\theta_1)\}.$$

The distance between their ends is now

$$d_2 = l_2-l'_2 = l_1-l'_1+(l_1\lambda-l'_1\lambda')(\theta_2-\theta_1),$$

or

$$d_2 = d_1+(l_1\lambda-l'_1\lambda')(\theta_2-\theta_1) \quad . \quad . \quad . \quad . \quad (4)$$

By a suitable choice of lengths and materials, the distance BB′ can be made to vary with temperature in any one of the following ways:

(1) The bar AB′ is made of invar, a nickel-steel whose linear expansivity is very small (p. 270). The point B′ then does not move with changes in temperature. In equation (4) we neglect λ', and find:

$$d_2 = d_1+l_1\lambda(\theta_2-\theta_1).$$

Thus the short distance BB′ expands by the same amount as the long distance AB. Consequently, the relative expansion of BB′ with temperature is much greater than that of a bar of length d_1.

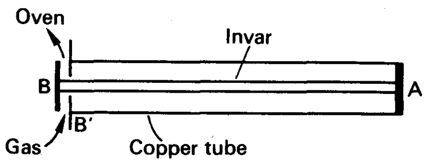

FIG. 11.7B. Thermostat principle.

(2) AB is made of invar, so that λ is negligible. The point B then does not move, and the distance BB′ *shrinks* rapidly as the temperature rises. This principle is used in the thermostats used to maintain gas ovens at constant temperatures (Fig. 11.7 (B)).

(3) The lengths and materials are chosen so that

$$\frac{l_1}{l'_1} = \frac{\lambda'}{\lambda},$$

or

$$l_1\lambda = l'_1\lambda'.$$

Then, by equation (4), d_1 or BB′ does not change with temperature. This principle is used in compensating clock pendula for temperature changes (p. 276).

Bimetal Strip

Fig. 11.8 (a) shows two strips of different metals, welded together along B, called a bimetal strip. The metal M_1 has a greater linear expansivity than the metal M_2. Therefore, when the strip is heated, M_1 will expand more than M_2, and the strip will curl with M_1 on the outside. The reverse is true when the strip is cooled, as M_1 then shrinks more than M_2 (see Fig. 11.8(a)).

Bi-metal strips are used in electrical thermostats for ovens, irons, laboratories, etc. The strip carries a contact, K in Fig. 11.8 (b), which presses against another contact K′ on the end of an adjusting screw S. When the strip warms, it tends to curl away from K′; the temperature at which the contact is broken can be set by turning the screw. When

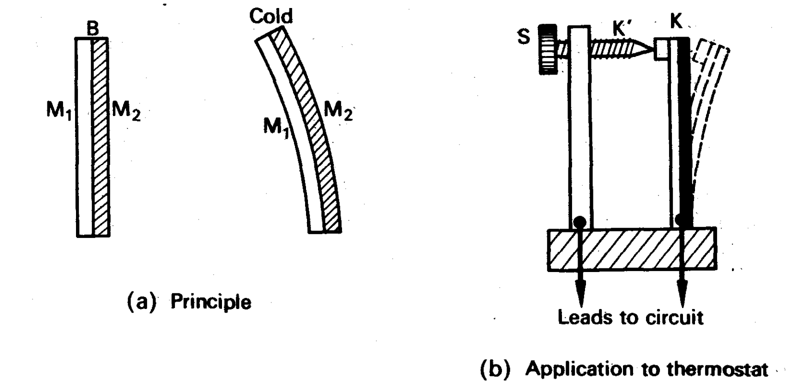

FIG. 11.8. Principle of gas thermostat.

the contacts open, they switch off the heating current. If the heating current is too great to be controlled by the contacts KK′, it is switched off by a relay, which is controlled by the contacts on the bimetal.

Let us now estimate the deflection of a heated bimetal strip. We suppose that the component strips have the same thickness d, and the same length l at a temperature θ_1 (Fig. 11.9 (a)). When they are heated, they are distorted, but to a first approximation we may assume that the mid-line of each, shown dotted, has the length which it would naturally have (Fig. 11.9 (b)). The difference in length of the mid-lines, p, is then the difference in their expansions. At a temperature θ_2,

$$p = l\lambda(\theta_2-\theta_1)-l\lambda'(\theta_2-\theta_1)$$
$$= l(\lambda-\lambda')(\theta_2-\theta_1).$$

The difference is taken up by the curvature of the strip. If α is the angle through which it bends, then, from the figure,

$$\alpha = \frac{p}{d} = \frac{l}{d}(\lambda-\lambda')(\theta_2-\theta_1).$$

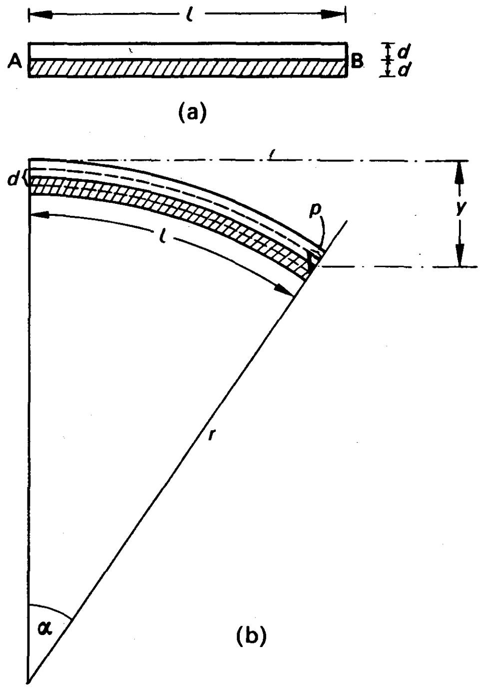

FIG. 11.9. Expansion of bimetal strip (exaggerated).

(The expansion of d is negligible, to a very good approximation.)

To find the radius r of the arc formed by the strip, we assume that the length of the arc is l; this we may do because the expansions in length are all small. Then

$$\alpha = \frac{l}{r};$$

and, by the above equation,

$$\frac{l}{r} = \frac{l}{d}(\lambda - \lambda')(\theta_2 - \theta_1),$$

whence

$$r = \frac{d}{(\lambda - \lambda')(\theta_2 - \theta_1)}.$$

The deflection y of the end of the strip is given by the approximate equation

$$2ry = l^2,$$

from the geometry of the circle.

Thus

$$y = \frac{l^2}{2r} = \frac{l^2(\lambda - \lambda')(\theta_2 - \theta_1)}{2d}.$$

Temperature Compensation

The rate of a clock varies considerably with temperature, unless arrangements are made to prevent its doing so. If the clock is governed by a pendulum, the length of the pendulum increases with temperature, its period therefore also increases, and the clock loses. A clock governed by a balance-wheel and hair-spring also loses as the temperature rises. For, as the temperature rises, the spring becomes less stiff, and the period of the balance-wheel increases. Also the spokes of the wheel expand a little, increasing the moment of inertia of the wheel, and thus further increasing its period.

A balance-wheel clock may be compensated against temperature changes by making the circumference of the wheel in the form of two or three bimetal strips, as shown in Fig. 11.10. The strips carry small weights W, to give the wheel the necessary moment of inertia. As the temperature rises, the strips curl inwards, and bring the weights nearer to the axle; thus the moment of inertia of the wheel decreases. In a correctly designed timing-system, the decrease in moment of inertia just offsets the decrease in stiffness of the spring, and then the period of the balance-wheel does not change with temperature.

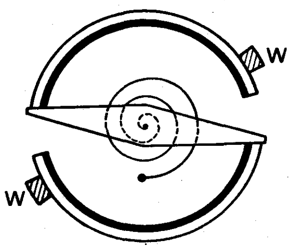

FIG. 11.10. Bimetal balance-wheel.

Many modern watches are not compensated. Their balance-wheels are made of invar, which, as we have seen, has a very small coefficient of expansion. Another nickel-steel, of slightly different composition, is used for their hair-springs. This alloy changes its elasticity very little with temperature, and is called *elinvar*. The combination of invar balance-wheel and elinvar hair-spring gives a rate which is nearly enough independent of temperature for everyday purposes.

Pendula

To compensate a pendulum clock against changes of temperature, the pendulum must be so made that its effective length remains constant. Fig. 11.11 (a) shows one way of doing this, in the so-called grid-iron pendulum (Harrison, 1761). Brass and steel rods are arranged so that the expansion of the brass rods raises the bob B of the pendulum, while the expansion of the steel rods lowers it. As explained on p. 274, the expansions can be made to cancel if

$$\frac{l_B}{l_S} = \frac{\lambda_S}{\lambda_B};$$

here l_B and l_S are the total lengths of brass and steel respectively, and λ_B, λ_S are their linear expansivities.

Fig. 11.11 (b) shows the same principle applied to a pendulum with a wooden rod and a cylindrical metal bob. To a first approximation the effective length of the pendulum, l, is the distance from its support to the centre of gravity, G, of the bob. The condition for constant period is then

$$\frac{\text{length of rod}}{\frac{1}{2}\text{ length of bob}} = \frac{\lambda_{\text{wood}}}{\lambda_{\text{metal}}}.$$

The bob may be made of lead or zinc, either of which is much more expansible than is wood, along its grain.

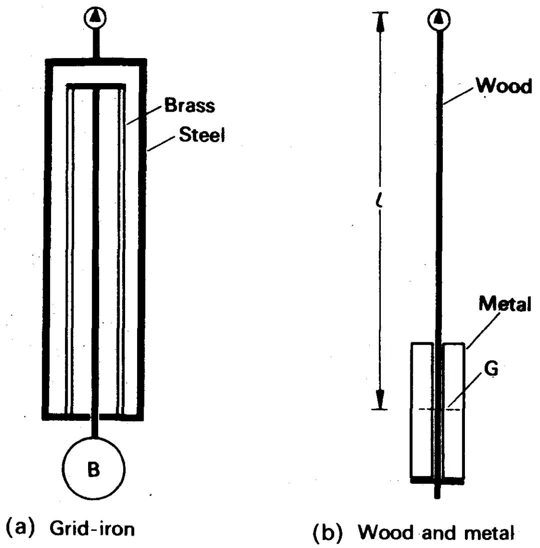

FIG. 11.11. Compensated pendula.

Metal-Glass Seals

In radio valves and many other pieces of physical apparatus, it is necessary to seal metal cones into glass tubes, with a vacuum-tight joint. The seal must be made at about 400°C, when the glass is soft; as it cools to room temperature, the glass will crack unless the glass and metal contract at the same rate. This condition requires that the metal and the glass have the same linear expansivity at every temperature between room temperature and the melting-point of glass. It is satisfied nearly enough by platinum and soda glass (mean $\lambda = 9$ and $8{\cdot}5 \times 10^{-6}$ per deg C, respectively), and by tungsten and some types of hard glass similar to pyrex (mean linear expansivity $\lambda = 3-4 \times 10^{-6}$ per deg C).

Modern seals through soft glass are not made with platinum, but with a wire of nickel-iron alloy, which has about the same linear coefficient as the glass. The wire has a thin coating of copper, which adheres to glass more firmly than the alloy. Also, being soft, the copper takes up small differences in expansion between the alloy and the glass.

In transmitting valves, and large vacuum plants, glass and metal tubes several centimetres in diameter must be joined end-to-end. The metal tubes are made of copper, chamfered to a fine taper at the end where the joint is to be made. The glass is sealed on to the edge of the chamfer; the copper there is thin enough to distort, with the difference in contraction, without cracking the glass.

Superficial Expansion

The increase in area of a body with temperature change is called the *superficial expansion* of the body. A rectangular plate, of sides a, b, at a given temperature, has an area

$$S_1 = ab.$$

If its temperature is increased by θ its sides become $a(1+\lambda\theta)$, $b(1+\lambda\theta)$ where λ is its mean linear expansivity. Thus its area becomes

$$\begin{aligned} S_2 &= a(1+\lambda\theta)\, b(1+\lambda\theta) \\ &= S_1(1+\lambda\theta)^2 \\ &= S_1(1+2\lambda\theta+\lambda^2\theta^2). \end{aligned}$$

In this expression, the term $\lambda^2\theta^2$ is small compared with $2\lambda\theta$; if λ is of the order of 10^{-5}, and θ of the order of 100, then $\lambda\theta \simeq 10^{-3}$, and $\lambda^2\theta^2 \simeq 10^{-6}$. Therefore we may neglect $\lambda^2\theta^2$ and write

$$S_2 = S_1(1+2\lambda\theta).$$

The superficial expansivity of the material of the plate is defined as

$$\frac{\text{increase of area}}{\text{original area} \times \text{temp. rise}} = \frac{S_2 - S_1}{S_1\theta}.$$

Its value is hence equal to 2λ, twice the linear expansivity. A hole in a plate changes its area by the same amount as would a plug that fitted the hole.

Cubic Expansivity

Cubic expansivity is expansion in volume. Consider a rectangular block of sides a, b, c, and therefore of volume

$$V_1 = abc.$$

If the block is raised in temperature by θ its sides expand, and its volume becomes

$$V_2 = a(1+\lambda\theta)\,b(1+\lambda\theta)\,c(1+\lambda\theta) = abc(1+\lambda\theta)^3$$
$$= abc(1+3\lambda\theta+3\lambda^2\theta^2+\lambda^3\theta^3).$$

Since $\lambda^2\theta^2$ and $\lambda^3\theta^3$ are small compared with $\lambda\theta$, we may in practice neglect them. We then have

$$V_2 = abc(1+3\lambda\theta) = V_1(1+3\lambda\theta).$$

The cubic expansivity of the solid is defined as

$$\gamma = \frac{\text{increase in volume}}{\text{original volume} \times \text{temp. rise}}$$
$$= \frac{V_2 - V_1}{V_1\theta}$$
$$= 3\lambda, \text{ to a very good approximation.}$$

Thus the cubic expansivity is *three* times the linear expansivity.

By imagining a block cut out of a larger block, we can see that the cubical expansion of a hollow vessel is the same as that of a solid plug which would fit into it.

LIQUIDS

Cubic Expansivity

The temperature of a liquid determines its volume, but its vessel determines its shape. The only expansivity which we can define for a liquid is therefore its cubic expansivity, γ. Most liquids, like most solids, do not expand uniformly, and γ is not constant over a wide range of temperature. Over a given range θ_1 to θ_2, the *mean coefficient* γ is defined as

$$\gamma = \frac{V_2 - V_1}{V_1(\theta_2 - \theta_1)},$$

where V_1 and V_2 are the volumes of a given mass of liquid at the temperatures θ_1 and θ_2.

MEAN EXPANSIVITIES OF LIQUIDS
(Near room temperature)

Liquid	γ (K^{-1}) $\times 10^{-4}$	Liquid		γ (K^{-1}) $\times 10^{-4}$
Alcohol (methyl)	12·2	Water:	5–10°C	0·53
Alcohol (ethyl)	11·0		10–20°C	1·50
Aniline	8·5		20–40°C	3·02
Ether (ethyl)	16·3		40–60°C	4·58
Glycerine	5·3		60–80°C	5·87
Olive oil	7·0	Mercury:	0–30°C	1·81
Paraffin oil	9·0		0–100°C	1·82
Toluene	10·9		0–300°C	1·87

True and Apparent Expansion: Change of Density

If we try to find the cubic expansivity of a liquid by warming it in a vessel, the vessel also expands. The expansion which we observe is the difference between the increases in volume of the liquid and the vessel. This is true whether we start with the vessel full, and catch the overflow, or observe the creep of the liquid up the vessel (Fig. 11.12). The expansion we observe we call the *apparent expansion*; it is always less than the true expansion of the liquid.

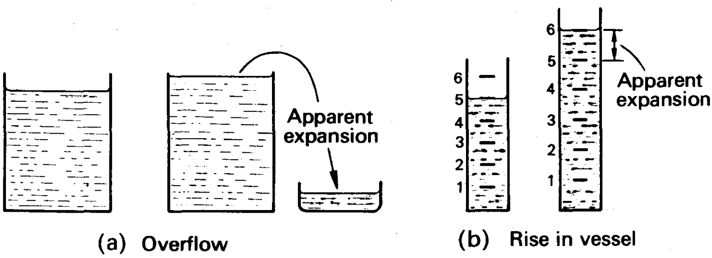

FIG. 11.12. Apparent expansion of a liquid.

Most methods of measuring the expansion of a liquid, whether true or apparent, depend on the change in density of the liquid when it expands. We therefore consider this change, before describing the measurements in detail. The mean true or absolute expansivity of a liquid γ, is defined in the same way as the mean cubic expansivity of a solid:

$$\gamma = \frac{\text{increase in volume}}{\text{initial volume} \times \text{temperature rise}}.$$

Thus, if V_1 and V_2 are the volumes of unit mass of the liquid at θ_1 and θ_2, then

$$V_2 = V_1\{1+\gamma(\theta_2-\theta_1)\}.$$

The densities of the liquid at the two temperatures are

$$\rho_1 = \frac{1}{V_1},$$

$$\rho_2 = \frac{1}{V_2};$$

so that

$$\frac{1}{\rho_2} = \frac{1}{\rho_1}\{1+\gamma(\theta_2-\theta_1)\}$$

or

$$\rho_2 = \frac{\rho_1}{1+\gamma(\theta_2-\theta_1)} \quad . \quad . \quad . \quad . \quad (5)$$

Measurement of True (Absolute) Expansivity

The first measurement of the true expansion of a liquid was made by Dulong and Petit in 1817. A simple form of their apparatus is shown in Fig. 11.13. It consists of a glass tube ABCD, a foot or two high, containing mercury, and surrounded by glass jackets XY. The jacket X contains ice-water, and steam is passed through the jacket Y.

For the mercury to be in equilibrium, its hydrostatic pressure at B must equal its hydrostatic pressure at C. Let h_0 be the height of the mercury in the limb at 0°C and ρ_0 its density; and let h and ρ be the corresponding quantities at the temperature θ of the steam.

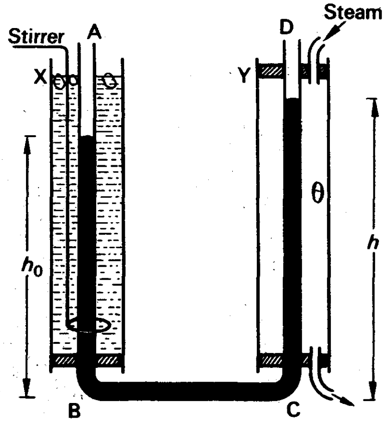

FIG. 11.13. Apparatus for true (absolute) expansivity of mercury.

Then $$g\rho_0 h_0 = g\rho h,$$

where g is the acceleration of gravity.

Hence $$\frac{\rho}{\rho_0} = \frac{h_0}{h}.$$

But, by equation (5),

$$\frac{\rho}{\rho_0} = \frac{1}{1+\gamma\theta}.$$

$$\therefore \frac{h_0}{h} = \frac{1}{1+\gamma\theta}$$

or $$h_0 + h_0\gamma\theta = h.$$

$$\therefore \gamma = \frac{h-h_0}{h_0\theta}.$$

The height $h-h_0$ is measured with a cathetometer (a travelling telescope on a vertical column).

This simple apparatus is inaccurate because:

(i) the expansion of CD throws BC out of the horizontal;

(ii) the wide separation of A and D makes the measurement of $(h-h_0)$ inaccurate;

(iii) surface tension causes a difference of pressure across each free surface of mercury; and these do not cancel one another, because the surface tensions are different at the temperatures of the hot and cold columns.

Regnault got round these difficulties with the apparatus shown, somewhat simplified, in Fig. 11.14. The points B and G are fixed at the same horizontal level, the join DE is made of flexible iron tubing, and the difference in height between its ends, h_2, is measured. The parts AB, GH are at room temperature θ_1; and to a fair approximation, the average temperature of DE is also θ_1. Suppose θ is the steam temperature.

If the density of mercury at θ_1, θ is ρ_1, ρ, respectively then equating the pressures on both sides at the horizontal level of E, we have

$$g\rho_1 h_1 + g\rho_0 h_0 + g\rho_1 h_2 = g\rho h + g\rho_1 h_3.$$

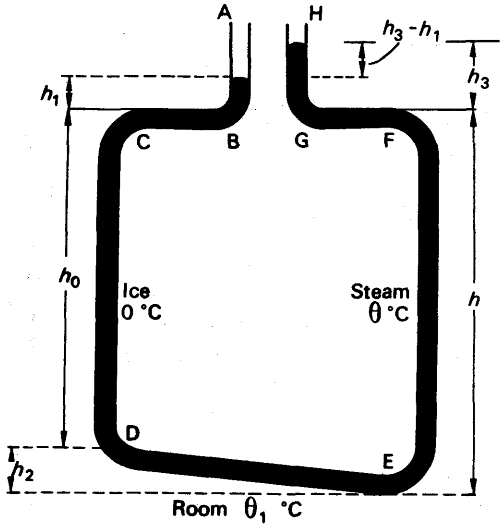

FIG. 11.14. Modified apparatus for true expansivity

Therefore*

$$\frac{\rho_0 h_1}{1+\gamma\theta_1} + \rho_0 h_0 + \frac{\rho_0 h_2}{1+\gamma\theta_1} = \frac{\rho_0 h}{1+\gamma\theta} + \frac{\rho_0 h_3}{1+\gamma\theta_1}. \qquad (6)$$

The uncertainty in the temperature of DE is not important, since the height h_2 is very small. Equation (6) gives

$$h_0 + \frac{h_2}{1+\gamma\theta_1} = \frac{h}{1+\gamma\theta} + \frac{h_3 - h_1}{1+\gamma\theta_1}. \qquad (7)$$

Equation (7) can be solved for γ: the quantities which need to be known accurately are h_0, h, and the difference $h_3 - h_1$. This difference can be measured accurately, because AB and GH are close together; and because they are at the same temperature there is no error due to surface tension. The heights h_0 and h are 1 or 2 metres, and so are easy to measure accurately.

Callender and Moss used six pairs of hot and cold columns, each 2 metres long, to increase the difference in level of the liquid. In this way they avoided the complication due to density change of the liquid under high pressure. All the hot columns were beside one another in a hot oil bath kept at a constant high temperature, while the cold columns were similarly placed in a bath of melting ice. Platinum resistance thermometers were used to measure the temperatures, and a cathetometer to measure the heights and difference in level.

* Strictly, equation (6) should be written

$$\frac{\rho_0 h_1}{1+\gamma_1\theta_1} + \rho_0 h_0 + \frac{\rho_0 h_2}{1+\gamma_1\theta_1} = \frac{\rho_0 h}{1+\gamma\theta} + \frac{\rho_0 h_3}{1+\gamma_1\theta_1},$$

where γ_1 is the mean coefficient between 0°C and θ_1, and γ is that between 0°C and θ. This equation can be solved for γ by successive approximations. (See Roberts-Miller, *Heat and Thermodynamics*, Chapter X. Blackie.)

Apparent Expansion: Weight Thermometer Method

The method of balancing columns for the absolute expansivity of a liquid is slow and awkward; it has only been applied to mercury. Routine measurements are more conveniently made by measuring the apparent expansion; from this, as we shall see, the absolute expansion can be calculated.

A *weight thermometer* is a bulb of fused quartz fitted with a fine stem (Fig. 11.15). It is filled with liquid at a low temperature, and then warmed; the liquid expands, and from the mass which flows out the apparent expansion of the liquid can be found. The weight thermometer is filled by warming it to expel air, and then dipping the stem into the liquid. The process has to be repeated many times; and for accurate work the liquid in the thermometer should be boiled at intervals during the filling, to expel dissolved air. In a laboratory experiment, a glass density bottle may be used, filled in the usual way.

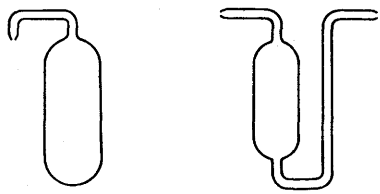

FIG. 11.15. Weight thermometers.

The weight thermometer must be filled at a temperature slightly below the lower limit, θ_1, of the range over which the expansion is to be measured. It is then kept in a bath at θ_1, until no more liquid flows out. Next it is weighed, and from its known mass when empty the mass of liquid in it is found. Let this be m_1. The weight thermometer is then placed in a bath at the higher temperature of the range, θ_2, and the mass remaining in it, m_2, is found by weighing.

If V_1 and V_2 are the volumes of the weight thermometer at θ_1 and θ_2, then

$$V_2 = V_1\{1+3\lambda(\theta_2-\theta_1)\},$$

where λ is the linear expansivity of quartz, the material of the weight thermometer. Now, if ρ denotes the density of the liquid, whose cubic expansivity is γ, we have

$$m_1 = V_1\rho_1,$$

$$m_2 = V_2\rho_2,$$

and $$\frac{\rho_2}{\rho_1} = \frac{1}{1+\gamma(\theta_2-\theta_1)}.$$

Therefore $$\frac{m_2}{m_1} = \frac{V_2\rho_2}{V_1\rho_1}$$

$$= \frac{1+3\lambda(\theta_2-\theta_1)}{1+\gamma(\theta_2-\theta_1)}.$$

Hence $m_2+m_2\gamma(\theta_2-\theta_1) = m_1+3\lambda m_1(\theta_2-\theta_1)$

or $m_2\gamma(\theta_2-\theta_1) = m_1-m_2+3\lambda m_1(\theta_2-\theta_1)$

and
$$\gamma = \frac{m_1-m_2}{m_2(\theta_2-\theta_1)}+3\lambda\frac{m_1}{m_2} \quad . \quad . \quad (11)$$

If we ignore the expansion of the solid, we obtain the apparent expansivity of the liquid, γ_a, relative to the solid.

Thus
$$\gamma_a = \frac{m_1-m_2}{m_2(\theta_2-\theta_1)} \quad . \quad . \quad . \quad . \quad (12)$$

The expression for the apparent expansivity may be expressed in words:

$$\gamma_a = \frac{\text{mass expelled}}{\text{mass left in} \times \text{temp. rise}}.$$

Attention is often drawn to the fact that the mass in the denominator is the mass at the higher temperature, and not the lower.

From equation (11),

$$\gamma = \gamma_a+\frac{m_1}{m_2}\gamma_g \quad . \quad . \quad . \quad . \quad (13a)$$

where $\gamma_g = 3\lambda$ = the cubical coefficient of the solid. Since m_1 and m_2 are nearly equal, then, to a good approximation,

$$\gamma = \gamma_a+\gamma_g \quad . \quad . \quad . \quad . \quad (13b)$$

Thus after the apparent coefficient has been calculated, the absolute coefficient is given to a good approximation by

absolute expansivity = apparent expansivity + cubic expansivity of vessel.

Coefficient of Weight-thermometer Material

The cubic expansivity of a glass weight-thermometer may not be accurately known, because glasses differ considerably in their physical properties. Also the expansivity may be changed when the glass is heated in the blowing of the bulb. The expansivity can conveniently be measured, however, by using the weight thermometer to find the apparent value for mercury; and then subtracting the value found from the known value of the absolute expansivity of mercury.

Expansion of a Powder

The weight thermometer can be used to find the cubic expansivity of a granular or powdery solid, such as sand. The procedure is the same as in finding the relative density of the solid, but is gone through at two known temperatures. From the change in relative density, and a knowledge of the expansions of the liquid and the weight thermometer, the change in absolute density of the powder can be found, and hence its cubic expansivity.

The Dilatometer

A dilatometer is an instrument for rapidly—but roughly—measuring the expansion of a liquid. It consists of a glass bulb B, with a graduated stem S (Fig. 11.16). The volume V_b of the bulb, up to the zero of S, is known, and S is graduated in cubic millimetres or other small units. The volume of the bulb, and the value of one scale division, vary with temperature; the dilatometer therefore measures apparent expansion.

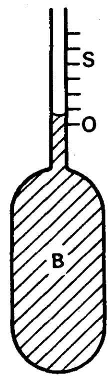

FIG. 11.16. Dilatometer.

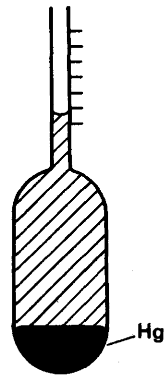

FIG. 11.17. Compensated dilatometer.

The dilatometer is filled with the liquid under test to a point just above the zero of the stem, at a temperature θ_1. The volume V_1 of the liquid is found by adding V_b to the stem-reading v_1. Next the dilatometer is warmed to θ_2, and the liquid rises to v_2. Then $(v_2 - v_1)$ is the apparent expansion of the liquid, and hence

$$\gamma_a = \frac{v_2 - v_1}{V_1(\theta_2 - \theta_1)}.$$

If γ_g is the cubical coefficient of the glass, then

$$\gamma = \gamma_a + \gamma_g.$$

For demonstration work, a dilatometer can be compensated so that it shows roughly the true expansion of a liquid. Mercury is introduced into the bulb, until it occupies 1/7th of the bulb's volume (Fig. 11.17). The expansion of the mercury is then about equal to the expansion of the glass, so that the free volume in the bulb is roughly constant. The cubic expansivities of mercury and glass are given respectively by

$$\gamma_{Hg} = 18{\cdot}1 \times 10^{-5}\ \mathrm{K}^{-1}$$

and $$\gamma_g = 3\lambda_g = 3 \times 8{\cdot}5 \times 10^{-6} = 2{\cdot}55 \times 10^{-5}\ \mathrm{K}^{-1}.$$

Thus $$\frac{\gamma_{Hg}}{\gamma_g} = \frac{18{\cdot}1}{2{\cdot}55} = 7{\cdot}1.$$

Thus the expansion of the mercury offsets that of the glass, within about $1\frac{1}{2}$ per cent.

The space above the mercury, whose volume is constant, is filled with the liquid to be examined. When the bulb is warmed, the movement of the liquid up the stem shows the liquid's true expansion. This device may be used to show the anomalous expansion of water (p. 288).

Correction of the Barometer

The hydrostatic pressure of a column of mercury, such as that in a barometer, depends on its density as well as its height. When we speak of a pressure of 760 mm mercury, therefore, we must specify the temperature of the mercury; we choose 0°C. In practice barometers are generally warmer than that, and their readings must therefore be

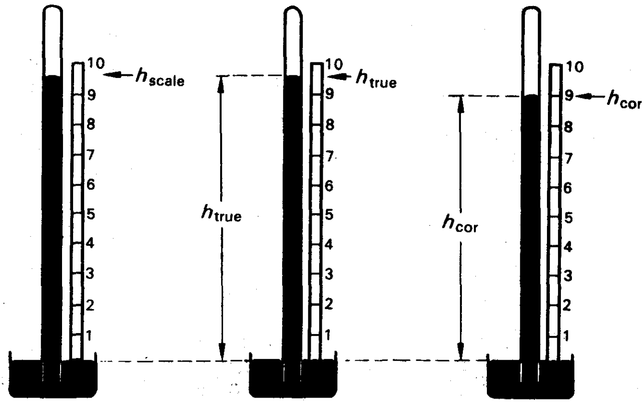

FIG. 11.18. Reduction of barometer height to 0°C.

reduced to what they would be at 0°C. Also we must allow for the expansion of the scale with which the height is measured.

The scale of a barometer may be calibrated at 0°C. At any higher temperature, θ, the height which it indicates, h_{scale}, is less than the true height, h_{true}, of the mercury meniscus above the free surface in the reservoir (Fig. 11.18 (a) (b)). The true height is given by equation (3) of p. 273.

$$h_{true} = h_{scale}(1+\lambda\theta) \qquad . \quad . \quad . \quad (14)$$

where λ is the linear expansivity of the scale. If ρ_θ is the density of mercury at θ, then the pressure of the atmosphere is

$$p = g\rho_\theta h_{true}.$$

The height of a mercury column at 0°C which would exert the same pressure is called the *corrected height*, $h_{cor.}$ (Fig. 11.18 (b) (c)). It is given by

$$p = g\rho_0 h_{cor.},$$

where ρ_0 is the density of mercury at 0°C.

Therefore

$$\rho_\theta h_{true} = \rho_0 h_{cor.}.$$

If γ is the coefficient of expansion of mercury, then

$$\rho_\theta = \frac{\rho_0}{1+\gamma\theta}.$$

Hence $$\frac{\rho_0}{1+\gamma\theta} h_{true} = \rho_0 h_{cor.}$$

and $$h_{cor.} = \frac{h_{true}}{1+\gamma\theta}.$$

Therefore, by equation (13),

$$h_{cor.} = \frac{h_{scale}(1+\lambda\theta)}{1+\gamma\theta}.$$

Let us write this as

$$h_{cor.} = h_{scale}(1+\lambda\theta)(1+\gamma\theta)^{-1}.$$

Then, if we ignore $\gamma^2\theta^2$ and higher terms, we may write

$$h_{cor.} = h_{scale}(1+\lambda\theta)(1-\gamma\theta).$$

Hence $$h_{cor.} = h_{scale}(1+\lambda\theta-\gamma\theta+\gamma\lambda\theta^2),$$

and ignoring $\gamma\lambda\theta^2$ we find

$$h_{cor.} = h_{scale}\{1+(\lambda-\gamma)\theta\}.$$

The coefficient γ is greater than λ, and the corrected height is less than the scale height. It is convenient therefore, to write

$$h_{cor.} = h_{scale}\{1-(\gamma-\lambda)\theta\}.$$

The reader should notice that the correction depends on the difference between the cubic expansivity of the mercury, and the linear expansivity of the scale; as in Dulong and Petit's experiment, there is no question of apparent expansion.

The Anomalous Expansion of Water

If we nearly fill a tall jar with water, and float lumps of ice on it, the water at the base of the jar does not cool below 4°C, although the water at the top soon reaches 0°C. At 4°C water has its greatest density; as it cools to this temperature it sinks to the bottom. When the water at the top cools below 4°C, it becomes less dense than the water below, and stays on the top. Convection ceases, and the water near the bottom of the jar can lose heat only by conduction. Since water is a bad conductor, the loss by conduction is extremely small, and in practice the water at the bottom does not cool below 4°C. The same happens in a pond, cooled at the top by cold air. Ice forms at the surface, but a little below it the water remains at 4°C, and life in the pond survives. It could not if the water contracted in volume continuously to 0°C; for then convection would always carry the coldest water to the bottom, and the pond would freeze solid. In fact, lakes and rivers, unless they are extremely shallow, never do freeze solid; even in arctic climates, they take only a crust of ice.

Fig. 11.19 shows how the volume of 1 g of water varies with temperature. The decrease from 0°C to 4°C is called *anomalous expansion*; it can be shown with a compensated dilatometer (p. 285). Water also has the unusual property of expanding when it freezes, hence burst

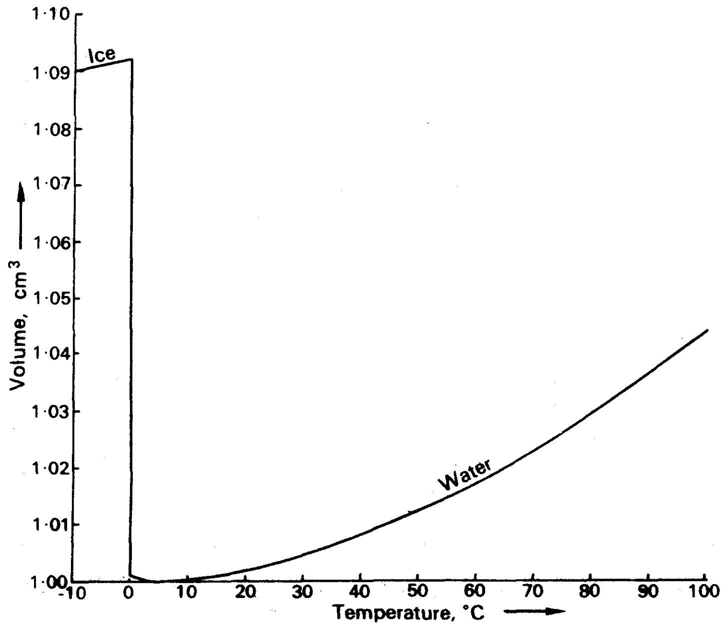

FIG. 11.19. Volume variation of 1 g of water.

pipes. As the figure shows, the expansion is about 9 per cent. If ice were not less dense than water, it would sink, and so, despite the anomalous expansion of water, lakes and rivers would freeze solid in winter.

The following table gives the densities of ice and water at various temperatures. Accurate experiments show that the temperature of maximum density is 3·98°C.

DENSITY OF ICE AND WATER

Temperature °C .	0	2	4	6	8
ρ_{water} kg m^{-3} .	999·84	999·94	999·97	999·94	999·85
ρ_{ice} kg m^{-3} .	916·0 (volume of 1 g ice at 0°C = 1·092 cm^3)				

Temperature °C .	10	20	40	60	80	100
ρ_{water} kg m^{-3} .	999·70	998·20	992·2	983·2	971·8	958·4

The temperature of maximum density was first measured in 1804, by Hope, whose apparatus is shown in Fig. 11.20. Both vessels are made of metal; the inner one contains water, and the outer one contains a mixture of ice and salt, which has a temperature below 0°C. After several hours, the water at the top cools to 0°C—it may even freeze—but the water at the bottom does not fall below 4°C.

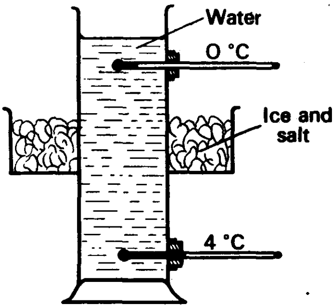

FIG. 11.20. Hope's experiment.

The contraction of water from 0 to 4°C is explained by supposing that the molecules form clusters, such as H_4O_2, H_6O_3. At first, the contraction due to the formation of these more than offsets the expansion due to the rise of temperature; but above 4°C the expansion prevails. The metal antimony behaves like water: it expands on freezing, and contracts at first when warmed above its melting-point. Because it expands on freezing, it makes sharp castings, and is used as a constituent of type-metal.

EXAMPLES

1. Describe how you would use a specific gravity bottle to find the coefficient of expansion of paraffin oil relative to glass between 0° and 50°C.

A specific gravity bottle contains 44·25 g of a liquid at 0°C and 42·02 g at 50°C. Assuming that the linear expansivity of the glass is 0·00001 K^{-1}, (*a*) compare the densities of the liquid at 0° and 50°C, (*b*) deduce the real expansivity of the liquid. Prove any formula employed. (*N.*)

First part. See text.

Second part. The apparent expansivity of the liquid, γ_a, is given by

$$\gamma_a = \frac{\text{mass expelled}}{\text{mass left} \times \text{temp. rise}} = \frac{44{\cdot}25-42{\cdot}02}{42{\cdot}02\times 50}.$$

$$\therefore \gamma_a = \frac{2{\cdot}23}{42{\cdot}02\times 50} = 0{\cdot}00106\ K^{-1}.$$

Now cubic expansivity, γ_g, of glass $= 3\times 0{\cdot}00001 = 0{\cdot}00003\ K^{-1}$.

$$\therefore \text{real coefficient, } \gamma, = \gamma_a+\gamma_g = 0{\cdot}00106+0{\cdot}00003 = 0{\cdot}00109\ K^{-1}.$$

Also, if ρ_0, ρ_t are the densities at 0°C, 50°C respectively,

$$\frac{\rho_0}{\rho_t} = 1+\gamma t$$

$$= 1+0{\cdot}00109\times 50$$

$$= 1{\cdot}055.$$

2. Define the linear expansivity of a solid, and show how it is related to the cubic expansivity. Describe an accurate method of determining the coefficient of expansion of a solid.

In order to make connexion to the carbon anode of a transmitting valve it is required to thread a copper wire (2 mm in diameter at 20°C) through a smaller hole in the carbon block. If the process can just be carried out by immersing both specimens in dry ice (solid CO_2) at -80°C and the coefficients of linear expansion of copper and carbon are 17×10^{-6} and 5×10^{-6} K^{-1}, calculate the size of the hole at 20°C. (*O. & C.*)

First part. The linear expansivity is the increase in length per unit length per degree rise in temperature. It is one-third of the cubic expansivity, proved on p. 279. An accurate method of measuring the linear expansivity is the comparator method, described on p. 270.

Second part. Suppose r cm is the diameter of the hole at 20°C. Then, from the formula

$$l_2 = l_1[1+\lambda(\theta_2-\theta_1)]. \text{ where } (\theta_2-\theta_1) = -80 - 20 = -100°\text{C},$$

$$\text{diameter at } -80°\text{C} = r(1-5\times 10^{-6}\times 100) = 0{\cdot}9995\, r \text{ cm}.$$

Similarly, the diameter of the copper wire at -80°C

$$= 0{\cdot}2(1-17\times 10^{-6}\times 100) = 0{\cdot}19966 \text{ cm}.$$

$$\therefore 0{\cdot}9995\, r = 0{\cdot}19966$$

$$\therefore r = \frac{0{\cdot}19966}{0{\cdot}9995} = 0{\cdot}19976 \text{ cm} = 1{\cdot}998 \text{ mm}.$$

3. Describe in detail how the cubic expansivity of a liquid may be determined by the weight thermometer method.

The height of the mercury column in a barometer provided with a brass scale correct at 0°C is observed to be 749·0 mm on an occasion when the temperature is 15°C. Find (*a*) the true height of the column at 15°C, (*b*) the height of a column of mercury at 0°C which would exert an equal pressure. Assume that the cubic expansivities of brass and of mercury are respectively 0·000054 and 0·000181 K^{-1}. (*N.*)

First part. The weight thermometer method gives the apparent expansivity of the liquid, which is calculated from:

$$\frac{\text{mass expelled}}{\text{mass left} \times \text{temperature rise}}.$$

The true expansivity is obtained by adding the cubic expansivity of the container's material to the apparent expansivity.

Second part. (*a*) The linear expansivity of brass, λ,

$$= \tfrac{1}{3} \times \text{volume expansivity}$$

$$= \tfrac{1}{3} \times 0{\cdot}000054 = 0{\cdot}000018 \text{ K}^{-1}.$$

$$\therefore \text{true height, } h_\theta, = 749{\cdot}0\,(1+\lambda\theta) = 749{\cdot}0\,(1+0{\cdot}000018\times 15)$$

$$= 749{\cdot}2 \text{ mm}.$$

(*b*) Suppose h_0 is the required height at 0°C. Then, since pressure $= h\rho g$,

$$h_0\rho_0 g = h_\theta\rho_\theta g$$

$$\therefore h_0 = h_\theta\frac{\rho_\theta}{\rho_0}.$$

But $$\frac{\rho_\theta}{\rho_0} = \frac{1}{1+\gamma\theta} = \frac{1}{1+0{\cdot}000181\times 15} = \frac{1}{1{\cdot}0027}$$

$$\therefore\ h_0 = 749{\cdot}0\times\frac{1{\cdot}00027}{1{\cdot}0027}$$

$$= 749\ [1-(0{\cdot}0027-0{\cdot}00027)]$$

$$= 747{\cdot}2\ \text{mm}.$$

EXERCISES 11

1. Describe and explain how the absolute expansivity of a liquid may be determined without a previous knowledge of any other expansivity.

Aniline is a liquid which does not mix with water, and when a small quantity of it is poured into a beaker of water at 20°C it sinks to the bottom, the densities of the two liquids at 20°C being 1021 and 998 kg m^{-3} respectively. To what temperature must the beaker and its contents be uniformly heated so that the aniline will form a globule which just floats in the water? (The mean absolute expansivity of aniline and water over the temperature range concerned are 0·00085 and 0·00045 K^{-1}, respectively.) (*L.*)

2. Define the *linear expansivity* of a solid, and describe a method by which it may be measured.

Show how the superficial expansivity can be derived from this value.

A 'thermal tap' used in certain apparatus consists of a silica rod which fits tightly inside an aluminium tube whose internal diameter is 8 mm at 0°C. When the temperature is raised, the fit is no longer exact. Calculate what change in temperature is necessary to produce a channel whose cross-section is equal to that of a tube of 1 mm internal diameter. [Linear expansivity of silica = 8×10^{-6} K^{-1}. Linear expansivity of aluminium = 26×10^{-6} K^{-1}.] (*O. & C.*)

3. Define the *cubic expansivity* of a liquid. Find an expression for the variation of the density of a liquid with temperature in terms of its expansivity.

Describe, without experimental details, how the cubic expansivity of a liquid may be determined by the use of balanced columns.

A certain Fortin barometer has its pointers, body and scales made from brass. When it is at 0°C it records a barometric pressure of 760 mm Hg. What will it read when its temperature is increased to 20°C if the pressure of the atmosphere remains unchanged?

[Cubic expansivity of mercury = $1{\cdot}8\times 10^{-4}$ K^{-1}; linear expansivity of brass = 2×10^{-5} K^{-1}.] (*O. & C.*)

4. Describe an experiment to determine the absolute (real) expansivity of paraffin between 0°C and 50°C, using a weight thermometer made of glass of known cubic expansivity. Derive the formula used to calculate the result.

A glass capillary tube with a uniform bore contains a thread of mercury 100·0 cm long, the temperature being 0°C. When the temperature of the tube and mercury is raised to 100°C the thread is increased in length by 1·65 cm. If the mean coefficient of absolute expansion of mercury between 0°C and 100°C is 0·000182 K^{-1}, calculate a value for the linear expansivity of glass (*N.*)

5. Define the linear and cubic expansivity, and derive the relation between them for a particular substance. Describe, and give the theory of, a method for finding *directly* the absolute coefficient of expansion of a liquid. The bulb of a mercury-in-glass thermometer has a volume of 0·5 cm^3 and the distance between progressive degree marks is 2 mm. If the linear expansivity of glass is 10^{-5} K^{-1}, and the cubic expansivity of mercury is $1{\cdot}8 \times 10^{-4}$ K^{-1}, find the cross-sectional area of the bore of the stem. (*C.*)

6. Define the terms 'apparent' and 'absolute' expansivity of a liquid, and show how the former is found by means of a weight thermometer. A litre flask, which is correctly calibrated at 4°C, is filled to the mark with water at 80°C. What is the weight of water in the flask? [Linear expansivity of the glass of the flask = $8{\cdot}5 \times 10^{-6}$ K^{-1}; mean cubic expansivity of water = $5{\cdot}0 \times 10^{-4}$ K^{-1}.] (*O. & C.*)

7. Describe how to measure the apparent expansivity of a liquid using a weight thermometer. Show how the result can be calculated from the observations.

A specific gravity bottle of volume 50·0 cm^3 at 0°C is filled with glycerine at 20°C. What mass of glycerine is contained in the bottle if the density of glycerine at 0°C is 1·26 g per cm^3, and its real expansivity is $5{\cdot}2 \times 10^{-4}$ K^{-1}? Assume that the linear expansivity of the glass is 8×10^{-6} K^{-1}. (*N.*)

8. Describe in detail how a reliable value for the expansivity of mercury may be found by a method independent of the expansion of the containing vessel. Give the necessary theory.

A silica bulb of negligible expansivity holds 340·0 g of mercury at 0°C when full. Some steel balls are introduced and the remaining space is occupied at 0°C by 255·0 g of mercury. On heating the bulb and its contents to 100°C, 4·800 g of mercury overflow. Find the linear expansivity of the steel.

[Assume that the expansivity of mercury is 180×10^{-6} K^{-1}.] (*L.*)

9. Describe an accurate method for determining the linear expansivity of a solid in the form of a rod. The pendulum of a clock is made of brass whose linear expansivity is $1{\cdot}9 \times 10^{-5}$ per deg C. If the clock keeps correct time at 15°C, how many seconds per day will it lose at 20°C? (*O. & C.*)

10. A steel wire 8 metres long and 4 mm in diameter is fixed to two rigid supports. Calculate the increase in tension when the temperature falls 10°C. [Linear expansivity of steel = 12×10^{-6} K^{-1}, Young's modulus for steel = 2×10^{11} N m^{-2}.] (*O. & C.*)

11. How can you show that the density of water does not fall steadily as the temperature is raised from 0°C to 100°C? What does your experiment indicate about the expansion of water? What importance has this result in nature? (*C.*)

12. Why is mercury used as a thermometric fluid? Compare the advantages and disadvantages of the use of a mercury-in-glass thermometer and a platinum resistance thermometer to determine the temperature of a liquid at about 300°C.

A dilatometer having a glass bulb and a tube of uniform bore contains 150 g of mercury which extends into the tube at 0°C. How far will the meniscus rise up the tube when the temperature is raised to 100°C if the area of cross-section of the bore is 0·8 mm^2 at 0°C? Assume that the density of mercury at 0°C is 13·6 g cm^{-3}, that the expansivity of mercury is $1{\cdot}82 \times 10^{-4}$ K^{-1}, and that the linear expansitivity of glass is $1{\cdot}1 \times 10^{-5}K^{-1}$. (*N.*)

13. Describe in detail how you would determine the linear expansivity of a metal rod or tube. Indicate the chief sources of error and discuss the accuracy you would expect to obtain.

A steel cylinder has an aluminium alloy piston and, at a temperature of 20°C when the internal diameter of the cylinder is exactly 10 cm, there is an all-round clearance of 0·05 mm between the piston and the cylinder wall. At what temperature will the fit be perfect? (The linear expansivity of steel and the aluminium alloy are $1{\cdot}2 \times 10^{-5}$ and $1{\cdot}6 \times 10^{-5}$ K^{-1} respectively.) (*O. & C.*)

14. Explain the statement: *the absolute expansivity of mercury is* $1{\cdot}81 \times 10^{-4}$ K^{-1}. Describe an experiment to test the accuracy of this value. Why is a knowledge of it important?

Calculate the volume at 0°C required in a thermometer to give a degree of length 0·15 cm on the stem, the diameter of the bore being 0·24 mm. What would be the volume of this mercury at 100°C? [The linear expansivity of the glass may be taken as $8{\cdot}5 \times 10^{-6}$ K^{-1}.] (*L.*)