Preface to Second Edition

In this edition I have added an introduction to Atomic Structure, which covers the Advanced level syllabus on this topic. I am particularly indebted to Mr. J. Yarwood, M.Sc., F.Inst.P., head of the physics and mathematics department, Regent Street Polytechnic, London, for reading this section and for valuable advice, and to Prof. L. Pincherle, Bedford College, London University, for his kind assistance in parts of the text.

I am also indebted to G. Ullyott, Charterhouse School and L. G. Mead, Wellington School, Somerset, for their helpful comments on dynamics and optics respectively.

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PART ONE

Mechanics and Properties of Matter

chapter one

Dynamics

Motion in a Straight Line. Velocity

IF a car travels steadily in a constant direction and covers a distance s in a time t, then its *velocity* in that direction $= s/t$. If the car does not travel steadily, then s/t is its average velocity, and

$$\textit{distance } s = \textit{average velocity} \times t.$$

We are here concerned with motion in a constant direction. The term 'displacement' is given to the distance moved in a constant direction, for example, from L to C in Fig. 1.1 (i). Velocity may therefore be defined as the *rate of change of displacement.*

Velocity can be expressed in *centimetres per second* (cm/s or cm s^{-1}) or *metres per second* (m/s or m s^{-1}) or *kilometres per hour* (km/h or km h^{-1}). By calculations, 36 km h^{-1} = 10 m s^{-1}. It should be noted that complete information is provided for a velocity by stating its direction in addition to its magnitude, as explained shortly.

If an object moving in a straight line travels equal distances in equal times, no matter how small these distances may be, the object is said to be moving with *uniform* velocity. The velocity of a falling stone increases continuously, and so is a *non-uniform* velocity.

If, at any point of a journey, Δs is the small change in displacement in a small time Δt, the velocity v is given by $v = \Delta s/\Delta t$. In the limit, using calculus notation,

$$v = \frac{ds}{dt}.$$

Vectors

Displacement and *velocity* are examples of a class of quantities called *vectors* which have both magnitude and direction. They may therefore be represented to scale by a line drawn in a particular direction. Thus

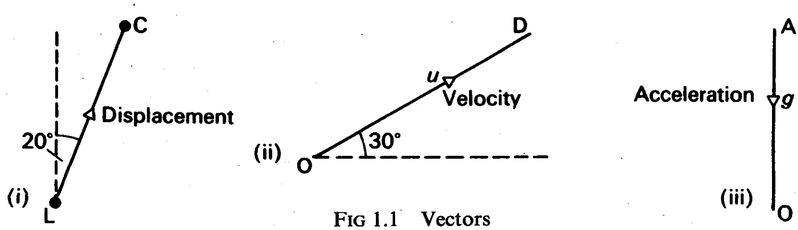

FIG 1.1 Vectors

Cambridge is 80 km from London in a direction 20° E. of N. We can therefore represent the displacement between the cities in magnitude and direction by a straight line LC 4 cm long 20° E. of N., where 1 cm represents 20 km, Fig. 1.1 (i). Similarly, we can represent the velocity u of a ball initially thrown at an angle of 30° to the horizontal by a straight line OD drawn to scale in the direction of the velocity u, the arrow on the line showing the direction, Fig. 1.1 (ii). The acceleration due to gravity, g, is always represented by a straight line AO to scale drawn vertically downwards, since this is the direction of the acceleration, Fig. 1.1 (iii). We shall see later that 'force' and 'momentum' are other examples of vectors.

Speed and Velocity

A car moving along a winding road or a circular track at 80 km h^{-1} is said to have a *speed* of 80 km h^{-1}. 'Speed' is a quantity which has no direction but only magnitude, like 'mass' or 'density' or 'temperature'. These quantities are called *scalars.*

The distinction between speed and velocity can be made clear by reference to a car moving round a circular track at 80 km h^{-1} say. Fig. 1.2. At every point on the track the *speed* is the same—it is 80 km h^{-1}.

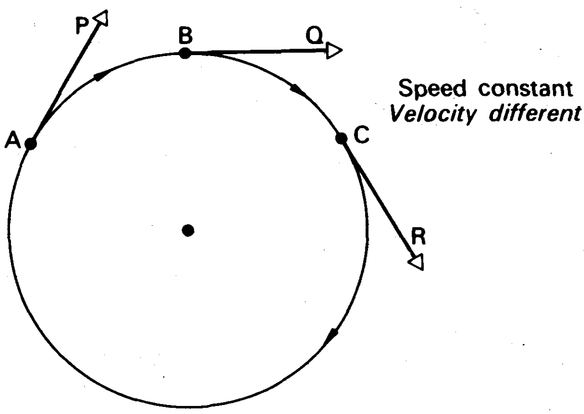

FIG. 1.2. Velocity and speed

At every point, however, the *velocity* is different. At A, B or C, for example, the velocity is in the direction of the particular tangent, AP, BQ or CR, so that even though the magnitudes are the same, the three velocities are all different because they point in different directions. Generally, vector quantities can be represented by a line drawn in the direction of the vector and whose length represents its magnitude.

Distance-Time Curve

When the displacement, or distance, s of a moving car from some fixed point is plotted against the time t, a *distance-time* ($s-t$) *curve* of the motion is obtained. The velocity of the car at any instant is given by the change in distance per second at that instant. At E, for example, if the change in distance s is Δs and this change is made in a time Δt,

$$\text{velocity at E} = \frac{\Delta s}{\Delta t}.$$

In the limit, then, when Δt approaches zero, the velocity at E becomes equal to the *gradient of the tangent to the curve* at E. Using calculus notation, $\Delta s/\Delta t$ then becomes equal to ds/dt (p. 1).

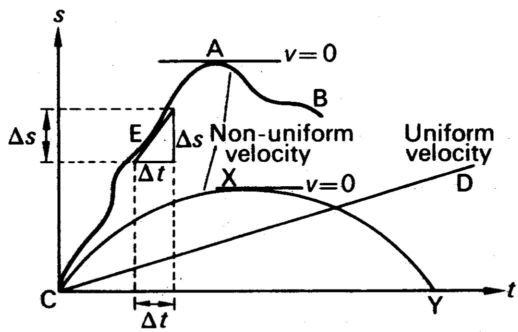

FIG. 1.3 Displacement (s)–time (t) curves

If the distance-time curve is a straight line CD, the gradient is constant at all points; it therefore follows that the car is moving with a *uniform* velocity, Fig. 1.3. If the distance-time curve is a curve CAB, the gradient varies at different points. The car then moves with non-uniform velocity. We may deduce that the velocity is zero at the instant corresponding to A, since the gradient at A to the curve CAB is zero.

When a ball is thrown upwards, the height s reached at any instant t is given by $s = ut - \frac{1}{2}gt^2$, where u is the initial velocity and g is the constant equal to the acceleration due to gravity (p. 8). The graph of s against t is represented by the parabolic curve CXY in Fig. 1.3; the gradient at X is zero, illustrating that the velocity of the ball at its maximum height is zero.

Velocity-Time Curves

When the velocity of a moving train is plotted against the time, a 'velocity-time (v-t) curve' is obtained. Useful information can be deduced from this curve, as we shall see shortly. If the velocity is uniform, the velocity-time graph is a straight line parallel to the time-axis, as shown by line (1) in Fig. 1.4. If the train accelerates uniformly from rest, the velocity-time graph is a straight line, line (2), inclined to the time-axis. If the acceleration is not uniform, the velocity-time graph is curved.

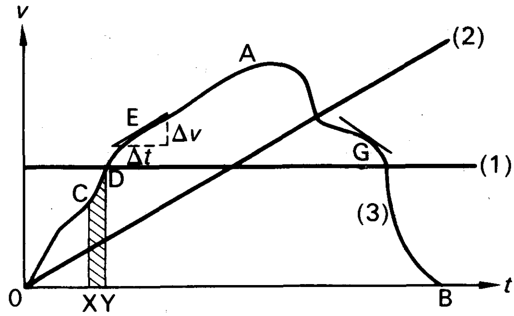

FIG. 1.4 Velocity (v)–time (t) curves

In Fig. 1.4, the velocity-time graph OAB represents the velocity of a train starting from rest which reaches a maximum velocity at A, and then comes to rest at the time corresponding to B; the acceleration and retardation are both not uniform in this case.

Acceleration is the 'rate of change of velocity', i.e. the change of velocity per second. *The acceleration of the train at any instant is given by the gradient to the velocity-time graph* at that instant, as at E. At the peak point A of the curve OAB the gradient is zero, i.e., the acceleration is then zero. At any point, such as G, between A, B the gradient to the curve is negative, i.e., the train undergoes retardation.

The gradient to the curve at any point such as E is given by:

$$\frac{\text{velocity change}}{\text{time}} = \frac{\Delta v}{\Delta t}$$

where Δv represents a small change in v in a small time Δt. In the limit, the ratio $\Delta v/\Delta t$ becomes dv/dt, using calculus notation.

Area Between Velocity-Time Graph and Time-Axis

Consider again the velocity-time graph OAB, and suppose the velocity increases in a very small time-interval XY from a value represented by XC to a value represented by YD, Fig. 1.4. Since the small distance travelled = average velocity × time XY, the distance travelled is represented by the *area* between the curve CD and the time-axis, shown shaded in Fig. 1.4. By considering every small time-interval between OB in the same way, it follows that *the total distance travelled by the train in the time OB is given by the area between the velocity-time graph and the time-axis.* This result applies to any velocity-time graph, whatever its shape.

Fig. 1.5 illustrates the velocity-time graph AB of an object moving with uniform acceleration a from an initial velocity u. From above, the distance s travelled in a time t or OC is equivalent to the area OABC. The area OADC $= u\,.\,t$. The area of the triangle ABD =

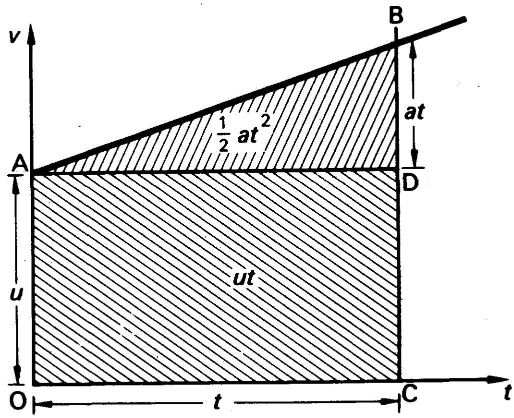

FIG. 1.5 Uniform acceleration

$\frac{1}{2}$AD . BD $= \frac{1}{2}t$. BD. Now BD = the increase in velocity in a time t $= at$. Hence area of triangle ABD $= \frac{1}{2}t \cdot at = \frac{1}{2}at^2$

$$\therefore \text{ total area OABC} = s = ut + \tfrac{1}{2}at^2.$$

This result is also deduced on p. 6.

Acceleration

The *acceleration* of a moving object at an instant is the *rate of change of its velocity* at that instant. In the case of a train accelerating steadily from 36 km h^{-1} (10 m s^{-1}) to 54 km h^{-1} (15 m s^{-1}) in 10 second, the uniform acceleration

$$= (54-36) \text{ km h}^{-1} \div 10 \text{ seconds} = 1{\cdot}8 \text{ km h}^{-1} \text{ per second,}$$

or

$$(15-10) \text{ m s}^{-1} \div 10 \text{ seconds} = 0{\cdot}5 \text{ m s}^{-1} \text{ per second.}$$

Since the time element (second) is repeated twice in the latter case, the acceleration is usually given as 0·5 m s^{-2}. Another unit of acceleration is 'cm s^{-2}'. In terms of the calculus, the acceleration a of a moving object is given by

$$a = \frac{dv}{dt}$$

where dv/dt is the velocity change per second.

Distance Travelled with Uniform Acceleration. Equations of Motion

If the velocity changes by equal amounts in equal times, no matter how small the time-intervals may be, the acceleration is said to be *uniform*. Suppose that the velocity of an object moving in a straight line with uniform acceleration a increases from a value u to a value v in a time t. Then, from the definition of acceleration,

$$a = \frac{v-u}{t},$$

from which $$\mathbf{v = u + at} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Suppose an object with a velocity u accelerates with a uniform acceleration a for a time t and attains a velocity v. The distance s travelled by the object in the time t is given by

$$s = \text{average velocity} \times t$$

$$= \tfrac{1}{2}(u+v) \times t$$

But $$v = u + at$$

$$\therefore s = \tfrac{1}{2}(u+u+at)t$$

$$\therefore \mathbf{s = ut + \tfrac{1}{2}at^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If we eliminate t by substituting $t = (v-u)/a$ from (1) in (2), we obtain, on simplifying,

$$\mathbf{v^2 = u^2 + 2as} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equations (1), (2), (3) are the equations of motion of an object moving in a straight line with uniform acceleration. When an object undergoes a uniform *retardation*, for example when brakes are applied to a car, a has a *negative* value.

EXAMPLES

1. A car moving with a velocity of 54 km h^{-1} accelerates uniformly at the rate of 2 m s^{-2}. Calculate the distance travelled from the place where acceleration began to that where the velocity reaches 72 km h^{-1}, and the time taken to cover this distance.

(i) 54 km h^{-1} = 15 m s^{-1}, 72 km h^{-1} = 20 m s^{-1}, acceleration a = 2 m s^{-2}.

Using $$v^2 = u^2 + 2as,$$

$$\therefore 20^2 = 15^2 + 2 \times 2 \times s$$

$$\therefore s = \frac{20^2 - 15^2}{2 \times 2} = 43\tfrac{3}{4} \text{ m.}$$

(ii) Using $$v = u + at$$

$$\therefore 20 = 15 + 2t$$

$$\therefore t = \frac{20-15}{2} = 2{\cdot}5 \text{ s.}$$

2. A train travelling at 72 km h^{-1} undergoes a uniform retardation of 2 m s^{-2} when brakes are applied. Find the time taken to come to rest and the distance travelled from the place where the brakes were applied.

(i) 72 km h^{-1} = 20 m s^{-1}, and $a = -2$ m s^{-2}, $v = 0$.

Using
$$v = u + at$$
$$\therefore 0 = 20 - 2t$$
$$\therefore t = 10 \text{ s}$$

(ii) The distance, s, $= ut + \frac{1}{2}at^2$.
$$= 20 \times 10 - \tfrac{1}{2} \times 2 \times 10^2 = 100 \text{ m}.$$

Motion Under Gravity

When an object falls to the ground under the action of gravity, experiment shows that the object has a constant or uniform acceleration of about 980 cm s^{-2}, while it is falling (see p. 49). In SI units this is 9·8 m s^{-2} or 10 m s^{-2} approximately. The numerical value of this acceleration is usually denoted by the symbol g. Suppose that an object is dropped from a height of 20 m above the ground. Then the initial velocity $u = 0$, and the acceleration $a = g = 10$ m s^{-2} (approx). Substituting in $s = ut + \frac{1}{2}at^2$, the distance fallen s in metres is calculated from
$$s = \tfrac{1}{2}gt^2 = 5t^2.$$
When the object reaches the ground, $s = 20$ m.
$$\therefore 20 = 5t^2, \text{ or } t = 2 \text{ s}$$
Thus the object takes 2 seconds to reach the ground.

If a cricket-ball is thrown vertically upwards, it slows down owing to the attraction of the earth. The ball is thus retarded. The magnitude of the retardation is 9·8 m s^{-2}, or g. Mathematically, a retardation can be regarded as a negative acceleration in the direction along which the object is moving; and hence $a = -9{\cdot}8$ m s^{-2} in this case.

Suppose the ball was thrown straight up with an initial velocity, u, of 30 m s^{-1}. The time taken to reach the top of its motion can be obtained from the equation $v = u + at$. The velocity, v, at the top is zero; and since $u = 30$ m and $a = -9{\cdot}8$ or 10 m s^{-2} (approx), we have
$$0 = 30 - 10t.$$
$$\therefore t = \frac{30}{10} = 3 \text{ s}.$$
The highest distance reached is thus given by
$$s = ut + \tfrac{1}{2}at^2$$
$$= 30 \times 3 - 5 \times 3^2 = 45 \text{ m}.$$

Resultant. Components

If a boy is running along the deck of a ship in a direction OA, and the ship is moving in a different direction OB, the boy will move relatively to the sea along a direction OC, between OA and OB, Fig. 1.6 (i). Now in one second the boat moves from O to B, where OB represents the velocity of the boat, a vector quantity, in magnitude and direction. The boy moves from O to A in the same time, where OA represents the velocity of the boy in magnitude and direction. Thus in one second the net effect relative to the sea is that the boy moves from O to C. It can now be seen that if lines OA, OB are drawn to represent in magnitude and direction the respective velocities of the boy and the ship, the magnitude and direction of the *resultant* velocity of the boy is represented by the diagonal OC of the completed parallelogram having OA, OB as two of its sides; OACB is known as a *parallelogram of velocities.* Conversely, a velocity represented completely by OC can be regarded as having an 'effective part', or *component* represented by OA, and another component represented by OB.

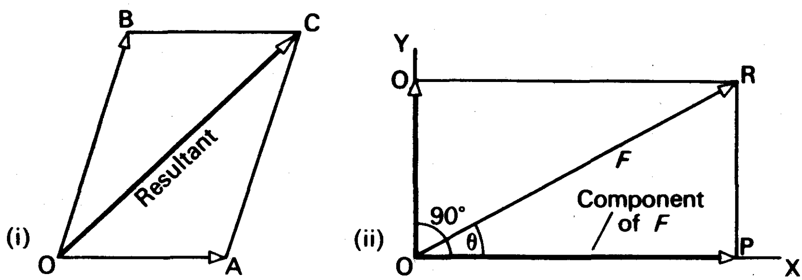

FIG. 1.6. Resultant and component.

In practice, we often require to find the component of a vector quantity in a certain direction. Suppose OR represents the vector F, and OX is the direction, Fig. 1.6 (ii). If we complete the parallelogram OQRP by drawing a perpendicular RP from R to OX, and a perpendicular RQ from R to OY, where OY is perpendicular to OX, we can see that OP, OQ represent the components of F along OX, OY respectively. Now the component OQ has no effect in a perpendicular direction; consequently OP represents the total effect of F along the direction OX. OP is called the 'resolved component' in this direction. If θ is the angle ROX, then, since triangle OPR has a right angle at P,

$$OP = OR \cos\theta = F \cos\theta \quad . \quad . \quad . \quad (4)$$

Components of g

The acceleration due to gravity, g, acts vertically downwards. In free fall, an object has an acceleration g. An object sliding freely down an inclined plane, however, has an acceleration due to gravity equal to the component of g down the plane. If it is inclined at 60° to the vertical, the acceleration down the plane is then $g \cos 60°$ or $9{\cdot}8 \cos 60°$ m s^{-2}, which is 4·9 m s^{-2}.

Consider an object O thrown forward from the top of a cliff OA with a horizontal velocity u of 15 m s^{-1}. Fig. 1.7. Since u is horizontal, it has no component in a *vertical* direction. Similarly, since g acts vertically, it has no component in a *horizontal* direction.

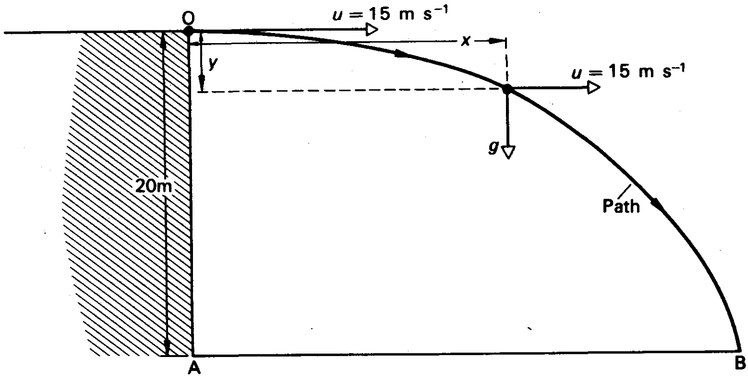

FIG. 1.7 Motion under gravity

We may thus treat vertical and horizontal motion independently. Consider the vertical motion from O. If OA is 20 m, the ball has an initial vertical velocity of zero and a vertical acceleration of g, which is 9·8 m s^{-2} (10 m s^{-2} approximately). Thus, from $s = ut + \frac{1}{2}at^2$, the time t to reach the bottom of the cliff is given, using $g = 10$ m s^{-2}, by

$$20 = \tfrac{1}{2} . 10 . t^2 = 5t^2, \text{ or } t = 2 \text{ s}.$$

So far as the horizontal motion is concerned, the ball continues to move forward with a constant velocity of 15 m s^{-1} since g has no component horizontally. In 2 seconds, therefore,

$$\text{horizontal distance AB} = \text{distance from cliff} = 15 \times 2 = 30 \text{ m}.$$

Generally, in a time t the ball falls a vertical distance, y say, from O given by $y = \frac{1}{2}gt^2$. In the same time the ball travels a horizontal distance, x say, from O given by $x = ut$, where u is the velocity of 15 m s^{-1}. If t is eliminated by using $t = x/u$ in $y = \frac{1}{2}gt^2$, we obtain $y = gx^2/2u$. This is the equation of a *parabola*. It is the path OB in Fig. 1.7.

Addition of Vectors

Suppose a ship is travelling due east at 30 km h^{-1} and a boy runs across the deck in a north-west direction at 6 km h^{-1}, Fig. 1.8 (i). We

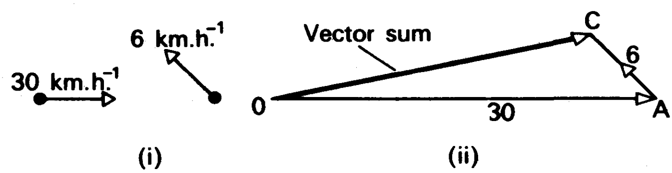

FIG. 1.8 Addition of vectors

can find the velocity and direction of the boy relative to the sea by adding the two velocities. Since velocity is a vector quantity, we draw a line OA to represent 30 km h^{-1} in magnitude and direction, and then, from the end of A, draw a line AC to represent 6 km h^{-1} in magnitude and direction, Fig. 1.8 (ii). The sum, or resultant, of the velocities is now represented by the line OC in magnitude and direction, because a distance moved in one second by the ship (represented by OA) together with a distance moved in one second by the boy (represented by AC) is equivalent to a movement of the boy from O to C relative to the sea.

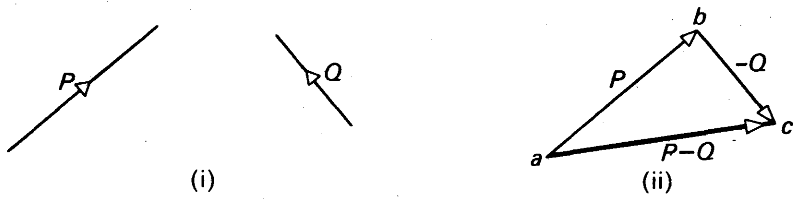

FIG. 1.9 Subtraction of velocities

In other words, the difference between the vectors $\vec{P}$, $\vec{Q}$ in Fig. 1.9 (i) is the *sum* of the vectors $\vec{P}$ and $(-\vec{Q})$. Now $(-\vec{Q})$ is a vector drawn exactly equal and opposite to the vector $\vec{Q}$. We therefore draw *ab* to represent $\vec{P}$ completely, and then draw *bc* to represent $(-\vec{Q})$ completely, Fig. 1.9 (ii). Then $\vec{P}+(-\vec{Q})$ = the vector represented by $ac = \vec{P}-\vec{Q}$.

Relative Velocity and Relative Acceleration

If a car A travelling at 50 km h^{-1} is moving in the same direction as another car B travelling at 60 km h^{-1}, the *relative velocity* of B to A = $60-50 = 10$ km h^{-1}. If, however, the cars are travelling in opposite directions, the relative velocity of B to A = $60-(-50) = 110$ km h^{-1}.

Suppose that a car X is travelling with a velocity v along a road 30° east of north, and a car Y is travelling with a velocity u along a road due east, Fig. 1.10 (i). Since 'velocity' has direction as well as magnitude, i.e., 'velocity' is a vector quantity (p. 1), we cannot subtract u and v numerically to find the relative velocity. We must adopt a method which takes into account the direction as well as the magnitude of the velocities, i.e., a vector subtraction is required.

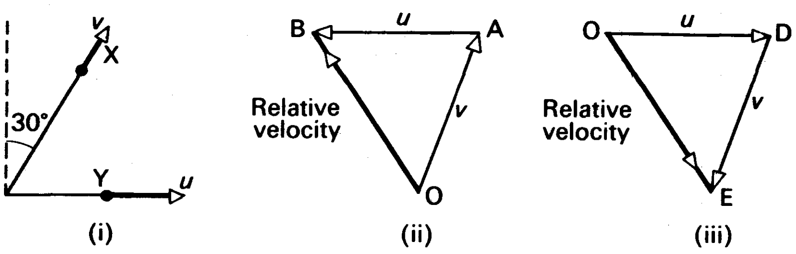

FIG. 1.10. Relative velocity.

The velocity of X relative to Y $= \vec{v} - u = \vec{v} + (-\vec{u})$. Suppose OA represents the velocity, v, of X in magnitude and direction, Fig. 1.10 (ii). Since Y is travelling due east, a velocity AB numerically equal to u but in the due *west* direction represents the vector $(-\vec{u})$. The vector sum of OA and AB is OB from p. 0, which therefore represents in magnitude and direction the velocity of X relative to Y. By drawing an accurate diagram of the two velocities, OB can be found.

The velocity of Y relative to X $= \vec{u} - \vec{v} = \vec{u} + (-\vec{v})$, and can be found by a similar method. In this case, OD represents the velocity, u, of Y in magnitude and direction, while DE represents the vector $(-\vec{v})$, which it is drawn numerically equal to v but in the *opposite* direction, Fig. 1.10 (iii). The vector sum of OD and DE is OE, which therefore represents the velocity of Y relative to X in magnitude and direction.

When two objects P, Q are each accelerating, the acceleration of P relative to Q = acceleration of P − acceleration of Q. Since 'acceleration' is a vector quantity, the relative acceleration must be found by vector subtraction, as for the case of relative velocity.

EXAMPLE

Explain the difference between a scalar and a vector quantity.

What is meant by the relative velocity of one body with respect to another? Two ships are 10 km apart on a line running S. to N. The one farther north is steaming W. at 20 km h^{-1}. The other is steaming N. at 20 km h^{-1}. What is their distance of closest approach and how long do they take to reach it? (*C*.)

Suppose the two ships are at X, Y, moving with velocities u, v respectively, each 20 km h^{-1} Fig. 1.11 (i). The velocity of Y relative to X $= \vec{v} - \vec{u} = \vec{v} + (-\vec{u})$. We therefore draw OA to represent $\vec{v}$ (20) and add to it AB, which represents $(-\vec{u})$, Fig. 1.11 (ii). The relative velocity is then represented by OB.

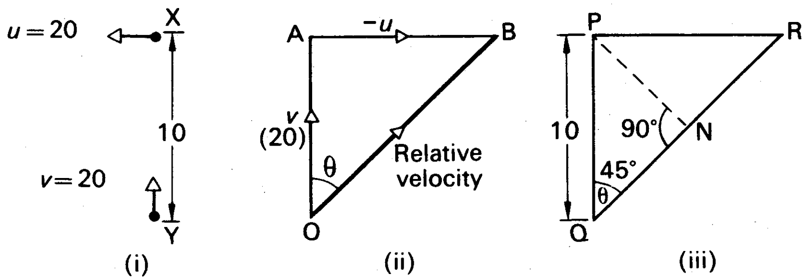

FIG. 1.11 Example

Since OAB is a right-angled triangle,

$$OB = \sqrt{OA^2 + AB^2} = \sqrt{20^2 + 20^2} = 28{\cdot}28 = 28{\cdot}3 \text{ km h}^{-1} \quad \text{(i)}$$

Also,
$$\tan\theta = \frac{AB}{OA} = \frac{20}{20} = 1, \text{ i.e., } \theta = 45° \quad \text{(ii)}$$

Thus the ship Y will move along a direction QR relative to the ship X, where QR is at 45° to PQ, the north-south direction, Fig. 1.11(iii). If PQ = 10 km, the distance of closest approach is PN, where PN is the perpendicular from P to QR.

$$\therefore \text{PN} = \text{PQ} \sin 45° = 10 \sin 45° = 7{\cdot}07 \text{ km}.$$

The distance QN = 10 cos 45° = 7·07 km. Since, from (i), the relative velocity is 28·28 km h^{-1}, it follows that

$$\text{time to reach N} = \frac{7{\cdot}07}{28{\cdot}28} = \tfrac{1}{4} \text{ hour}.$$

LAWS OF MOTION. FORCE AND MOMENTUM

Newton's Laws of Motion

In 1686 Sir Isaac Newton published a work called *Principia*, in which he expounded the Laws of Mechanics. He formulated in the book three 'laws of motion':

Law I. *Every body continues in its state of rest or uniform motion in a straight line, unless impressed forces act on it.*

Law II. *The change of momentum per unit time is proportional to the impressed force, and takes place in the direction of the straight line along which the force acts.*

Law III. *Action and reaction are always equal and opposite.*

These laws cannot be proved in a formal way; we believe they are correct because all the theoretical results obtained by assuming their truth agree with the experimental observations, as for example in astronomy (p. 58).

Inertia. Mass

Newton's first law expresses the idea of **inertia**. The inertia of a body is its reluctance to start moving, and its reluctance to stop once it has begun moving. Thus an object at rest begins to move only when it is pushed or pulled, i.e., when a *force* acts on it. An object O moving in a

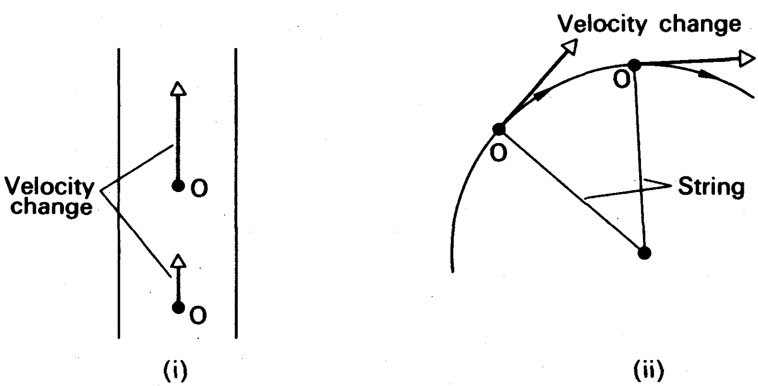

FIG. 1.12 Velocity changes

straight line with constant velocity will change its direction or move faster only if a new force acts on it. Fig. 1.12 (i). This can be demonstrated by a puck moving on a cushion of gas on a smooth level sheet of glass. As the puck slides over the glass, photographs taken at successive equal times by a stroboscopic method show that the motion is practically that of uniform velocity. Passengers in a bus or car are jerked forward when the vehicle stops suddenly. They continue in their state of motion until brought to rest by friction or collision. The use of safety belts reduces the shock.

Fig. 1.12 (ii) illustrates a velocity change when an object O is whirled at constant speed by a string. This time the magnitude of the velocity v is constant but its direction changes.

'Mass' is a measure of the inertia of a body. If an object changes its direction or its velocity slightly when a large force acts on it, its inertial mass is high. The mass of an object is constant all over the world; it is the same on the earth as on the moon. Mass is measured in kilogrammes (kg) or grammes (g) by means of a chemical balance, where it is compared with standard masses based on the International Prototype Kilogramme (see also p. 14).

Force. The newton

When an object X is moving it is said to have an amount of *momentum* given, by definition, by

$$momentum = mass\ of\ X \times velocity \quad . \quad . \quad (1)$$

Thus an object of mass 20 kg moving with a velocity of 10 m s^{-1} has a momentum of 200 kg m s^{-1}. If another object collides with X its velocity alters, and thus the momentum of X alters. From Newton's second law, a *force* acts on X which is equal to the change in momentum per second.

Thus if F is the magnitude of a force acting on a constant mass m,

$$F \propto m \times \text{change of velocity per second}$$

$$\therefore F \propto ma,$$

where a is the *acceleration* produced by the force, by definition of a.

$$\therefore F = kma \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where k is a constant.

With SI units, the **newton** (N) is the unit of force. It is defined as the force which gives a mass of 1 kilogramme an acceleration of 1 metre s^{-2}. Substituting $F = 1N$, $m = 1$ kg and $a = 1$ m s^{-2} in the expression for F in (i), we obtain $k = 1$. Hence, with units as stated, $k = 1$.

$$\therefore \mathbf{F = ma},$$

which is a standard equation in dynamics. Thus if a mass of 200 g is acted upon by a force F which produces an acceleration a of 4 m s^{-2}, then, since $m = 200$ g $= 0{\cdot}2$ kg,

$$F = ma = 0{\cdot}2(\text{kg}) \times 4(\text{m s}^{-2}) = 0{\cdot}8 \text{ N}.$$

C.g.s. units of force

The *dyne* is the unit of force in the centimetre-gramme-second system; it is defined as the force acting on a mass of 1 gramme which gives it an acceleration of 1 cm s^{-2}. The equation $F = ma$ also applies when m is in grammes, a is in cm s^{-2}, and F is in dynes. Thus if a force of 10000 dynes acts on a mass of 200 g, the acceleration a is given by

$$F = 10000 = 200 \times a, \quad \text{or} \quad a = 50 \text{ cm s}^{-2}.$$

Suppose $m = 1$ kg $= 1000$ g, $a = 1$ m $s^{-2} = 100$ cm s^{-2}. Then, the force F is given by

$$F = ma = 1000 \times 100 \text{ dynes} = 10^5 \text{ dynes}.$$

But the force acting on a mass of 1 kg which gives it an acceleration of 1 m s^{-2} is the *newton*, N. Hence

$$1 \text{ N} = 10^5 \text{ dynes}$$

Weight. Relation between newton, kgf and dyne, gf

The *weight* of an object is defined as the *force* acting on it due to gravity; the weight of an object can hence be measured by attaching it to a spring-balance and noting the extension, as the latter is proportional to the force acting on it (p. 50).

Suppose the weight of an object of mass m is denoted by W. If the object is released so that it falls to the ground, its acceleration is g. Now $F = ma$. Consequently the force acting on it, i.e., its weight, is given by

$$W = mg.$$

If the mass is 1 kg, then, since $g = 9{\cdot}8$ m s^{-2}, the weight $W = 1 \times 9{\cdot}8 = 9{\cdot}8$ N (newton). The force due to gravity on a mass of 1 kg where g has the value 9·80665 m s^{-2} is called a 1 *kilogramme force* or 1 kgf (this is roughly equal to 1 kilogramme weight or 1 kg wt, which depends on the value of g and thus varies from place to place). Hence it follows that

$$1 \; kgf = 9{\cdot}8 \; N = 10 \; N \text{ approximately.}$$

A weight of 5 kgf is thus about 50 N. Further, 1 N $= \frac{1}{10}$ kgf approx $=$ 100 gf. The weight of an apple is about 1 newton.

The weight of a mass of 1 gramme is called *gramme-force* (1 gf); it was formerly called '1 gramme wt'. From $F = ma$, it follows that

$$1 \text{ gf} = 1 \times 980 = 980 \text{ dynes.}$$

since $g = 980$ cm s^{-2} (approx).

The reader should note carefully the difference between the 'kilogramme' and the 'kilogramme force'; the former is a *mass* and is therefore constant all over the universe, whereas the kilogramme force is a *force* whose magnitude depends on the value of g. The acceleration due to gravity, g, depends on the distance of the place considered from the centre of the earth; it is slightly greater at the poles than at the equator, since the earth is not perfectly spherical (see p. 41). It therefore follows that the weight of an object differs in different parts of the world. On the moon, which is smaller than the earth and has a smaller density, an object would weigh about one-sixth of its weight on the earth.

The relation $F = ma$ can be verified by using a ticker-tape and timer to measure the acceleration of a moving trolley. Details are given in a more basic text, such as *Fundamentals of Physics* (Chatto and Windus) by the author.

The following examples illustrate the application of $F = ma$. It should be carefully noted that (i) F represents the *resultant* force on the object of mass m, (ii) F must be expressed in the appropriate units of a 'force' and m in the corresponding units of a 'mass'.

EXAMPLES

1. A force of 20 kgf pulls a sledge of mass 50 kg and overcomes a constant frictional force of 4 kgf. What is the acceleration of the sledge?

$$\text{Resultant force, } F, = 20\text{ kgf} - 4\text{ kgf} = 16\text{ kgf}.$$

To change this to units of newtons, use 1 kgf = 9·8 N = 10 N approx.

$$\therefore 16\text{ kgf} = 160\text{ N approx.}$$

From $F = ma$,

$$\therefore 160 = 50 \times a$$

$$\therefore a = 3{\cdot}2\text{ m s}^{-2}.$$

2. An object of mass 2·00 kg is attached to the hook of a spring-balance, and the latter is suspended vertically from the roof of a lift. What is the reading on the spring-balance when the lift is (i) ascending with an acceleration of 20 cm s^{-2}, (ii) descending with an acceleration of 10 cm s^{-2}, (iii) ascending with a uniform velocity of 15 cm s^{-1}.

Suppose T is the tension (force) in the spring-balance in kgf.

(i) The object is acted upon two forces: (*a*) The tension T kgf in the spring-balance, which acts upwards, (*b*) its weight, 2 kgf, which acts downwards. Since the object moves upwards, T is greater than 2 kgf. Hence the net force, F, acting on the object = $(T-2)$ kgf = $(T-2) \times 10$ N, approx. Now

$$F = ma,$$

where a is the acceleration in m s^{-2}.

$$\therefore (T-2) \times 10 = 2 \times a = 2 \times 0{\cdot}2$$

$$\therefore T = 2{\cdot}04\text{ kgf} \quad . \quad . \quad . \quad . \quad (1)$$

(ii) When the lift descends with an acceleration of 10 cm s^{-2} or 0·1 m s^{-2}, the weight, 2 kgf, is now greater than T_1 kgf, the tension in the spring-balance.

$$\therefore \text{resultant force} = (2 - T_1)\text{ kgf} = (2 - T_1) \times 10\text{ N approx.}$$

$$\therefore F = (2 - T_1) \times 10 = ma = 2 \times 0{\cdot}1$$

$$\therefore T_1 = 2 - 0{\cdot}02 = 1{\cdot}98\text{ kgf}.$$

(iii) When the lift moves with constant velocity, the acceleration is zero. In this case the reading on the spring-balance is exactly equal to the weight, 2 kgf.

Linear Momentum

Newton defined the force acting on an object as the rate of change of its momentum, the momentum being the product of its mass and velocity (p. 13). *Momentum is thus a vector quantity.* Suppose that the mass of an object is m, its initial velocity is u, and its final velocity due to a force F acting on it for a time t is v. Then

$$\text{change of momentum} = mv - mu,$$

and hence

$$F = \frac{mv - mu}{t}$$

$$\therefore\ Ft = mv - mu = \textit{momentum change} \quad . \quad . \quad (1)$$

The quantity Ft (force × time) is known as the *impulse* of the force on the object, and from (1) it follows that the units of momentum are the same as those of Pt, i.e., *newton second* (N s). From 'mass × velocity', alternative units are 'kg m s^{-1}'.

Force and momentum change

A person of mass 50 kg who is jumping from a height of 5 metres will land on the ground with a velocity $= \sqrt{2gh} = \sqrt{2 \times 10 \times 5}$ $= 10$ m s^{-1}, assuming $g = 980$ cm $s^{-2} = 10$ m s^{-2} approx. If he does not flex his knees on landing, he will be brought to rest very quickly, say in $\frac{1}{10}$th second. The force F acting is then given by

$$F = \frac{\text{momentum change}}{\text{time}}$$

$$= \frac{50 \times 10}{\frac{1}{10}} = 5000\ \text{N} = 500\ \text{kgf (approx).}$$

This is a force of about 10 times the person's weight and this large force has a severe effect on the body.

Suppose, however, that the person flexes his knees and is brought to rest much more slowly on landing, say in 1 second. Then, from above, the force F now acting is 10 times less than before, or 50 kgf (approx). Consequently, much less damage is done to the person on landing.

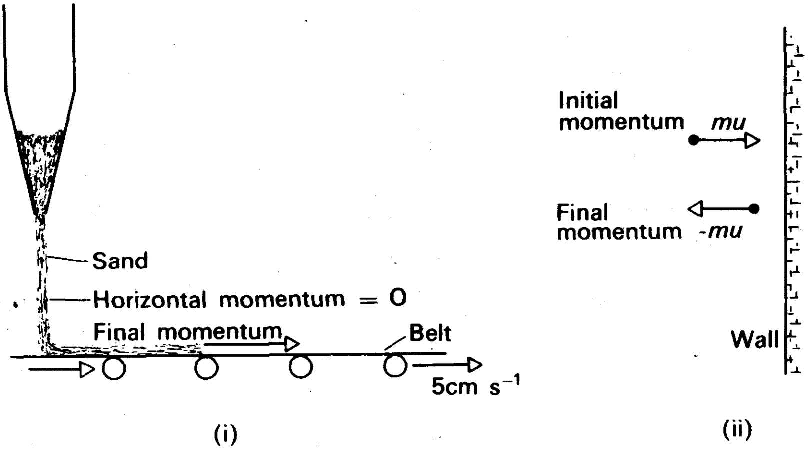

FIG. 1.13 Linear momentum

Suppose sand is allowed to fall vertically at a steady rate of 100 g s^{-1} on to a horizontal conveyor belt moving at a steady velocity of 5 cm s^{-1}. Fig. 1.13 (i). The initial horizontal velocity of the sand is zero. The final horizontal velocity is 5 cm s^{-1}. Now

$$\text{mass} = 100\text{ g} = 0{\cdot}1\text{ kg, velocity} = 5\text{ cm s}^{-1} = 5\times10^{-2}\text{ m s}^{-1}$$

$$\therefore \text{momentum change per second} = 0{\cdot}1\times5\times10^{-2} = 5\times10^{-3}\text{ newton}$$
$$= \text{force on belt}$$

Observe that this is a case where the *mass* changes with time and the velocity gained is constant. In terms of the calculus, the force is the rate of change of momentum mv, which is $v \times dm/dt$, and dm/dt is 100 g s^{-1} in this numerical example.

Consider a molecule of mass m in a gas, which strikes the wall of a vessel repeatedly with a velocity u and rebounds with a velocity $-u$. Fig. 1.13 (ii). Since momentum is a vector quantity, the momentum change = final momentum − initial momentum = $mu-(-mu) = 2mu$. If the containing vessel is a cube of side l, the molecule repeatedly takes a time $2l/u$ to make an impact with the same side.

$$\therefore \text{average force on wall due to molecule}$$
$$= \frac{\text{momentum change}}{\text{time}}$$
$$= \frac{2mu}{2l/u} = \frac{mu^2}{l}.$$

The total gas pressure is the average force per unit area on the walls of the container due to all the numerous gas molecules.

EXAMPLES

1. A hose ejects water at a speed of 20 cm s^{-1} through a hole of area 100 cm^2. If the water strikes a wall normally, calculate the force on the wall in newton, assuming the velocity of the water normal to the wall is zero after collision.

The volume of water per second striking the wall = $100\times20 = 2000$ cm^3.

$$\therefore \text{mass per second striking wall} = 2000\text{ g s}^{-1} = 2\text{ kg s}^{-1}.$$

Velocity change of water on striking wall = $20-0 = 20$ cm s^{-1} = 0·2 m s^{-1}.

$$\therefore \text{momentum change per second} = 2\,(\text{kg s}^{-1})\times0{\cdot}2\,(\text{m s}^{-1}) = 0{\cdot}4\text{ newton.}$$

2. Sand drops vertically at the rate of 2 kg s^{-1} on to a conveyor belt moving horizontally with a velocity of 0·1 m s^{-1}. Calculate (i) the extra power needed to keep the belt moving, (ii) the rate of change of kinetic energy of the sand. Why is the power twice as great as the rate of change of kinetic energy?

(i) Force required to keep belt moving = rate of increase of horizontal momentum of sand = mass per second $(dm/dt)\times$ velocity change = $2\times0{\cdot}1$ = 0·2 newton.

$$\therefore \text{power} = \text{work done per second} = \text{force}\times\text{rate of displacement}$$
$$= \text{force}\times\text{velocity} = 0{\cdot}2\times0{\cdot}1 = 0{\cdot}02\text{ watt (p. 25).}$$

(ii) Kinetic energy of sand $= \frac{1}{2}mv^2$.

$$\therefore \text{ rate of change of energy} = \tfrac{1}{2}v^2 \times \frac{dm}{dt}, \text{ since } v \text{ is constant,}$$

$$= \tfrac{1}{2} \times 0{\cdot}1^2 \times 2 = 0{\cdot}01 \text{ watt.}$$

Thus the power supplied is twice as great as the rate of change of kinetic energy. The extra power is due to the fact that the sand does not immediately assume the velocity of the belt, so that the belt at first moves relative to the sand. The extra power is needed to overcome the friction between the sand and belt.

Conservation of Linear Momentum

We now consider what happens to the linear momentum of objects which *collide* with each other.

Experimentally, this can be investigated by several methods:

1. Trolleys in collision, with ticker-tapes attached to measure velocities.
2. Linear Air-track, using perspex models in collision and stroboscopic photography for measuring velocities.

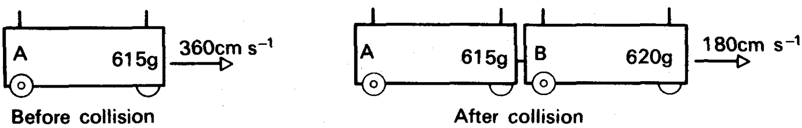

FIG. 1.14 Linear momentum experiment

As an illustration of the experimental results, the following measurements were taken in trolley collisions (Fig. 1.14):

Before collision.

Mass of trolley A = 615 g; initial velocity = 360 cm s^{-1}.

After collision.

A and B coalesced and both moved with velocity of 180 cm s^{-1}.

Thus the total linear momentum of A and B before collision $= 0{\cdot}615 \text{ (kg)} \times 3{\cdot}6 \text{ (m s}^{-1}) + 0 = 2{\cdot}20 \text{ kg m s}^{-1}$ (approx). The total momentum of A and B after collision $= 1{\cdot}235 \times 1{\cdot}8 = 2{\cdot}20 \text{ kg m s}^{-1}$ (approx).

Within the limits of experimental accuracy, it follows that *the total moment of A and B before collision = the total momentum after collision.* Similar results are obtained if A and B are moving with different speeds after collision, or in opposite directions before collision.

Principle of Conservation of Linear Momentum

These experimental results can be shown to follow from Newton's second and third laws of motion (p. 12).

Suppose that a moving object A, of mass m_1 and velocity u_1, collides with another object B, of mass m_2 and velocity u_2, moving in the same direction, Fig. 1.15. By Newton's law of action and reaction, the force F exerted by A on B is equal and opposite to that exerted by B on A. Moreover, the time t during which the force acted on B is equal to the time during which the force of reaction acted on A. Thus the magnitude of the impulse, Ft, on B is equal and opposite to the magnitude of the impulse on A. From equation (1), p. 16, the impulse is equal to the change of momentum. It therefore follows that the change in the total momentum of the two objects is *zero*, i.e., the total momentum of the two objects is constant although a collision had occurred. Thus if A moves with a reduced velocity v_1 after collision, and B then moves with an increased velocity v_2,

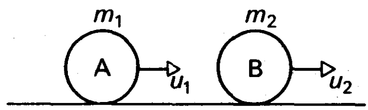

FIG. 1.15
Conservation of linear momentum

$$m_1u_1+m_2u_2 = m_1v_1+m_2v_2.$$

The *principle of the conservation of linear momentum* states that, *if no external forces act on a system of colliding objects, the total momentum of the objects remains constant.*

EXAMPLES

1. An object A of mass 2 kg is moving with a velocity of 3 m s^{-1} and collides head on with an object B of mass 1 kg moving in the opposite direction with a velocity of 4 m s^{-1}. Fig. 1.16 (i). After collision both objects coalesce, so that they move with a common velocity v. Calculate v.

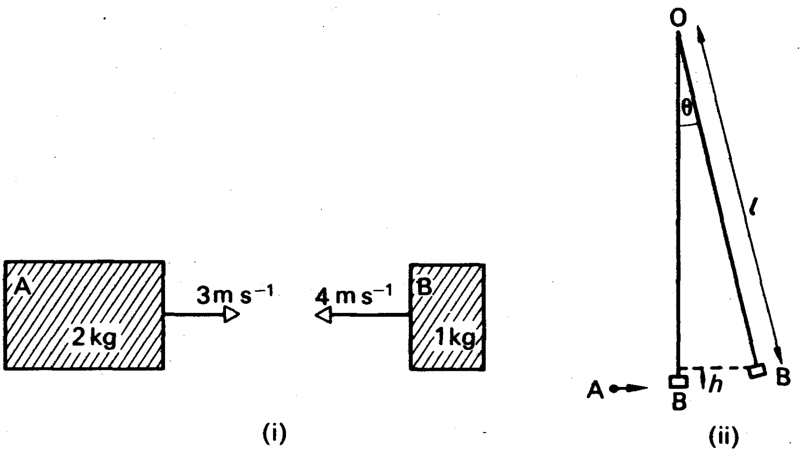

FIG. 1.16 Examples

Total momentum before collision of A and B in the direction of A

$$= 2\times3-1\times4 = 2\,\text{kg m s}^{-1}.$$

Note that momentum is a vector and the momentum of B is of opposite sign to A.

After collision, momentum of A and B in the direction of A $= 2v+1v = 3v$.

$$\therefore 3v = 2$$

$$\therefore v = \tfrac{2}{3}\ \text{m s}^{-1}.$$

2. What is understood by (*a*) the principle of the *conservation of energy*, (*b*) the principle of the *conservation of momentum*?

A bullet of mass 20 g travelling horizontally at 100 m s^{-1}, embeds itself in the centre of a block of wood of mass 1 kg which is suspended by light vertical strings 1 m in length. Calculate the maximum inclination of the strings to the vertical.

Describe in detail how the experiment might be carried out and used to determine the velocity of the bullet just before the impact of the block. (*N.*)

Second part. Suppose A is the bullet, B is the block suspended from a point O, and θ is the maximum inclination to the vertical, Fig. 1.16(ii). If v cm s^{-1} is the common velocity of block and bullet when the latter is brought to rest relative to the block, then, from the principle of the conservation of momentum, since 20 g = 0·02 kg,

$$(1+0{\cdot}02)v = 0{\cdot}02\times 100$$

$$\therefore v = \frac{2}{1{\cdot}02} = \frac{100}{51}\ \text{m s}^{-1}.$$

The vertical height risen by block and bullet is given by $v^2 = 2gh$, where $g =$ 9·8 m s^{-2} and $h = l-l\cos\theta = l(1-\cos\theta)$.

$$\therefore v^2 = 2gl(1-\cos\theta).$$

$$\therefore \left(\frac{100}{51}\right)^2 = 2\times 9{\cdot}8\times 1(1-\cos\theta).$$

$$\therefore 1-\cos\theta = \left(\frac{100}{51}\right)^2\times\frac{1}{2\times 9{\cdot}8} = 0{\cdot}1962.$$

$$\therefore \cos\theta = 0{\cdot}8038,\ \text{or}\ \theta = 37^\circ\ \text{(approx.)}.$$

The velocity, v, of the bullet can be determined by applying the conservation of momentum principle.

Thus $mv = (m+M)V$, where m is the mass of the bullet, M is the mass of the block, and V is the common velocity. Then $v = (m+M)V/m$. The quantities m and M can be found by weighing. V is calculated from the horizontal displacement a of the block, since (i) $V^2 = 2gh$ and (ii) $h(2l-h) = a^2$ from the geometry of the circle, so that, to a good approximation, $2h = a^2/l$.

Inelastic and elastic collisions

In collisions, the total momentum of the colliding objects is always conserved. Usually, however, their total kinetic energy is not conserved. Some of it is changed to heat or sound energy, which is not recoverable. Such collisions are said to be *inelastic*. If the total kinetic energy is conserved, the collision is said to be *elastic*. The collision between two smooth billiard balls is approximately elastic. Many atomic collisions are elastic. Electrons may make elastic or inelastic collisions with atoms of a gas. As proved on p. 28, the kinetic energy of a mass m moving with a velocity v has kinetic energy equal to $\frac{1}{2}mv^2$.

As an illustration of the mechanics associated with elastic collisions, consider a sphere A of mass m and velocity v incident on a stationary sphere B of equal mass m. (Fig. 1.17 (i). Suppose the collision is elastic, and after collision let A move with a velocity v_1 at an angle of 60° to its original direction and B move with a velocity v_2 at an angle θ to the direction of v.

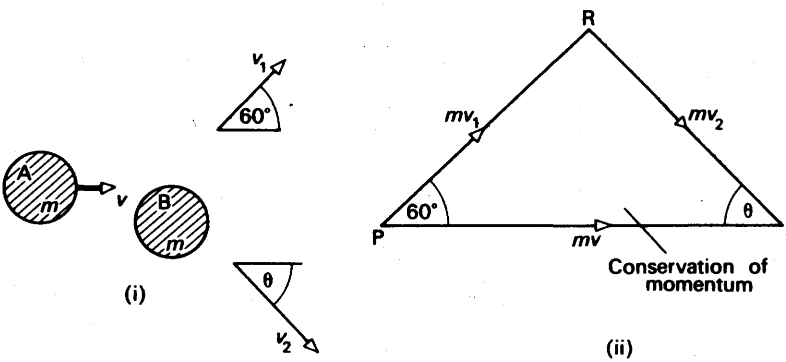

FIG. 1.17 Conservation of momentum

Since momentum is a vector (p. 17), we may represent the momentum mv of A by the line PQ drawn in the direction of v. Fig. 1.17 (ii). Likewise, PR represents the momentum mv_1 of A after collision. *Since momentum is conserved, the vector RQ must represent the momentum mv_2 of B after collision*, that is,

$$m\vec{v} = m\vec{v}_1 + m\vec{v}_2.$$

Hence
$$\vec{v} = \vec{v}_1 + \vec{v}_2,$$

or PQ represents v in magnitude, PR represents v_1 and RQ represents v_2. But if the collision is elastic,

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_1{}^2 + \tfrac{1}{2}mv_2{}^2$$

$$\therefore\ v^2 = v_1{}^2 + v_2{}^2.$$

Consequently, triangle PRQ is a right-angled triangle with angle R equal to 90°.

$$\therefore\ v_1 = v\cos 60° = \frac{v}{2}.$$

Also, $\theta = 90° - 60° = 30°$, and $v_2 = v\cos 30° = \frac{\sqrt{3}v}{2}$.

Coefficient of restitution

In practice, colliding objects do not stick together and kinetic energy is always lost. If a ball X moving with velocity u_1 collides head-on with a ball Y moving with a velocity u_2 in the same direction, then Y will move faster with a velocity v_1 say and X may then have a reduced velocity v_2 in the same direction. The coefficient of restitution, e, between X and Y is defined as the ratio:

$$\frac{\text{velocity of separation}}{\text{velocity of approach}} \quad \text{or} \quad \frac{v_2-v_1}{u_1-u_2}.$$

The coefficient of restitution is approximately constant between two given materials. It varies from $e = 0$, when objects stick together and the collision is completely inelastic, to $e = 1$, when objects are very hard and the collision is practically elastic. Thus, from above, if $u_1 = 4 \text{ m s}^{-1}$, $u_2 = 1 \text{ m s}^{-1}$ and $e = 0{\cdot}8$, then velocity of separation, $v_2-v_1 = 0{\cdot}8\times(4-1) = 2{\cdot}4 \text{ m s}^{-1}$.

Momentum and Explosive forces

There are numerous cases where momentum changes are produced by *explosive* forces. An example is a bullet of mass $m = 50$ g say, fired from a rifle of mass $M = 2$ kg with a velocity v of 100 m s^{-1}. Initially, the total momentum of the bullet and rifle is zero. From the principle of the conservation of linear momentum, when the bullet is fired the total momentum of bullet and rifle is still zero, since no external force has acted on them. Thus if V is the velocity of the rifle,

$$mv\ (\text{bullet})+MV(\text{rifle}) = 0$$

$$\therefore\ MV = -mv, \quad \text{or} \quad V = -\frac{m}{M}v.$$

The momentum of the rifle is thus *equal and opposite* to that of the bullet. Further, $V/v = -m/M$. Since $m/M = 50/2000 = 1/40$, it follows that $V = -v/40 = 2{\cdot}5 \text{ m s}^{-1}$. This means that the rifle moves back or *recoils* with a velocity only about $\frac{1}{40}$th that of the bullet.

If it is preferred, one may also say that the explosive force produces the same numerical momentum change in the bullet as in the rifle. Thus $mv = MV$, where V is the velocity of the rifle in the *opposite* direction to that of the bullet. The joule (J) is the unit of energy (p. 24).

The kinetic energy, E_1, of the bullet $= \frac{1}{2}mv^2 = \frac{1}{2}\,.\,0{\cdot}05\,.\,100^2 = 250\text{J}$

The kinetic energy, E_2, of the rifle $= \frac{1}{2}MV^2 = \frac{1}{2}\,.\,2\,.\,2{\cdot}5^2 = 6{\cdot}25\text{J}$

Thus the total kinetic energy produced by the explosion $= 256{\cdot}25$ J. The kinetic energy E_1 of the bullet is thus $250/256{\cdot}25$, or about 98%, of the total energy. This is explained by the fact that the kinetic energy depends on the *square* of the velocity. The high velocity of the bullet thus more than compensates for its small mass relative to that of the rifle. See also p. 26.

Rocket

Consider a rocket moving in outer space where no external forces act on it. Suppose its mass is M and its velocity is v at a particular instant. Fig. 1.18 (i). When a mass m of fuel is ejected, the mass of the rocket becomes $(M-m)$ and its velocity increases to $(v+\Delta v)$. Fig. 1.18 (ii).

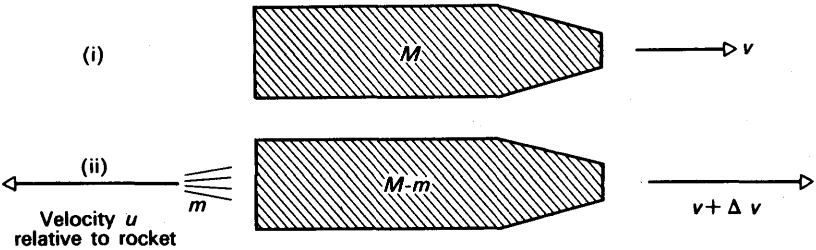

FIG. 1.18. Motion of rocket

Suppose the fuel is always ejected at a constant speed u relative to the rocket. Then the velocity of the mass $m = v+\frac{\Delta v}{2}-u$ in the direction of the rocket, since the initial velocity of the rocket is v and the final velocity is $v+\Delta v$, an average of $v+\Delta v/2$.

We now apply the principle of the conservation of momentum to the rocket and fuel. *Initially*, before m of fuel was ejected, momentum of rocket and fuel inside rocket $= Mv$.

After m is ejected, momentum of rocket $= (M-m)(v+\Delta v)$

$$\text{and momentum of fuel} = m\left(v+\frac{\Delta v}{2}-u\right).$$

$$\therefore\ (M-m)(v+\Delta v)+m\left(v+\frac{\Delta v}{2}-u\right) = Mv.$$

Neglecting the product of $m\,.\,\Delta v$, then, after simplification,

$$M.\Delta v-mu = 0,$$

$$\therefore\ \frac{m}{M} = \frac{\Delta v}{u}.$$

Now $$m = \text{mass of fuel ejected} = -\Delta M,$$

$$\therefore\ -\frac{\Delta M}{M} = \frac{\Delta v}{u}.$$

Integrating between limits of M, M_0 and v, v_0 respectively

$$\therefore \int_{M_0}^{M} -\frac{\Delta M}{M} = \frac{1}{u}\int_{v_0}^{v} \Delta v.$$

$$\therefore\ -\log_e\frac{M}{M_0} = \frac{v-v_0}{u}.$$

$$\therefore\ M = M_0 e^{-(v-v_0)/u} \quad . \quad . \quad . \quad . \quad (1)$$

or $$v = v_0-u\log_e(M/M_0) \quad . \quad . \quad . \quad (2)$$

When the mass M decreases to $M_0/2$

$$v = v_0+u\log_e 2.$$

Motion of centre of mass

If two particles, masses m_1 and m_2, are distances x_1, x_2 respectively from a given axis, their *centre of mass* is at a distance x from the axis given by $m_1x_1 + m_2x_2 = (m_1 + m_2)\bar{x}$. See p. 104. Since velocity, $v = dx/dt$ generally, the velocity $\bar{v}$ of the centre of mass in the particular direction is given by $m_1v_1 + m_2v_2 = (m_1 + m_2)\bar{v}$, where v_1, v_2 are the respective velocities of m_1, m_2. The quantity $(m_1v_1 + m_2v_2)$ represents the total momentum of the two particles. The quantity $(m_1 + m_2)\bar{v} = M\bar{v}$, where M is the total mass of the particles. Thus we can imagine that the total mass of the particles is concentrated at the centre of mass while they move, and that the velocity $\bar{v}$ of the centre of mass is always given by *total momentum* $= M\bar{v}$.

If *internal forces* act on the particles while moving, then, since action and reaction are equal and opposite, their resultant on the whole body is zero. Consequently the total momentum is unchanged and hence the velocity or motion of their centre of mass if unaffected. If an *external force*, however, acts on the particles, the total momentum is changed. The motion of their centre of mass now follows a path which is due to the external force.

We can apply this to the case of a shell fired from a gun. The centre of mass of the shell follows at first a parabolic path. This is due to the external force of gravity, its weight. If the shell explodes in mid-air, the fragments fly off in different directions. But the numerous internal forces which occur in the explosion have zero resultant, since action and reaction are equal and opposite and the forces can all be paired. Consequently *the centre of mass of all the fragments continues to follow the same parabolic path.* As soon as one fragment reaches the ground, an external force now acts on the system of particles. A different parabolic path is then followed by the centre of mass

If a bullet is fired in a horizontal direction from a rifle, where is their centre of mass while the bullet and rifle are both moving?

Work

When an engine pulls a train with a constant force of 50 units through a distance of 20 units in its own direction, the engine is said by definition to do an amount of *work* equal to 50×20 or 1000 units, the product of the force and the distance. Thus if W is the amount of work,

$$W = \text{force} \times \text{distance moved in direction of force.}$$

Work is a *scalar* quantity; it has no property of direction but only magnitude. When the force is one newton and the distance moved is one metre, then the work done is one *joule*. Thus a force of 50 N moving through a distance of 10 m does 50×10 or 500 joule of work. Note this is also a measure of the *energy* transferred to the object.

The force to raise steadily a mass of 1 kg is 1 kilogram force (1 kgf), which is about 10 N (see p. 14). Thus if the mass of 1 kg is raised vertically through 1 m, then, approximately, work done $= 10\,(\text{N}) \times 1\,(\text{m}) = 10$ joule.

The c.g.s. unit of work is the *erg*; it is the work done when a force of 1 dyne moves through 1 cm. Since 1 N = 10^5 dynes and 1 m = 100 cm, then 1 N moving through 1 m does an amount of work = 10^5 (dyne) × 100 (cm) = 10^7 ergs = 1 joule, by definition of the joule (p. 24).

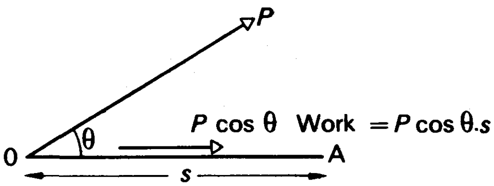

FIG. 1.19 Work

Before leaving the topic of 'work', the reader should note carefully that we have assumed the force to move an object in its own direction. Suppose, however, that a force P pulls an object a distance s along a line OA acting at an angle θ to it, Fig. 1.19. The component of P along OA is P cos θ (p. 8), and this is the effective part of P pulling along the direction OA. The component of P along a direction perpendicular to OA has no effect along OA. Consequently

$$\text{work done} = P\cos\theta \times s.$$

In general, the work done by a force is equal to the product of the force and the displacement of its point of application in the direction of the force.

Power

When an engine does work quickly, it is said to be operating at a high *power*; if it does work slowly it is said to be operating at a low power. 'Power' is defined as the *work done per second*, i.e.,

$$\text{power} = \frac{\text{work done}}{\text{time taken}}.$$

The practical unit of power, the SI unit, is 'joule per second' or *watt* (W); the watt is defined as the rate of working at 1 joule per second.

$$1 \text{ horse-power (hp)} = 746 \text{ W} = \tfrac{3}{4} \text{ kW (approx)},$$

where 1 kW = 1 kilowatt of 1000 watt. Thus a small motor of $\frac{1}{6}$ hp in a vacuum carpet cleaner has a power of about 125 W.

Kinetic Energy

An object is said to possess *energy* if it can do work. When an object possesses energy because it is moving, the energy is said to be *kinetic*, e.g., a flying stone can disrupt a window. Suppose that an object of mass m is moving with a velocity u, and is gradually brought to rest in a distance s by a constant force F acting against it. The kinetic energy originally possessed by the object is equal to the work done against F, and hence

$$\text{kinetic energy} = F \times s.$$

But $F = ma$, where a is the retardation of the object. Hence $F \times s = mas$. From $v^2 = u^2 + 2as$ (see p. 6), we have, since $v = 0$ and a is negative in this case,

$$0 = u^2 - 2as, \text{ i.e., } as = \frac{u^2}{2}.$$

$$\therefore \textit{ kinetic energy} = mas = \tfrac{1}{2}mu^2.$$

When m is in kg and u is in m s^{-1}, then $\frac{1}{2}mu^2$ is in *joule*. Thus a car of mass 1000 kg, moving with a velocity of 36 km h^{-1} or 10 m s^{-1}, has an amount W of kinetic energy given by

$$W = \tfrac{1}{2}mu^2 = \tfrac{1}{2} \times 1000 \times 10^2 = 50000 \text{ J}$$

Kinetic Energies due to Explosive Forces

Suppose that, due to an explosion or nuclear reaction, a particle of mass m breaks away from the total mass concerned and moves with velocity v, and a mass M is left which moves with velocity V in the opposite direction. Then

$$\frac{\text{kinetic energy, } E_1, \text{ of mass } m}{\text{kinetic energy, } E_2, \text{ of mass } M} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}MV^2} = \frac{mv^2}{MV^2} \quad . \quad (1)$$

Now from the principle of the conservation of linear momentum, $mv = MV$. Thus $v = MV/m$. Substituting for v in (1).

$$\therefore \frac{E_1}{E_2} = \frac{mM^2V^2}{m^2MV^2} = \frac{M}{m} = \frac{1/m}{1/M}.$$

Hence the energy is *inversely*-proportional to the masses of the particles, that is, the smaller mass, m say, has the larger energy. Thus if E is the total energy of the two masses, the energy of the smaller mass $= ME/(M+m)$. An α-particle has a mass of 4 units and a radium nucleus a mass of 228 units. If disintegration of a thorium nucleus, mass 232, produces an α-particle and radium nucleus, and a release of energy of 4·05 MeV, where 1 MeV $= 1{\cdot}6 \times 10^{-13}$J, then

$$\text{energy of } \alpha\text{-particle} = \frac{228}{(4+228)} \times 4{\cdot}05 = 3{\cdot}98 \text{ MeV}.$$

The α-particle thus travels a relatively long distance before coming to rest compared to the radium nucleus.

Potential Energy

A weight held stationary above the ground has energy, because, when released, it can raise another object attached to it by a rope passing over a pulley, for example. A coiled spring also has energy, which is released gradually as the spring uncoils. The energy of the weight or spring is called *potential energy*, because it arises from the position or arrangement of the body and not from its motion. In the case of the weight, the energy given to it is equal to the work done by the person or machine which raises it steadily to that position against the force of attraction of the earth. In the case of the spring, the energy is equal to the work done in displacing the molecules from their normal equilibrium positions against the forces of attraction of the surrounding molecules.

If the mass of an object is m, and the object is held stationary at a height h above the ground, the energy released when the object falls to the ground is equal to the work done

$$= \text{force} \times \text{distance} = \text{weight of object} \times h.$$

Suppose the weight is 5 kgf and h is 4 metre. Then, since 1 kgf = 9·8 N = 10 N approx, then

$$\text{potential energy P.E.} = 50\ (\text{N}) \times 4\ (\text{m}) = 200\ \text{J}$$

$$(\text{more accurately, P.E.} = 192\ \text{J}).$$

Generally, at a height of h,

$$\textit{potential energy} = mgh,$$

where m is in kg, h is in metre, $g = 9{\cdot}8$.

EXAMPLE

Define *work*, *kinetic energy*, *potential energy*. Give *one* example of *each* of the following: (*a*) the conversion into kinetic energy of the work done on a body and (*b*) the conversion into potential energy of the work done on a body.

A rectangular block of mass 10 g rests on a rough plane which is inclined to the horizontal at an angle $\sin^{-1}(0{\cdot}05)$. A force of 0·03 newton, acting in a direction parallel to a line of greatest slope, is applied to the block so that it moves up the plane. When the block has travelled a distance of 110 cm from its initial position, the applied force is removed. The block moves on and comes to rest again after travelling a further 25 cm. Calculate (i) the work done by the applied force, (ii) the gain in potential energy of the block and (iii) the value of the coefficient of sliding friction between the block and the surface of the inclined plane. How would the coefficient of sliding friction be measured if the angle of the slope could be altered? (*O.* and *C.*)

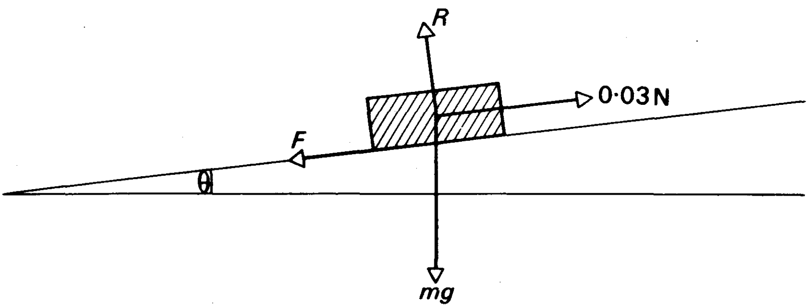

FIG. 1.20 Example

(i) Force = 0·03 newton; distance = 110 cm = 1·1 m.

$$\therefore \text{work} = 0{\cdot}03 \times 1{\cdot}1 = 0{\cdot}033\ \text{J}.$$

(ii) Gain in P.E. = wt × height moved = 0·01 kgf × 1·35 sin θ m,

$$= 0{\cdot}01 \times 9{\cdot}8 \text{ newton} \times 1{\cdot}35 \times 0{\cdot}05 \text{ m} = 0{\cdot}0066 \text{ J (approx.)}.$$

(iii) Work done against frictional force F = work done by force − gain in P.E.

$$= 0{\cdot}033 - 0{\cdot}0066 = 0{\cdot}0264 \text{ J}.$$

$$\therefore F \times 1{\cdot}35 = 0{\cdot}0264.$$

$$\therefore F = \frac{0{\cdot}0264}{1{\cdot}35} \text{ newton}.$$

Normal reaction, $R = mg\cos\theta = mg$ (approx.), since θ is so small

$$\therefore \mu = \frac{F}{R} = \frac{0{\cdot}0264}{1{\cdot}35 \times 0{\cdot}01 \times 9{\cdot}8} = 0{\cdot}2 \text{ (approx.)}.$$

Conservative Forces

If a ball of weight W is raised steadily from the ground to a point X at a height h above the ground, the work done is $W.h$. The potential energy, P.E., of the ball is thus $W.h$. Now whatever route is taken from ground level to X, the work done is the same—if a *longer* path is chosen, for example, the component of the weight in the particular direction must then be overcome and so the force required to move the ball is correspondingly smaller. The P.E. of the ball at X is thus independent of the route to X. This implies that if the ball is taken in a closed path round to X again, *the total work done is zero.* Work has been expended on one part of the closed path, and regained on the remaining part.

When the work done in moving round a closed path in a field to the original point is zero, the forces in the field are called *conservative forces.* The earth's gravitational field is an example of a field containing conservative forces, as we now show.

Suppose the ball falls from a place Y at a height h to another X at a height of x above the ground. Fig. 1.21. Then, if W is the weight of the ball and m its mass,

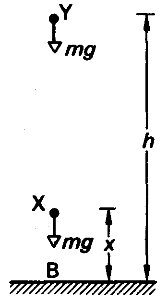

FIG. 1.21. Mechanical energy

$$\text{P.E. at X} = Wx = mgx$$

and $$\text{K.E. at X} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\,.\,2g(h-x) = mg(h-x),$$

using $v^2 = 2as = 2g(h - x)$. Hence

$$\text{P.E.} + \text{K.E.} = mgx + mg(h-x) = mgh.$$

Thus at any point such as X, the total mechanical energy of the falling ball is equal to the original energy. The mechanical energy is hence constant or conserved. This is the case for a conservative field.

Non-Conservative forces. Principle of Conservation of Energy

The work done in taking a mass m round a closed path in the conservative earth's gravitational field is zero. Fig. 1.22 (i). If the work done in taking an object round a closed path to its original position is

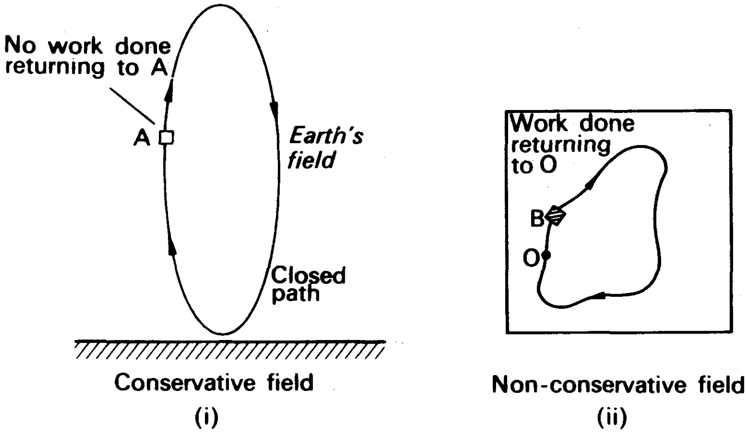

FIG. 1.22 Non-conservative and conservative fields

not zero, the forces in the field are said to be *non-conservative.* This is the case, for example, when a wooden block B is pushed round a closed path on a rough table to its initial position O. Work is therefore done against friction, both as A moves away from O and as it returns. In a conservative field, however, work is done during part of the path and regained for the remaining part.

When a body falls in the earth's gravitational field, a small part of the energy is used up in overcoming the resistance of the air. This energy is dissipated or lost as heat—it is not regained in moving the body back to its original position. This resistance is another example of the action of a non-conservative force.

Although energy may be transformed from one form to another, as in the last example from mechanical energy to heat, *the total energy in a given system is always constant.* If an electric motor is supplied with 1000 joule of energy, 850 joule of mechanical energy, 140 joule of heat energy and 10 joule of sound energy may be produced. This is called the *Principle of the Conservation of Energy* and is one of the key principles in science.

Mass and Energy

Newton said that the 'mass' of an object was 'a measure of the quantity of matter' in it. In 1905, Einstein showed from his Special Theory of Relativity that energy is released from an object when its mass decreases. His mass-energy relation states that if the mass decreases by Δm kg, the energy released in joule, ΔW, is given by

$$\Delta W = \Delta m \, . \, c^2,$$

where c is the numerical value of the speed of light in m s^{-1}, which is 3×10^8. Experiments in Radioactivity on nuclear reactions showed that Einstein's relation was true. Thus mass is a form of energy.

Einstein's relation shows that even if a small change in mass occurs, a relatively large amount of energy is produced. Thus if $\Delta m = 1$ milli-gramme $= 10^{-6}$ kg, the energy ΔW released

$$= \Delta m \,.\, c^2 = 10^{-6} \times (3 \times 10^8)^2 = 9 \times 10^{10} \text{ J}.$$

This energy will keep 250000 100-W lamps burning for about an hour. In practice, significant mass changes occur only in nuclear reactions.

The internal energy of a body of mass m may be considered as $E_{int} = mc^2$, where m is its rest mass. In nuclear reactions where two particles collide, a change occurs in their total kinetic energy and in their total mass. The increase in total kinetic energy is accompanied by an equal decrease in internal energy, $\Delta m \,.\, c^2$. Thus the total energy, kinetic plus internal, remains constant.

Before Einstein's mass-energy relation was known, two independent laws of science were:

(1) *The Principle of the Conservation of Mass* (the total mass of a given system of objects is constant even though collisions or other actions took place between them);

(2) *The Principle of the Conservation of Energy* (the total energy of a given system is constant). From Einstein's relation, however, the two laws can be combined into one, namely, the Principle of the Conservation of Energy.

The summary below may assist the reader; it refers to the units of some of the quantities encountered, and their relations.

Quantity	SI	C.G.S.	Relations
Force (vector)	newton (N)	dyne	10^5 dyne = 1 N 1 kgf = 9·8 N (approx, 10 N) 1 gf = 0·0098 N (approx, 0·01 N)
Mass (scalar)	kilogramme (kg)	gramme (g)	1000 g = 1 kg
Momentum (vector)	newton second (Ns)	dyne second	10^5 dyn s = 1 N s
Energy (scalar)	joule (J)	erg	10^7 erg = 1 J
Power (scalar)	watt (W)	erg s^{-1}	1 W = 1 J s^{-1} 1 h.p. = 746 W

Dimensions

By the *dimensions* of a physical quantity we mean the way it is related to the fundamental quantities mass, length and time; these are usually denoted by M, L, and T respectively. An area, length × breadth, has dimensions L × L or L^2; a volume has dimensions L^3; density, which is mass/volume, has dimensions M/L^3 or ML^{-3}; relative density has no dimensions, since it is the ratio of similar quantities, in this case two masses (p. 114); an angle has no dimensions, since it is the ratio of two lengths.

As an area has dimensions L^2, the *unit* may be written in terms of the metre as 'm^2'. Similarly, the dimensions of a volume are L^3 and hence the unit is 'm^3'. Density has dimensions ML^{-3}. The density of mercury is thus written as '13600 kg m^{-3}'. If some physical quantity has dimensions $ML^{-1}T^{-1}$, its unit may be written as 'kg m^{-1} s^{-1}'.

The following are the dimensions of some quantities in Mechanics:

Velocity. Since velocity $= \dfrac{\text{distance}}{\text{time}}$, its dimensions are L/T or LT^{-1}.

Acceleration. The dimensions are those of velocity/time, i.e., L/T^2 or LT^{-2}.

Force. Since force = mass × acceleration, its dimensions are MLT^{-2}.

Work or Energy. Since work = force × distance, its dimensions are ML^2T^{-2}.

EXAMPLE

In the gas equation $(p+\frac{a}{V^2})(V-b) = RT$, what are the dimensions of the constants a and b?

p represents pressure, V represents volume. The quantity a/V^2 must represent a pressure since it is added to p. The dimensions of p = [force]/[area] = $MLT^{-2}/L^2 = ML^{-1}T^{-2}$; the dimensions of $V = L^3$. Hence

$$\frac{[a]}{L^6} = ML^{-1}T^{-2}, \text{ or } [a] = ML^5T^{-2}.$$

The constant b must represent a volume since it is subtracted from V. Hence

$$[b] = L^3.$$

Application of Dimensions. Simple Pendulum

If a small mass is suspended from a long thread so as to form a simple pendulum, we may reasonably suppose that the period, T, of the oscillations depends only on the mass m, the length l of the thread, and the acceleration, g, due to gravity at the place concerned. Suppose then that

$$T = km^x l^y g^z \quad . \quad . \quad . \quad . \quad \text{(i)}$$

where x, y, z, k are unknown numbers. The dimensions of g are LT^{-2} from above. Now the dimensions of both sides of (i) must be the same.

$$\therefore T = M^x L^y (LT^{-2})^z.$$

Equating the indices of M, L, T on both sides, we have

$$x = 0,$$

$$y+z = 0,$$

and

$$-2z = 1.$$

$$\therefore z = -\tfrac{1}{2}, y = \tfrac{1}{2}, x = 0.$$

Thus, from (i), the period T is given by

$$T = kl^{\frac{1}{2}}g^{-\frac{1}{2}},$$

or

$$T = k\sqrt{\frac{l}{g}}.$$

We cannot find the magnitude of k by the method of dimensions, since it is a number. A complete mathematical investigation shows that $k = 2\pi$ in this case, and hence $T = 2\pi\sqrt{l/g}$. (See also p. 48).

Velocity of Transverse Wave in a String

As another illustration of the use of dimensions, consider a wave set up in a stretched string by plucking it. The velocity, V, of the wave depends on the tension, F, in the string, its length l, and its mass m, and we can therefore suppose that

$$V = kF^x l^y m^z, \quad \ldots \quad \text{(i)}$$

where x, y, z are numbers we hope to find by dimensions and k is a constant.

The dimensions of velocity, V, are LT^{-1}, the dimensions of tension, F, are MLT^{-2}, the dimension of length, l, is L, and the dimension of mass, m, is M. From (i), it follows that

$$LT^{-1} \equiv (MLT^{-2})^x \times L^y \times M^z.$$

Equating powers of M, L, and T on both sides,

$$\therefore 0 = x + z, \quad \ldots \quad \text{(i)}$$

$$1 = x + y, \quad \ldots \quad \text{(ii)}$$

and

$$-1 = -2x, \quad \ldots \quad \text{(iii)}$$

$$\therefore x = \tfrac{1}{2},\ z = -\tfrac{1}{2},\ y = \tfrac{1}{2}.$$

$$\therefore V = k \,.\, F^{\frac{1}{2}} l^{\frac{1}{2}} m^{-\frac{1}{2}},$$

$$\text{or } V = k\sqrt{\frac{Fl}{m}} = k\sqrt{\frac{F}{m/l}} = k\sqrt{\frac{\text{Tension}}{\text{mass per unit length}}}$$

A complete mathematical investigation shows that $k = 1$.

The method of dimensions can thus be used to find the relation between quantities when the mathematics is too difficult. It has been extensively used in hydrodynamics, for example. See also pp. 176, 181.

EXERCISES 1

(Assume $g = 10\ m\,s^{-2}$, unless otherwise given)

What are the missing words in the statements 1–10?

1. The dimensions of velocity are . . .

2. The dimensions of force are . . .

3. Using 'vector' or 'scalar', (i) mass is a . . . (ii) force is a . . . (iii) energy is a . . . (iv) momentum is a . . .

4. Linear momentum is defined as . . .

5. An 'elastic' collision is one in which the . . . and the . . . are conserved.

6. When two objects collide, their . . . is constant provided no . . . forces act.

7. One newton × one metre = . . .

8. 1 kilogram force = ... newton, approx.

9. The momentum of two different bodies must be added by a ... method.

10. Force is the ... of change of momentum.

Which of the following answers, A, B, C, D or E, do you consider is the correct one in the statements 11–14?

11. When water from a hosepipe is incident horizontally on a wall, the force on the wall is calculated from *A* speed of water, *B* mass × velocity, *C* mass per second × velocity, *D* energy of water, *E* momentum change.

12. When a ball of mass 2 kg moving with a velocity of 10 m s^{-1} collides head-on with a ball of mass 3 kg and both move together after collision, the common velocity is *A* 5 m s^{-1} and energy is lost, *B* 4 m s^{-1} and energy is lost, *C* 2 m s^{-1} and energy is gained, *D* 6 m s^{-1} and momentum is gained, *E* 6 m s^{-1} and energy is conserved.

13. An object of mass 2 kg moving with a velocity of 4 m s^{-1} has a kinetic energy of *A* 8 joule, *B* 16 erg, *C* 4000 erg, *D* 16 joule, *E* 40000 joule.

14. The dimensions of work are *A* ML^2T^{-2} and it is a scalar, *B* ML^2T^{-2} and it is a vector, *C* MLT^{-1} and it is a scalar, *D* ML^2T and it is a scalar, *E* MLT and it is a vector.

15. A car moving with a velocity of 36 km h^{-1} accelerates uniformly at 1 m s^{-2} until it reaches a velocity of 54 km h^{-1}. Calculate (i) the time taken, (ii) the distance travelled during the acceleration, (iii) the velocity reached 100 m from the place where the acceleration began.

16. A ball of mass 100 g is thrown vertically upwards with an initial speed of 72 km h^{-1}. Calculate (i) the time taken to return to the thrower, (ii) the maximum height reached, (iii) the kinetic and potential energies of the ball half-way up.

17. The velocity of a ship A relative to a ship B is 10·0 km h^{-1} in a direction N. 45° E. If the velocity of B is 20·0 km h^{-1} in a direction N. 60° W., find the actual velocity of A in magnitude and direction.

18. Calculate the energy of (i) a 2 kg object moving with a velocity of 10 m s^{-1}, (ii) a 10 kg object held stationary 5 m above the ground.

19. A 4 kg ball moving with a velocity of 10·0 m s^{-1} collides with a 16 kg ball moving with a velocity of 4·0 m s^{-1} (i) in the same direction, (ii) in the opposite direction. Calculate the velocity of the balls in each case if they coalesce on impact, and the loss of energy resulting from the impact. State the principle used to calculate the velocity.

20. A ship X moves due north at 30·0 km h^{-1}; a ship Y moves N. 60° W. at 20·0 km h^{-1}. Find the velocity of Y relative to X in magnitude and direction. If Y is 10 km due east of X at this instant, find the closest distance of approach of the two ships.

21. Two buckets of mass 6 kg are each attached to one end of a long inextensible string passing over a fixed pulley. If a 2 kg mass of putty is dropped from a height of 5 m into one bucket, calculate (i) the initial velocity of the system, (ii) the acceleration of the system, (iii) the loss of energy of the 2 kg mass due to the impact.

22. A bullet of mass 25 g and travelling horizontally at a speed of 200 m s^{-1} imbeds itself in a wooden block of mass 5 kg suspended by cords 3 m long. How far will the block swing from its position of rest before beginning to return? Describe a suitable method of suspending the block for this experiment and explain briefly the principles used in the solution of the problem. (*L.*)

23. State the principle of the conservation of linear momentum and show how it follows from Newton's laws of motion.

A stationary radioactive nucleus of mass 210 units disintegrates into an alpha particle of mass 4 units and a residual nucleus of mass 206 units. If the kinetic energy of the alpha particle is *E*, calculate the kinetic energy of the residual nucleus. (*N.*)

24. Define linear momentum and state the principle of conservation of linear momentum. Explain briefly how you would attempt to verify this principle by experiment.

Sand is deposited at a uniform rate of 20 kilogramme per second and with negligible kinetic energy on to an empty conveyor belt moving horizontally at a constant speed of 10 metre per minute. Find (*a*) the force required to maintain constant velocity, (*b*) the power required to maintain constant velocity, and (*c*) the rate of change of kinetic energy of the moving sand. Why are the latter two quantities unequal? (*O. & C.*)

25. What do you understand by the *conservation of energy*? Illustrate your answer by reference to the energy changes occurring (*a*) in a body whilst falling to and on reaching the ground, (*b*) in an X-ray tube.

The constant force resisting the motion of a car, of mass 1500 kg, is equal to one-fifteenth of its weight. If, when travelling at 48 km per hour, the car is brought to rest in a distance of 50 m by applying the brakes, find the additional retarding force due to the brakes (assumed constant) and the heat developed in the brakes. (*N.*)

26. Define *uniform acceleration.* State, for each case, *one* set of conditions sufficient for a body to describe (*a*) a parabola, (*b*) a circle.

A projectile is fired from ground level, with velocity 500 m s^{-1} at 30° to the horizontal. Find its horizontal range, the greatest vertical height to which it rises, and the time to reach the greatest height. What is the least speed with which it could be projected in order to achieve the same horizontal range? (The resistance of the air to the motion of the projectile may be neglected.) (*O.*)

27. Define *momentum* and state the *law of conservation of linear momentum.*

Discuss the conservation of linear momentum in the following cases (*a*) a freely falling body strikes the ground without rebounding, (*b*) during free flight an explosive charge separates an earth satellite from its propulsion unit, (*c*) a billiard ball bounces off the perfectly elastic cushion of a billiard table.

A bullet of mass 10 g travelling horizontally with a velocity of 300 m s^{-1} strikes a block of wood of mass 290 g which rests on a rough horizontal floor. After impact the block and bullet move together and come to rest when the block has travelled a distance of 15 m. Calculate the coefficient of sliding friction between the block and the floor. (*O. & C.*)

28. Explain the distinction between *fundamental* and *derived* units, using two examples of each.

Derive the dimensions of (*a*) *the moment of a couple* and *work*, and comment on the results, (*b*) the constants *a* and *b* in van der Waals' equation $(p+a/v^2)(v-b) = rT$ for unit mass of a gas. (*N.*)

29. Explain what is meant by the relative velocity of one moving object with respect to another.

A ship A is moving eastward with a speed of 15 km h^{-1} and another ship B, at a given instant 10 km east of A, is moving southwards with a speed of 20 km h^{-1}. How long after this instant will the ships be nearest to each other, how far apart will they be then, and in what direction will B be sighted from A? (*C.*)

30. Define *momentum* and state the *law of conservation of linear momentum.*

Outline an experiment to demonstrate momentum conservation and discuss the accuracy which could be achieved.

Show that in a collision between two moving bodies in which no external act, the conservation of linear momentum may be deduced directly from Newton's laws of motion.

A small spherical body slides with velocity v and without rolling on a smooth horizontal table and collides with an identical sphere which is initially at rest on the table. After the collision the two spheres slide without rolling away from the point of impact, the velocity of the first sphere being in a direction at 30° to its previous velocity. Assuming that energy is conserved, and that there are no horizontal external forces acting, calculate the speed and direction of travel of the target sphere away from the point of impact. (*O. & C.*)

31. Answer the following questions making particular reference to the physical principles concerned (*a*) explained why the load on the back wheels of a motor car increases when the vehicle is accelerating, (*b*) the diagram, Fig. 1.23, shows a painter in a crate which hangs alongside a building. When the painter who weighs 100 kgf pulls on the rope the force he exerts on the floor of the crate is 45 kgf. If the crate weighs 25 kgf find the acceleration. (*N.*)

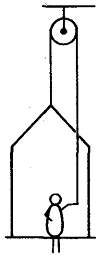

FIG. 1.23

32. Derive an expression for the kinetic energy of a moving body.

A vehicle of mass 2000 kg travelling at 10 m s^{-1} on a horizontal surface is brought to rest in a distance of 12·5 m by the action of its brakes. Calculate the average retarding force. What horse-power must the engine develop in order to take the vehicle up an incline of 1 in 10 at a constant speed of 10 m s^{-1} if the frictional resistance is equal to 20 kgf? (*L.*)

33. Explain what is meant by the principle of conservation of energy for a system of particles not acted upon by any external forces. What modifications are introduced when external forces are operative?

A bobsleigh is travelling at 10 m s^{-1} when it starts ascending an incline of 1 in 100. If it comes to rest after travelling 150 m up the slope, calculate the proportion of the energy lost in friction and deduce the coefficient of friction between the runners and the snow. (*O. & C.*)

34. State Newton's Laws of Motion and deduce from them the relation between the distance travelled and the time for the case of a body acted upon by a constant force. Explain the units in which the various quantities are measured.

A fire engine pumps water at such a rate that the velocity of the water leaving the nozzle is 15 m s^{-1}. If the jet be directed perpendicularly on to a wall and the rebound of the water be neglected, calculate the pressure on the wall (1 m^3 water weighs 1000 kg). (*O. & C.*)