Uganda Advanced Certificate of Education

Subsidiary Mathematics

TEACHING SYLLABUS

CONTENTS

Acknowledgements

The National Curriculum Development Centre (NCDC) would like to thank everyone who worked tirelessly towards the production of the Advanced Level (UACE) curriculum Teaching Syllabi.

Special thanks go to the Ministry of Education and Sports for supporting the work. NCDC would also like to thank the following for their professional input: Secondary Schools, Universities, National Teachers' Colleges, Uganda National Examinations Board (UNEB), Directorate of Educational Standards (DES), Secondary Science and Mathematics Teachers' Programme (SESEMAT), Technical and Business Institutions, Religious Organisations, Professional Organisations as well as retired and practising teachers.

Last but not least, NCDC would like to thank all those who worked behind the scenes to finalise the documents.

NCDC takes full responsibility for any errors and omissions in the documents and welcomes suggestions to address them.

Introduction

Subsidiary Mathematics has been part of the Advanced Level curriculum for a long time but the Teaching Syllabus has not been in place. The National Mathematics Panel for Subsidiary Mathematics has designed a Teaching Syllabus to ease the teaching and learning of the subject. The content of Subsidiary Mathematics has been reviewed. Some content in the previous syllabus has been removed while new content has been brought on board to address learning needs of the 21st Century student. The review has focused on:

i) the mathematics that is applicable in the world of work

ii) reducing content to make it suitable to all categories of learners

iii) equipping learners with mathematical skills that are desirable for further learning and in the world of work.

The current societal needs in Uganda have been a major factor during the identification of content to be added or removed from this subject. Only the relevant content has been retained.

The area of application of knowledge acquired in each topic is provided in the background.

Purpose of the Teaching Syllabus

The Subsidiary Mathematics Teaching Syllabus is meant to guide teachers handling the subject at Advanced Level. It is designed to achieve the aims of teaching Subsidiary Mathematics, standardise teaching of the subject across the country and guide the teacher during the teaching and learning process. It is also meant to guide on how the mathematical skills and competences in this subject can be developed among the learners across the subject topics.

Broad Aims of Education in Uganda

This syllabus contributes towards achieving the broad aims of education listed in the Government White Paper on Education of 1992 as follows:

1. To promote understanding and appreciation of the value of national unity, patriotism and cultural heritage, with due consideration of internal relations and beneficial inter-dependence.

2. To inculcate moral, ethical and spiritual values in the individual and to develop self-discipline, integrity, tolerance and human fellowship.
3. To inculcate a sense of service, duty and leadership for participation in civic, social and national affairs through group activities in educational institutions and the community.
4. To promote scientific, technical and cultural knowledge, skills and attitudes needed to promote development.
5. To eradicate illiteracy and to equip the individual with basic skills and knowledge to exploit the environment for self-development as well as national development, for better health, nutrition and family life and the capacity for continued learning.
6. To contribute to the building of an integrated, self-sustaining and independent national economy.

Aims and Objectives of Secondary Education in Uganda

The Government White Paper on Education provides the following aims and objectives of secondary education in Uganda;

1. Instilling and promoting national unity and an understanding of social and civic responsibilities; strong love and care for others and respect for public property as well as an appreciation of international relations and beneficial international co-operation.
2. Promoting an appreciation and understanding of the cultural heritage of Uganda including languages.
3. Imparting and promoting a sense of self-discipline, ethical and spiritual values, personal responsibility and initiative.
4. Enabling individuals to acquire and develop knowledge and an understanding of emerging needs of society and the economy.
5. Providing up-to-date and comprehensive knowledge in theoretical and practical aspects of innovative production, modern management methods in the field of commerce and industry and their application in the context of social-economic development of Uganda.
6. Enabling individuals to develop basic scientific, technological, agricultural, and commercial skills required for self-development.

7. Enabling individuals to develop personal skills of problem-solving, information-gathering and interpretation, independent reading and writing, self-improvement through learning and development of social, physical and leadership skills such as are obtained through games, sports, societies and clubs.
8. Laying the foundation for further education.
9. Enabling the individual to apply acquired skills in solving problems of the community.
10. Instilling positive attitudes towards productive work and strong respect for the dignity of labour and those who engage in productive labour activities.

Aims of Teaching Subsidiary Mathematics

Teaching Subsidiary Mathematics is aimed at:

a) enabling learners acquire a range of mathematical skills that are applicable in everyday situations and other subjects they may be studying.

b) equipping learners to use mathematics as a means of communication with emphasis on clear expression.

c) inspiring learners to develop an attitude of logical thought.

d) building on the basic mathematical concepts for better understanding of the subject by every learner at A level.

e) empowering learners to construct mathematical models by:

i) developing mathematics to the limits of their ability.

ii) applying mathematics with confidence to unfamiliar real situations.

iii) specialising in mathematical techniques required for further education or vocation.

iv) having a positive attitude towards mathematical problem-solving.

v) appreciating as far as possible the satisfaction and enjoyment that may be gained from pursuing the subject for its own sake.

vi) presenting and interpreting mathematical information in diagrammatic, tabular and graphic form.

Target

This Teaching Syllabus is intended for the Subsidiary Mathematics A Level secondary school teacher. It can also be used by the learner for guidance of personal learning and practice of mathematical concepts identified in this syllabus.

Scope and Depth

The Teaching Syllabus covers Pure Mathematics, Mechanics, Probability and Statistics.

Teaching Sequence

The topics have been arranged in chronological order of skills acquisition. You are advised to follow the teaching sequence for effective teaching and learning of the subject.

Time Allocation

Subsidiary Mathematics should be allocated 6 **periods**, each of **40** or **45 minutes** a week on the timetable. This is to allow enough time for learners to engage in a variety of learning activities and develop problem-solving skills. You are advised to expose learners to real world circumstances so that they are motivated to apply mathematical knowledge and reasoning in real life as much as possible. `Learning by doing' should be practised.

How to Use the Syllabus

This syllabus focuses on developing mathematical skills for day-to-day application and further learning. Teaching and learning of this subject should focus more on skill acquisition development and application if the learner is to benefit from it.

The suggested teaching-learning approaches in this syllabus are not an end in themselves. The teachers are encouraged to devise other teaching strategies to enable the learners develop competences described in this syllabus.

Syllabus Features

The teaching syllabus for Subsidiary Mathematics has the following features:

Duration

This gives the proposed number of periods for each topic, each period being of 40 or 45 minutes duration. This is to guide the teacher cover the syllabus adequately.

Learning Outcome

This is the statement that specifies what the learner will be able to do upon successful completion of the topic.

Competences

These define a specific range of skills, knowledge, or ability to be acquired by the learners. The teacher should use the competences to plan the teaching-learning strategies suitable for the lesson. Competences also guide in evaluating whether learning actually took place.

Learning/Teaching Strategies

These provide the teacher with guidance on the proposed activities and (methods/methodology) that can be used in the teaching. The following are the suggested learning/teaching strategies;

i) **Teacher guided research:** An instructional technique where the teacher gives students areas to read individually or in groups and later have a class discussion.

ii) **Peer presentation:** An instructional technique where the students share the knowledge they have in class with the teacher's guidance.

iii) **Brainstorming:** A technique used to gather ideas spontaneously contributed by learners. It is an effective way to generate ideas on a specific issue and then determine which idea(s) is the best.

iv) **Teacher exposition:** An instructional technique where you put the topic into context for the learner to elicit his/her contributions.

v) **Simulations:** A representation of the behaviour or characteristics of one system through the use of another system especially a computer program designed for the purpose.

Note: The teacher is not restricted to the suggested learning/teaching strategies.

Guidance to the Teacher

This is to guide the teacher prepare for lessons on a given topic. It spells out the areas of emphasis and tools to be used in a given topic.

Mode of Assessment

Continuous Assessment

This shall be carried out by the subject teacher within the provided teaching time to determine the student's progress. It can be done in a variety of forms such as written, oral or practical, real life or abstract; completed individually or as a group. A variety of approaches should be used so that a learner can draw inferences about learning based on information obtained through broad approaches like observing, questioning and testing.

The assessment should equally focus on all the sections. Continuous Assessment should help the learner to:

i) Apply relevant mathematical concepts, terminologies and notations;

ii) Recall accurately and successfully use appropriate manipulative techniques;

iii) Recognise the appropriate mathematical procedure for a given situation;

iv) Apply combinations of mathematical skills and techniques in solving problems.

v) Present mathematical work, and communicate conclusions, in a clear and logical way.

Summative Assessment

This shall be done at the end of the two years of Advanced Level education by Uganda National Examinations Board. The examination will be formatted as follows:

There will be one paper of **2 hours 40 minutes**. The paper will consist of **two** sections: Section A and Section B.

Section A will comprise short questions on Pure Mathematics, Statistics and Mechanics while Section B will comprise longer questions. Section A will consist of **eight (**8) compulsory questions. Candidates will be required to attempt all questions each carrying 5 marks. Section B will consist of **six** (6) questions of which candidates will be required to attempt any **four** (4) each carrying 15 marks.

In Section B, **Six** (6) questions will be set from Pure Mathematics, Mechanics and Probability and Statistics.

Assessment Strategy

Exercises in Class

These should be done in the teacher's presence and marked.

Assignments

Learners should be given homework to enable mastery of the content, should be marked and corrections made. Remedies should be given where necessary.

Short Test

This should be at the end of the sub-topic or topic, as the case may be.

Outline of the Teaching Syllabus

SENIOR FIVE	**TERM ONE**	
SECTION A	**SECTION B**	
1. The Scientific Tool Calculator	4. Descriptive Statistics - Introduction to	5. Motion in a Straight Line

2. Indices - Logarithms - Surds 3. Polynomials	Statistics - Organisation of Data - Measures of Central Tendency - Measures of Dispersion	
SENIOR FIVE	**TERM TWO**	
6. Quadratics 7. Differentiation - Derivative of a Function - Tangents and Normals - Second Derivative -	8. Moving Averages 9. Index Numbers 10. Correlation	11. Resultant and Components of Forces
SENIOR FIVE	**TERM THREE**	
12. Trigonometry - Compound Angle Formulae - Factor Formulae 13. Solution of Triangle Vectors	14. Probability Theory 15. Permutations and Combinations	16. Friction 17. Newton's Laws of Motion
SENIOR SIX	**TERM ONE**	
18. Integration - Definite and Indefinite Integrals - Area under a Curve 19. Displacement, Velocity and Acceleration 20. Series - Arithmetic Progression - Geometric Progression	21. Random Variables - Discrete Random Variables - Continuous Random Variables - 22. Binomial Distribution	23. Work, Energy, Power

SENIOR SIX	**TERM TWO**	
24. Matrices and their Applications	25. Normal Distribution	
SENIOR SIX	**TERM THREE**	
26. Differential Equations		

SENIOR FIVE TERM ONE

PURE MATHEMATICS

Topic 1: The Scientific Calculator

Duration: 4 Periods

Background

A calculator is a digital device widely used to perform mathematical calculation. However, learners do not always use it effectively. Sometimes they use it inappropriately. Throughout this topic, the learner is expected to learn how to use a scientific calculator efficiently.

Learning Outcome

The learner should be able to use a scientific calculator to carry out mathematics calculations more effectively.

Sub-Topic: Mathematical Operations

Competences	Content
The learner: • identifies keys for various mathematical operations on a calculator. • uses a calculator to evaluate algebraic expressions (3-5 digits) • uses a calculator to evaluate mathematics expressions with mixed operations (up to 8 digits)	• The four operations $+, -, \times, \div$ • • Other operations: Inx *or* $\log_e x$ $\sqrt{x}, \log x, \sqrt[x]{y}, x^y$, trigonometric functions • Mixed operations (using BODMAS)

Teaching/Learning Strategies

- Guide learners on rounding up figures to the required degree of accuracy.
- Through peer presentation, learners practice and guide one another to use a calculator.

Guidance to the Teacher

- Encourage learners to have personal non programmable scientific calculators. It is advisable for the ease of teaching that learners use the same type of calculator.
- Algebraic expressions with mixed operations should include expressions with parenthesis, fractions and other operations mentioned in the content column.
- As a school, you are advised, as much as possible, to use the same type of calculator to ease the teaching and learning process.
- Ensure learners can give answers to the required degree of accuracy.
- Introduce the use of the word “exponent” and its symbol (^). The symbol should not be used in their answers.
- Review the concept of standard form and significant figures learnt at O Level.

Topic 2: Indices, Logarithms and Surds

Duration: 14 Periods

Background

Indices, logarithms and powers mean the same thing and can be used interchangeably. Quite often we encounter very big and very small numbers e.g. the distance from the earth to other planets and the size of a molecule or atom. Indices and logarithms help us to deal with such numbers more conveniently. Surds are irrational numbers which can be expressed as powers.

Knowledge of logarithms is:

i) essential for further learning in other related subjects.

ii) used to make predictions of, e.g. population growth, rate at which a disease spreads.

iii) commonly used to multiply and divide large numbers.

Learning Outcome

The learner should be able to simplify and evaluate expressions involving logarithms, indices and surds.

Sub-Topic 1: Indices

Competences	Content
The learner: • relates powers to indices. • applies the laws of indices. • simplifies expressions using laws of indices. • solves equations that require use of laws of indices.	 • Powers as indices • Laws of indices • Simplifying expressions • equations that require use of laws of indices

Learning/Teaching Strategies

- Through multiplying the number by itself, guide the learners to use powers.

- Use the division approach to introduce negative powers.
- In groups or individually use the laws of indices to simplify expressions and solve equations.

Sub-Topic 2: Logarithms

Competences	Content
The learner: • relates logarithms to indices. • applies laws of logarithms to numbers • changes from one base to another. • simplifies logarithmic expressions in different bases. • solves equations involving bases and logarithms	 • Logarithm as an index • Laws of logarithms • Change of base • Simplification of logarithmic expressions • Simple equations involving indices and logarithms

Teaching/Learning Strategies

- Through exposition, guide the learners to state the laws of logarithms.
- Provide worksheets for learners to practice application of the laws of logarithms, simplify and evaluate expressions.

Sub-Topic 3: Surds

Competences	Content
The learner: • identifies rational and irrational numbers. • identifies and simplifies numbers in surd form. • rationalises surds. • solves simple linear equations with surds. • performs operations (adds, subtracts multiplies).	 • Rational and irrational numbers • Simplification of surds • Rationalisation of surds • Simple equations involving surds

Teaching/Learning Strategies

- Through teacher exposition, define rational numbers with examples and explain the significance of rationalisation.
- Guide learners to rationalise the denominator.
- Provide worksheets/activities for learners to practise and develop skills in rationalising the denominator.

Guidance to the Teacher

- Students should be taken through this topic slowly because the content is abstract. Guide the learners to identify where this knowledge can be applied.
- Where applicable, let the learners understand how the laws of indices and logarithms are applied.
- Some learners could be well versed with this topic. Group the learners and distribute fast learners to tutor the rest during group activities. Emphasise on group presentations so that the weak learners get more time to internalise the content and the approaches used.
- The required level of accuracy should not exceed four significant figures unless otherwise stated in a particular question or situation.

Topic 3: Polynomials

Duration: 6 Periods

Background

Polynomials are algebraic expressions that include real numbers and variables. They contain more than one term. Polynomials are the sums of monomials.

A monomial has one term for example $5y$ or $-8x^2$ or 3 are monomials.

A sum of two monomials which are not like terms for example; $3x^2 + 8$, or $9y + y^2$, is a special polynomial called a binomial. Similarly, a sum of three monomials is a trinomial for example; $3x^2 + 8x + 5$.

The degree of the polynomial is the highest exponent of the variable for example; $3x^2$ has a degree of 2, $3x^5 + 5x$ has a degree 5. When the variable does not have an exponent, always understand that there is a '1' e.g., $3x$ *is the same as* $3x^1$.

A polynomial of degree n in a variable x is a sum of any number of monomials and has the following form $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ where the coefficients a_k, $[k = n-1,\ n-2, \ldots, 0]$, are constants. a_k may be zero, positive or negative, a_n **not** zero.

Learning Outcome

The learner should be able to simplify and evaluate expressions involving polynomials.

Sub-Topic: Polynomials

Competences	Content
The learner: • forms polynomials. • identifies the order of a polynomial.	• Formation of polynomial • Order of a polynomial

Competences	Content
• performs operations on polynomials.	• Operations on polynomials (addition, subtraction, multiplication and Division)
• evaluates polynomials at a given point by substitution.	• evaluation of a polynomial
• factorises polynomials.	• Factorisation of polynomials up to degree 2
• solves simple polynomials.	• Solving a polynomial

Teaching/ Learning Strategies

- Guide the learners to identify monomials and binomials.
- Allow learners to form polynomials using knowledge from the teacher exposition.

Guide learners through multiplication and division of polynomials.

Guidance to the Teacher

- Polynomials can be formed by using sides of a regular object say; a rectangle, whose sides are given in form of monomials. Allow learners to find the perimeter as a way of forming polynomials from monomials. The same methodology can be used to introduce subtraction multiplication and division of polynomials if the sides of a regular object are given as polynomials.
- A Polynomial $p(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ can be expressed as a function

  $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$, [a_n **not** zero]

- You should emphasise to learners that $f(x)$ is **not** 'f multiplied by x' **but** means the 'value of the expression' for example: $f(a)$ means 'the value of the expression when $x = a$.

PROBABILITY AND STATISTICS

Topic 4: Descriptive Statistics

Duration: 20 Periods

Background

Statistics is a set of concepts, rules, and procedures that help us to:

- **organise** numerical information in the form of tables, graphs, and charts;
- **understand** statistical techniques underlying decisions that affect our lives and well-being; and
- **make** informed decisions basing on information generated from processed data.

Statistics plays a vital role in every field of human activity. For example, it has an important role in determining the existing position of per capita income, unemployment, population growth rate, housing, schooling, medical facilities, etc, in a country. Statistics is a vital tool in fields like industry, commerce, trade, Physics, Chemistry, Geography, Economics, Mathematics, Biology, Botany, Psychology, and Astronomy. Statistical methods are used in research to collect, analyse, and formulate research findings in every field at higher institutions of learning.

Learning Outcome

The learner should be able to collect, present and analyse or interpret data using measures of central tendency and measures of dispersion.

Sub-Topic 1: Introduction to Statistics

Competences	Content
The learner: • identifies discrete and continuous data with examples. • identifies ungrouped and grouped data. • groups data.	• Discrete and continuous raw data • Types of data: - ungrouped data - grouped data

Teaching/ Learning Strategies

- Guide learners to collect data for example they can take measurements of their height, waist, fingers, shoe size or any other variable.
- Guide learners to group their data.

Sub-Topic 2: Organisation of Data

Competences	Content
The learner: • identifies data presentation methods. • selects a suitable way of presenting raw statistical data. • presents data using any of the methods including a frequency distribution table for ungrouped and grouped data. • interprets statistical diagrams.	• Data presentation methods: - Frequency tables - Histograms • Cumulative frequency graph (Ogive) • Interpretation of statistical diagrams (Frequency tables, Histograms and Ogives)

Teaching/ Learning Strategies

- Guide learners on different forms of data presentation.
- In groups or individually, guide learners to make presentations of their data making use of various charts and diagrams.

Sub-Topic 3: Measures of Central Tendency

Competences	Content
The learner: • calculates the mean , mode and median of grouped data. • estimates the mode using the histogram.	• Mean, mode and median

• estimates the median using the Ogive. • uses different measures of central tendency (mean, mode, median) to analyse data.	

Teaching/ Learning Strategies

- Using a number of examples from practical examples, guide the learners to calculate measures of central tendency.
- Guide learners to compare and contrast sets of data and make meaningful conclusions on the data collected at the beginning of the topic.
- Guide learners on use of a calculator for statistical computations.

Sub-Topic 4: Measures of Variation (Dispersion)

Competences	Content
The learner: • determines the range. • uses the Ogive to estimate the quartiles, percentiles and interquartile range. • calculates the variance and standard deviation. • analyses data using variance and standard deviation.	• Range • Quartiles/percentiles • Variance and standard deviation • Data analysis

Teaching/ Learning Strategies

- Guide learners to determine measures of dispersion.
- Guide learners to use measures of dispersion to make conclusions on sets of data.
- Guide learners on use of a calculator for statistical computations

Guidance to the Teacher

- At this level, most learners would have had some reasonable background knowledge on this topic. The methodologies used should aim at clarifying concepts already learned.
- You are advised to teach this topic as a project covering areas of data collection, presentation and analysis.
- Data presentation and analysis should look at constructing histograms (with equal class widths only).
- You should use the data familiar to the learner's environment. This way you will help learners create interest in the topic/subject.
- You should also focus on the ability to interpret graphs and charts and the importance of central tendency values.
- Train the learner to use different measures of variation (dispersion) in comparing and contrasting sets of data.

- A histogram is a special bar graph with the heights of the bars representing the frequencies of the groups and NO gaps between the bars.
- While dealing with practical activities, guide learners to use suitable scales and label the axes properly. Shading of the histogram is not important but if used it should be uniform.
- If computers are available, guide learners to use statistical packages to perform calculations and generate graphs and charts.

MECHANICS

Topic 5: Motion in a Straight Line

Duration: 6 Periods

Background

Mechanics is a branch of the physical sciences concerned with the state of rest or motion of bodies that are subjected to the action of forces. A body is said to be in motion if its position changes with time. The position of the object can be specified with respect to a conveniently chosen origin. A number of terms are used to describe motion in a straight line. These include, speed, velocity, displacement, distance, instantaneous rest and time taken.

In most cases, speed is referred to even if it is velocity. Speed should be differentiated from velocity and distance differentiated from displacement.

Knowledge acquired in this topic is required to plan journeys especially when integrated with map reading.

Learning Outcome

The learner should be able to plan journeys and determine missing variables for any journey using equations of motion.

Sub-Topic: Distance, Velocity and Acceleration

Competences	Content
The learner: • differentiates: - distance from displacement. - speed from velocity. • sketches and interprets displacement-time graphs. • sketches and interprets velocity-time graphs • applies equations of linear motion	• Distance, displacement, velocity, speed, average speed, acceleration • Displacement-time graphs • Velocity-time graphs

Competences	Content
to solve mathematical problems.	• Equations of linear motion • Numerical problems

Teaching/ Learning Strategies

- Brainstorm with learners the difference between speed and velocity; displacement and distance.
- Guide learners to present linear motions graphically.
- Provide a variety of graphs for learners to interpret and draw inferences.
- Using appropriate graphs, guide learners to derive equations of motion.
- Relate the gradient of a distance-time graph to speed and the gradient of the velocity time graph to acceleration.
- Provide learners worksheets to practice solving numerical problems.

Guidance to the Teacher

- Help the learners to:
  i) identify distance and speed as scalar quantities; displacement, velocity and acceleration as vector quantities.
  ii) sketch and interpret displacement-time graphs and velocity-time graphs, and in particular appreciate that:
    - the area under a velocity-time graph represents displacement.
    - the gradient of a displacement-time graph represents velocity.
    - the gradient of a velocity-time graph represents acceleration.
- Derivation of equations of motion is not necessary. The learner needs to simply quote and use them.

SENIOR FIVE TERM TWO

PURE MATHEMATICS

Topic 6: Differentiation

Duration: 12 Periods

Background

In mathematics, differential calculus is a subfield of calculus concerned with the study of the rates at which quantities change. It is one of the two traditional divisions of calculus, the other being integral calculus.

The primary objective of studying in differential calculus is the derivative of a function, related notions such as the differential, and their applications. The derivative of a function at a chosen input value describes the rate of change of the function near that input value. The process of finding a derivative is called differentiation. Geometrically, the derivative at a point equals the slope of the tangent line to the graph of the function at that point. For a real-valued function of a single real variable, the derivative of a function at a point generally determines the best linear approximation to the function at that point.

Differential calculus and integral calculus are connected by the fundamental theorem of calculus, which states that differentiation is the reverse process to integration.

Knowledge of differentiation is applicable in analysis of finance and economics. One important application of differentiation is in the area of optimisation, which means finding the condition for a maximum (or minimum) to occur. This is important in business (cost reduction, profit increase) and engineering (maximum strength, minimum cost).

Determination of the **maximum** and **minimum** points is a valuable aid in sketching a curve. **Maximum** and **minimum** points are also known as **turning points** of a curve. The first derivative is used to identify these turning points of the curve. Finding maximum and minimum can be applied to maximise revenue and profit and to minimise average cost when the appropriate functions are known. Maximising also helps us in optimisation

say in production and consumption, to test for sufficiency in production and consumption, monopoly markets and the like.

We shall also apply differentiation technique to find the relationship between displacement, velocity and acceleration.

Learning Outcome

The learner should be able to obtain derivatives of simple polynomials including finding the gradient of a line from the equation $y = mx + c$.

Sub-Topic 1: Derivative of a Function

Competences	Content
The learner: • differentiates polynomials. • determines the gradient of a line. • determines the gradient of a curve/function at a given point.	• Derivatives of polynomials • Gradient of a line. • Gradient of a curve

Teaching/ Learning Strategies

- Through teacher exposition, guide learners to use the notations $\frac{dy}{dx}$ and $f'(x)$.
- Take learners through the process of determining derivatives of polynomials.
- Allow learners time to practice how to determine derivatives of a function and slope of a curve at a point and also conduct peer presentation.

Sub-Topic 2: Second Derivative

Competences	Content
• The learner finds the second derivative using the notations $f''(x)$ or $\frac{d^2y}{dx^2}$.	• Second derivative

Teaching/ Learning Strategies

- Through exposition the teacher relates the first derivative to the second derivative.

Sub-Topic 3: Curve Sketching

Competences	Content
The learner: • uses intuition to sketch quadratic curves. • finds the turning points. • uses in table form the sign of the first derivative to determine the nature of the turning points. • uses the second derivative to distinguish between the maximum and minimum points. • sketches the curves. • uses differentiation to find solutions of word problems.	• Maximum and minimum points. (Do not include points of Inflexion) • Curve sketching

Teaching/ Learning Strategies

- Make a review of the shapes of quadratic curves through by sketching several curves.
- Prepare work on functions to be differentiated by the learners and find the roots for which the first derivative is zero.
- Through exposition, help the learners on how to use the signs of the gradients of the tangents.
- Through guided discovery, let the learners find out the nature of the turning points using the sign change method. (Second derivative)
- Through guided discovery, help the learners to sketch the curves.
- Through question and answer, let the learners use differentiation to solve word problems.

Guidance to the Teacher

- Use other variables like change in profit with change in number of employees to allow learners develop their own understanding of a derivative of a function, differentiation and rate of change.
- For a function $y = ax^n$, the formula $\frac{dy}{dx} = nax^{n-1}$ can be used to guide learners obtain derivatives of simple algebraic functions e.g. x^2, $3x^2$, $4x^3 \pm 2x^2$
- The variables x and y may be replaced by any other relevant variables.
- Differentiation of polynomials from first principles is not necessary.
- Use of product and quotient rule is not expected at this point.
- The second derivative should be mentioned. DO NOT mention distinguishing of the nature of the turning points at this point, but strictly lead the learners to only find the second derivative of simple functions. For example: Find the second derivative of:

  i) $y = 3x^3 + 4x^2 + 5$ ii) $y = 3x^4 + 8x^3 - 4x^2 + 5x + 7$

  ii) Emphasise on the difference between **sketching** and **plotting** a curve.

  iii) Remind the students about the general shape of the graph in the form $y = ax^2 + bx + c$ for $a \langle 0$ land $a \rangle 0$.

  iv) Expose the learner to real life situations.

Topic 7: Quadratics

Duration: 8 Periods

Background

Quadratic expressions are polynomials in which the highest power/degree of the variable is 2. A general quadratic expression in x is in the form $ax^2 + bx + c$ and the corresponding general quadratic equation takes on the form $ax^2 + bx + c = 0$ where a, b, c are constants and a is a non-zero integer. The solutions (values of x) for $ax^2 + bx + c = 0$ are referred to as **roots** of the equation. Applications of quadratics include, determining the trajectory of a projectile and expressing Production functions, Cost functions and Utility functions.

Learning Outcome

The learner should be able to apply knowledge of quadratic expressions.

Sub-Topic: Quadratic Equations

Competences	Content
The learner: • solves quadratic equations using factorisation and completing squares. • identifies the roots of a given quadratic equation. • forms quadratic equations using given roots. • solves the equations involving indices. • solves simple equations reducing to quadratics. • interprets roots • identifies/finds minimum/maximum values by	• Methods of solving quadratic equations • Sum and product of roots of a quadratic equation • Formation of quadratic equations • Equations reducing to quadratics • Maximum and minimum

Competences	Content
completing squares	values (of the quadratic function)

Teaching/ Learning Strategies

- Guide the learners to discover the different methods of solving quadratic equations i.e. graphical method, factorisation, use of formulae, completing squares.
- Guide the learners to use the discriminant to determine whether an equation has real roots.

- Use the method of completing the square to determine the maximum and minimum values of a quadratic expression.
- With students ensure students can sketch identify graphs of quadratic function.

Guidance to the Teacher

- Formation of equations that require use of symmetrical functions is beyond the scope of this syllabus.
- Remind the learners of the use of the identities at O Level where applicable.

$$x^2 - y^2 = (x + y)(x - y)$$
$$(x \pm y)^2 = (x^2 \pm 2xy + y^2)$$
$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

- Consider situations where not all the two roots of a quadratic equation may be applicable.

STATISTICS

Topic 8: Moving Averages

Duration: 8 Periods

Background

Moving averages is one of the methods used in business and other areas of life for forecasting numerical data. Forecasting gives a business the opportunity to plan for the future as well as planning for several changes. It is important to have an idea about the **trend**, that is, the underlying movement of the data in consideration. For example, there could be particular points during a year when sales are lower than previous time periods. The time period could be seconds, minutes, days, weeks, months, years, decades.

The process of moving averages means the averages are taken from the data.

Learning Outcome

The learner should be able to use data to forecast the trend.

Sub-Topic: Moving Averages

Competences	Content
The learner: • identifies the trend. • calculates odd or even moving totals. • uses these totals to calculate odd or even moving averages. • plots the moving averages. • uses the graph to forecasts the trend.	• The cycle of the data • Even and odd moving totals • Odd and even moving averages • Graphs of moving averages

Teaching/ Learning Strategies

- Guide the learners in identifying the cycle if not stated.

- Learners practise to calculate moving totals and the corresponding moving averages depending on the cycle.
- Guide learners to plot graphs of moving averages.

Guidance to the Teacher

- Moving averages requires a student to have knowledge about **extrapolation** which is vital for forecasting.
- Calculations are better done when in tabular form.

Topic 9: Index Numbers

Duration: 6 Periods

Background

Index numbers are statistical economic indicators which provide a measure of the relative change in some variable or group of variables at a specified date when completed with some fixed period in the past.

Index numbers are also widely used by business operators, to evaluate their trading positions in relation to competitors and rely on the national indices for wages, production, prices, sells, transport charges and share prices to provide simple background information against which objective decisions may be taken. For instance, they may be used to compare the present agricultural production or industrial production, price fluctuations of commodities, with those of the past years.

The simplest example of an index number is a **price relative** or **price index**. A **base year** is always chosen and this is the year on which the price changes are based.

For a price index to be realistic it should take into account the relative importance of the commodities. The method of **weighting** is used to cater for this.

Learning Outcome

The learner should be able to calculate simple and weighted price index numbers and use them to compare relative changes in a particular situation.

Sub-Topic: Index Numbers

Competences	Content
The learner: • calculates the price index. • calculates the: - simple price index number.	• Concept of price indices (price relatives) • Un-weighted price indices • Weighted price index:

Competences	Content
- simple aggregate price index. • calculates the: - weighted price index. - weighted aggregate price index. • determines the value index.	- Weighted aggregate price index. - Value index

Teaching/ Learning Strategies

- Through exposition, define the basic terminologies used e.g. Base year, Current year.
- Engages the learners in tasks to use the formula to calculate the simple price index number and simple aggregate price index.
- Provides activities to the learners to practice and develop skills to calculate the weighted price index and weighted aggregate price index.
- Through explanation, guides the learner to draw conclusions using the calculated index numbers.

Guidance to the Teacher

- Emphasise that when we are given the number, such as 120, referring to a price index, the learner must remember that the % sign is implied, that is to say the learner must use 120% or 1.2 in the calculation.
- You are advised to use Business/ Economics textbooks.

Topic 10: Correlation

Duration: 10 Periods

Background

Scatter diagram is a tool for analysing relationships between two variables. One variable is plotted on the horizontal axis and the other is plotted on the vertical axis. The pattern of their intersecting points can graphically show relationship patterns. Scatter diagrams are used in research to investigate relationships between two variables such as cause-and-effect relationships.

Learning Outcome

The learner will be able to interpret scatter diagrams for bivariate data and use it to draw suitable conclusions.

Sub-Topic: Scatter Diagrams

Competences	Content
The learner: • determines the nature of correlation between variables. • draws scatter diagrams and line of best fit. • determines coefficient of correlations. • draws conclusion using the coefficient of correlation.	• Concept of correlation • Scatter diagrams • Rank correlation coefficient • Applications of coefficient of correlation

Teaching/ Learning Strategies

- Prepare a project for learners to investigate the correlation between two variables.
- Guide learners to plot a scatter diagram, draw line of best fit, and use it to make conclusions.
- Guide the learner to calculate the correlation coefficient using Spearman's coefficient of rank correlation (r_s).

Guidance to the Teacher

You are advised to prepare a worksheet to guide learners to investigate the relationship between two variables.

MECHANICS

Topic 11: Resultants and Components of Forces

Duration: 12 Periods

Background

A force vector can be expressed in two dimension on the $(x,\ y)$ plane, for example, imagine the surface of a table top to be an $(x,\ y)$ plane. Objects can be pushed across this table surface in several different directions, not just parallel to the length or width of the table. Objects can be pushed across a table top at a slanted direction relative to the edges of the table top.

A force may be thought of as having a part that pushes right or left, and another part that pushes up or down.

This is used in many applications that require use of forces such as navigation, constructions and lifting/moving objects in daily life.

Learning Outcome

The learner should be able to resolve and determine resultants of a system of forces in a plane.

Sub-Topic 1: Resultant of Forces

Competences	Content
• The learner determines the resultant of parallel and non parallel forces.	• Resultant of: - parallel forces - non parallel forces - any number of forces

Teaching/ Learning Strategies

- Start with practical examples that have parallel forces.

Sub-Topic 2: Components of a Force

Competences	Content
The learner: • resolves a force in two perpendicular directions. • applies resolution of forces to find the resultant force.	• Components of a force • Resultant of a number of forces.

Teaching/ Learning Strategies

- Guide learners to resolve forces in two perpendicular directions.
- Let the learners use trigonometrical ratios of sine and cosine.

Guidance to the Teacher

- Use of vector notation $\mathbf{i}, \mathbf{j}, \mathbf{k}$ is beyond the scope of this topic.
- Forces that require resolving along an inclined surface are not expected. All resolutions should stick to components in two perpendicular directions. Use of arrows to show the directions of components is important.
- The number of forces acting at a point should not exceed five (5).

SENIOR FIVE TERM THREE

PURE MATHEMATICS

Topic 12: Trigonometry

Duration: 18 Periods

Background

Trigonometry is a branch of mathematics that studies the relationship between the three sides and the three angles of a right angled triangle in terms of ratios and representing them as trigonometrical ratios; sine, cosine and tangent. It was developed for astronomy and geography, but scientists have been using it for centuries for other purposes, too.

Knowledge of trigonometry is applicable in fields like land surveys, engineering and navigation.

Learning Outcome:

The learner will be able to use trigonometry to solve problems involving circular functions and triangular shapes.

Sub-Topic: Trigonometrical Ratios

Competences	Content
The learner: • writes down the six trigonometrical ratios using a right angled triangle.	• Expressions of the six trigonometrical ratios
• obtains the trigonometrical ratios for the special angles.	• Special angles of 0^o, 30^o, 45^o, 60^o and 90^o.
• obtains the trigonometrical ratios for any angle (positive and negative)	• Trigonometrical ratios of $90^o \pm \theta$, $180^o \pm \theta$, $-\theta$

Competences	Content
• deduces the sine, cosine, tangent of an angle of any magnitude using the quadrants of a unit circle. • sketches trigonometrical graphs. • uses Pythagoras theorem to derive trigonometrical identities. • evaluates and simplifies trigonometric expressions.	• Graphs of $\sin\theta$ and $\cos\theta$. • The ratios of sine, cosine, tangent and their reciprocals, cosecant, secant, cotangent, respectively). • Trigonometric identities: $\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $\cot^2\theta + 1 = \cos ec^2\theta$ • Trigonometric expressions • Equations

Teaching/ Learning Strategies

- Guide the learners to form trigonometrical ratios from right angled triangles.
- Surd form of the trigonometrical ratios for special angles should be derived and simplified using appropriate triangles.
- Sketching graphs of the trigonometric functions ($\sin\theta, \cos\theta$) should be a learner-centred activity.
- Guide the learners on how to obtain all the possible angles in the given range.
- Guide the learners to evaluate and simplify trigonometric expressions without using tables or calculators and recognising max/min of simple expressions.
- Group work activity should be encouraged when deriving trigonometrical identities to emphasise learner participation.

Guidance to the Teacher

- Use of parenthesis to write double angle and compound angles should be emphasised.

Common mistakes

- When solving trigonometrical equations, learners cancel trigonometrical ratios instead of factorizing them. This leads to loss of some values of θ required in the given range, e.g.

  $\cos\theta + 2\sin\theta\cos\theta = 0$. In this case learners cancel out $\cos\theta$ instead of factoring it out.
- Guide the learners on how to obtain all the possible angles in the given range.
- Trigonometric functions $\sin x$, $\cos x$ and $\tan x$ are usually referred to as circular functions. Knowledge about their domains and range: their periodic nature is recommended.
- Guiding students to get solutions graphically may be essential.
- On examination papers, radian measure should be assumed unless otherwise indicated. For example $f(x) = \sin x^o$.
- Given $\sin\theta$, ensure learners can find possible values of $\cos\theta$ and $\tan\theta$ without finding θ.
- Lines may be expressed as $y = x\tan\theta + c$: $\tan\theta$ is the gradient.
- Most learners interpret the cosine rule wrongly, i.e., $a^2 = b^2 + c^2 - 2bc\cos A$, when computing, students often write it as $a^2 = (b^2 + c^2 - 2bc)\cos A$, so you are encouraged to emphasise $a^2 = b^2 + c^2 - (2bc\cos A)$

Topic 13: Vectors

Duration: 10 Periods

Background

A vector may be described as a quantity associated with a particular direction in space. Any vector may be represented by a directed line segment, whose direction is that of the vector and whose length represents the magnitude of the vector. Displacement is one of the simplest examples of a vector. Vectors can be added together (vector addition), subtracted (vector subtraction) and multiplied by scalars (scalar multiplication). A vector from a point A to a point B is denoted $\vec{AB}$. If coordinates are involved, we use x, y in 2 – dimensions, the unit vectors in the respective directions are $\mathbf{i}$, $\mathbf{j}$. Any other quantity with only magnitude is known as **a scalar** quantity.

Vectors are applied in various fields of science, mechanics that involve magnitude (size) and direction, for example, displacement, velocity, force, acceleration, have a size or magnitude, but also they have associated with them the idea of a direction. Vectors are also employed in navigation by people who operate ships and aircrafts.

Learning Outcome

The learner should be able to carry out mathematical operations involving vectors in 2 – dimensions; compute the angle between two given vectors and interpret appropriate quantities.

Sub-Topic: Vectors in 2 Dimensions

Competences	Content
The learner: • expresses a vector in a column form $\begin{pmatrix} x \\ y \end{pmatrix}$. • writes a column vector in the form $a\mathbf{i} + b\mathbf{j}$.	• Vector notations

Competences	Content
• determines the displacement vector.	• Displacement vector
• identifies a position vector.	• Position vector
• adds and subtracts vectors.	• Addition and subtraction of vectors
• multiplies a vector by a scalar.	• Multiplication by a scalar
• calculates the magnitude of a vector.	• Magnitude and direction of a vector
• determines a unit vector in a given direction.	• Unit vector
• identifies parallel vectors.	• Parallel vectors
• identifies equal vectors.	• Equal vectors
• finds the dot products of two vectors.	• Dot/scalar product of two vectors
• finds the angle between two vectors.	• Angle between vectors
• identifies perpendicular vectors.	• Perpendicular vectors
• applies vector algebra to simple navigation problems.	

Teaching/Learning Strategies

- In groups, guide the learner to apply the vector notations when representing given problems.
- Prepare work on determining the displacement vector and position vector.
- Engage learners in tasks in vector addition, subtraction and scalar multiplication.
- Involves the learners with exercises to find the magnitude of a vector using the formula $|\mathbf{P}| = \sqrt{x^2 + y^2}$.
- Through exposition, explain how to calculate the angle between two vectors by use of the dot product.

Guidance to the Teacher

- Help the learners to identify scalar quantities and vector quantities.
- You are advised to concentrate more on vectors in two dimensions particularly displacement vectors.
- You should make the students aware that the sum of two or more vectors is the "resultant" vector. Direction of a vector should be restricted to 2 – dimensional vectors **only**.

PROBABILITY AND STATISTICS

Topic 14: Probability Theory

Duration: 10 Periods

Background

Probability theory is the branch of mathematics concerned with prediction and uncertainty. It was developed from the theory of games of chance and gambling. It plays a very important role in astronomy, physics, chemistry, engineering, economics, business, social science, psychology and research.

The probability of an event is the measure of the likelihood that it will occur and it is given on a numerical scale from 0 to 1.The numbers representing probabilities can be written as percentages, fractions or decimals.

A probability of **zero** implies that the event is **impossible**.

A probability of **one** (100%) indicates that the event is **certain to occur**.

All other events have a probability between zero and one.

A probability event consists of one or more outcomes of a probability experiment. Two events: A and B are said to be **mutually exclusive** if they cannot occur at the same time. Events can also be classified as **independent** or **dependent**. Independent events are events such that the occurrence of one does not affect the occurrence of the other.

The **complement** of an event is the set of outcomes of the event in the sample space that are not included in the outcomes of the event itself.

The **conditional probability** of an event B in relation to an event A is the probability that event B occurs after event A has already occurred.

Probability problems can be worked out by using the addition rules, the multiplication rules and the complementary event rules.

Learning Outcome

The learner should be able to calculate probabilities from give/researched data and draw out relevant conclusions, as well as solve problems involving probability.

Sub-Topic: Probability Theory

Competences	Content
The learner: • lists down the possible outcomes of an event in the experiment. • finds probability of an event using classical probability formula. • uses Venn diagrams to solve probability problems. • determines the number of outcomes to a sequence of events using tree diagrams. • finds the probabilities involving independent events using the multiplication rule. • computes the probabilities involving mutually exclusive events using the additive rule. • calculates numerical problems related to conditional probability.	• Experimental probability • Terminologies in probability theory • Probability laws and notations in relation to set theory • Contingency table • Probability tree diagrams • Mutually exclusive and independent events • Probability situations i.e. AND and OR • The conditional probability

Teaching/Learning Strategies

- Expose the learners to the knowledge about the probability theory through group work and experiments on a die and a coin.
- Brainstorm on the use of the terminologies and notations in probability theory.
- Guide the learners to state and apply the laws of probability to related problems.

- Use the contingency table to workout probabilities.
- Use Venn diagrams to illustrate events.
- In groups or individuals, let learners show how probability tree diagrams are constructed from relevant data and their application.

Guidance to the Teacher

- Encourage learners to make use of Venn diagrams when explaining the concept of probability.
- Use of O level knowledge and textbooks is recommended.
- Emphasise the application of the probability laws i.e. (i) $0 \leq P(A) \leq 1$
- (ii) $P(A) + P(\overline{A}) = 1$ to related problems.
- Picking with OR without replacement should be restricted to only 2 pickings.

Topic 15: Permutations and Combinations

Duration: 6 Periods

Background

Many problems in probability and statistics require a careful analysis of the outcomes of the events. A sequence of events occurs when one or more events follow one another. Many times one wishes to list the sequence of events, and can use several rules of counting which may include the permutation rules and the combination rules.

We always try to arrange given objects in our homes, shops, schools and elsewhere in different ways for some reasons such as convenience and neatness. A **permutation** is an arrangement of distinct objects in a specific order. For example, suppose a photographer must arrange three girls: Ann (A), Bena (B) and Halima (H) in a row for a photograph. He can do this in six possible ways: $\{A, B, H\}, \{A, H, B\}, \{B, A, H\}, \{B, H, A\}, \{H, A, B\}$ and $\{H, B, A\}$.

A **combination** is the number of ways of selecting a group of objects from a given set of objects, e.g. an A level subject combination such as HEG, PCB, etc. In a combination, the order of selection is not important, that is GEH, GHE, HGE, EGH, EHG are all the same as HEG. The difference between a permutation and a combination is that in a combination, the order or arrangement of the objects is not important.

Learning Outcome

The learner should be able to determine the number of permutations or combinations of a given set of objects**.**

Sub-Topic 1: Permutations

Competences	Content
The learner: • arranges items in a row. • identifies a permutation.	• Arrangement of objects in a row • Concept of permutation

Competences	Content
• relates the number of permutations to the factorial notation. • uses the permutation formula of to find the number of ways that *r* can be selected from *n* objects.	• Factorial notation $n!$ • Permutation notation ${}^{n}P_{r}$ and the formula ${}^{n}P_{r} = \frac{n!}{(n-r)!}$

Teaching/Learning Strategies

- Through exposition, guide the learners to arrange objects in a row.
- Explain the concept of permutation and how to use the formula ${}^{n}P_{r} = \frac{n!}{(n-r)!}$.
- In groups, guide the learners to arrange different objects in a row, then in a circle.

Sub-Topic 2: Combinations

Competences	Content
The learner: • identifies the characteristics of a combination. • applies the combination notation and formula to solve related problems.	• Ways of selecting objects • Identify a combination • Combination notation $\binom{n}{r}$ *or* ${}^{n}C_{r}$ and the formula ${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$

Teaching/Learning Strategies

- Discuss with the learners their subject combinations.
- Through exposition, explain the combination notation

- Guide the learners with examples on how to use the formula ${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$ to compute different combinations.

Guidance to the Teacher

- Encourage to use the objects around the classroom like books, chairs, students, boxes and playing cards to illustrate different arrangements and selections.
- For systematic grouping of objects, you may use O level subjects to pick the subject combinations with or without restriction.
- Only simple cases of permutations and combinations should be considered for example formation of a committee from a group of members of the same or different sex.

MECHANICS

Topic 16: Friction

Duration: 6 Periods

Background

Friction is the force which opposes relative motion between two bodies in contact. The magnitude of the frictional force is just sufficient to prevent relative motion.

Suppose a horizontal force says P is applied to a body on a rough horizontal table, the body does not necessarily move. There is a frictional force say F that opposes the applied force P. This frictional force F for a particular surface is not constant. It increases as the applied force **P** increases until the force F reaches a maximum force $\mathbf{F_{max}}$ beyond which it cannot increase.

When the body is just about to move, it is then said to be in a **state of limiting equilibrium**. At this point $F_{\max} = \mu R$ where μ is the coefficient of friction and R is the normal reaction. When the body starts to move, the frictional force F takes its limiting value μR and acts in the direction opposite to that of relative motion.

Learning Outcome

The learner should be able to draw and use simple diagrams to find the magnitude of the frictional force acting on any body either at rest or in motion.

Sub-Topic: Friction

Competences	Content
The learner: • lists the forces acting on a body placed on a rough horizontal surface. • relates limiting equilibrium to maximum friction force.	• Concept of friction • Limiting equilibrium

Competences	Content
• calculates the frictional force using $F = \mu R$ • calculates the coefficient of friction between the body and an inclined plane using $F = \mu R$. • applies the laws of friction to different situations. • calculates the friction force or any other force acting on the body moving on: - Horizontal plane - Inclined plane	• Coefficient of friction • Friction on horizontal surfaces and inclined planes

Teaching/Learning Strategies

- With the aid of a diagram explain the forces acting on a body on a horizontal rough surface.
- Carry out a discussion on the terminologies: limiting equilibrium and coefficient of friction.
- With the aid of worked out examples, guide the learners to calculate frictional force on an object using simple diagrams on both the horizontal surfaces and inclined planes.

Guidance to the Teacher

- Let the learner identify where friction is applied in daily life.
- Guide the leathers to explain the advantages and disadvantages of friction.
- Use the knowledge they acquired from their O level physics lessons.

Topic 17: Newton's Laws of Motion

Duration: 6 Periods

Background

The three Newton's laws of motion, namely; **Newton's first law** states that a body will remain at rest or will continue to move with constant velocity in a straight line unless acted upon by a resultant force. **Newton's second law** states that the resultant force on a body is directly proportional to the rate of change of momentum of a body. **Newton's third law** states that every action has an equal and opposite reaction. The three laws are used in the theory of elementary **Dynamics**. They deal with the effect of forces acting on a body in motion. Common bodies include vehicles, lifts or pulleys, connected bodies like trucks and trains.

When two moving bodies are connected by a string which is light and inextensible, there will be a tension in a string. By Newton's third law, the forces acting on the bodies will have the same magnitude but will act in opposite directions.

Learning Outcome

The learner should be able to use Newton's laws of motion to find the acceleration of connected bodies and the tension in the string(s).

Sub-Topic: Connected Particles

Competences	Content
The learner: • applies Newton's laws of motion to pulley systems. • works out problems involving smooth and rough surfaces for horizontal and inclined planes.	• Newton's laws of motion • Force exerted by the engine of a vehicle pulling another. • Pulley systems: - Connected bodies along a horizontal smooth surface - Connected bodies on inclined planes.

Teaching/Learning Strategies

- State Newton's laws of motion by brainstorming with the learners.
- With the aid of diagrams, guide the learners to illustrate all forces acting on connected bodies.
- Prepare and engage the learners with examples and exercises to apply the Newton's laws of motion to connected bodies on horizontal surfaces and inclined planes.

Guidance to the Teacher

- Connecting strings should be light and inelastic.
- Clearly drawn diagrams showing all the forces acting on the connected bodies should be emphasised.
- Pulleys should be smooth.
- Inclined planes can be smooth or rough.
- Moving wedges should be avoided.

SENIOR SIX TERM ONE

PURE MATHEMATICS

Topic 18: Integration

Duration: 18 Periods

Background

Integration is the process of obtaining an original function from a given gradient function; hence, it is the reverse of differentiation. Thus, if the rate of variation of a function is known, integration process can enable us to get the function itself. Integration is used to compute such things as the areas and volumes of irregular shapes and solids.

We call $\int f(x)\,dx$ an **indefinite integral** because it does not give a definite answer and we add an arbitrary constant after integrating.

The indefinite integral can be used to derive total cost and profit function from the marginal cost and marginal revenue functions.

We call $\int_a^b f(x)\,dx$ a **definite integral** because it gives a definite answer, where a is the **lower limit** of the integral and b is the **upper limit** of the integral.

The definite integral can be used in a number of applications in all science related disciplines such as calculating work done, business and economics, including price discrimination, revenue verses cost, consumer's surplus and producer's surplus.

Learning Outcome

The learner should be able to relate integration to differentiation and find the indefinite and definite integrals of simple functions.

Sub-Topic 1: Definite and Indefinite Integrals

Competences	Content
The learner: • relates the limit of summation to the integral sign. • determines indefinite integrals with the constant of integration. • evaluates definite integrals.	• The reverse of differentiation • Indefinite integrals • Definite integrals

Teaching/Learning Strategies

- Through exposition, guide the learners to relate integration to differentiation.
- Guide the learners to use the rule of integration.
- Help the learners to distinguish between indefinite and definite integrals.
- Guide the learners through examples, exercises and assignments to conceptualise the basics of integration.

Sub-Topic 2: Area under a Curve

Competences	Content
The learner: • sketches a curve. • uses integration to find the area between the given curve and the X-axis.	• Area under a curve

Teaching/Learning Strategies

- Through various exercises helps the learner to find the area under a curve

Guidance to the Teacher

- You should stick to the rule of integration: "Add One To The Power And Divide By The New Power."

- Emphasise the integration sign as a long S, and also indicate that the integration is with respect to a variable say x.
- Emphasise the constant of integration with indefinite integrals. We know that $y = x^3$, $y = x^3 + 5$, $y = x^3 - 6$, all satisfy $\frac{dy}{dx} = 3x^2$, for this reason we write $y = x^3 + c$ after integrating because we do not know whether the original function had a constant term or not.
- Remind the learners that when working with definite integrals, the constants of integration cancel and are excluded in the final result.
- Emphasises sketching the curves (degree 2) to clearly show the area covered.

Topic 19: Displacement, Velocity and Acceleration

Duration: 6 Periods

Background

In Integration, we looked at determination of indefinite and definite integrals. Under this topic, we are going to apply these integrals in determining:

i) the velocity given the acceleration

ii) displacement given the velocity

iii) finding integrals of trigonometric functions.

Displacement, velocity and acceleration

We also learned that displacement, velocity and acceleration are linked together by the process of differentiation with respect to time. In the reverse order, acceleration (a), velocity (v) and displacement(s), are linked together by integration. When we integrate an expression for the acceleration of a body with respect with time, we obtain an expression for the velocity of a body at a time, t i.e. $v = \int a\,dt$. Similarly, if we integrate an expression for the velocity of the body with respect to time, we obtain an expression for the displacement of the body at a time, t, i.e. $s = \int v\,dt$. We use the knowledge of indefinite integrals with boundary conditions to determine the expressions for the velocity and displacement.

Learning Outcome

The learner should be able to determine velocity and displacement of a body from an acceleration function.

Sub-Topic: Displacement, Velocity and Acceleration

Competences	Content
• The learner uses integration to find displacement and velocity.	• Velocity and acceleration

Teaching/Learning Strategies

- Involve the learners to link together the terms displacement, velocity and acceleration to the process of differentiation w.r.t. time i.e. $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$.
- Lead the learners to link together the terminologies a, v and s to the process of integration i.e. $s = \int v\,dt$, $v = \int a\,dt$

Guidance to the Teacher

- Let the learners explain the relationship between displacement, velocity and acceleration.
- Explain to the learners how differentiation and integration are used in this topic.
- Identify where the knowledge achieved in this topic is applied.

Topic 20: Series

Duration: 8 Periods

Background

A series is the sum of the terms of a sequence where a sequence is a set of numbers expressed in a definite order. The concept of series is utilised in banks and insurance companies. Series are classified into two i.e. the Arithmetic progression (A.P) and the Geometric progression (G.P).

An A.P is a sequence of numbers in which any term can be obtained from the previous one by adding a certain number called the *common difference, d*.

A G.P is a sequence of numbers in which any term can be obtained from the previous one by multiplying a certain number called the *common ratio, r*.

Learning Outcome

The learner should be able to distinguish between Arithmetic Progressions and Geometric progressions, generate both the A.P. and G.P. and find their sums.

Sub-Topic 1: Sequences

Competences	Content
The learner: • identifies the characteristics of a sequence. • identifies the characteristics of a series. • writes a series using the summation notation.	• Sequences • Series • Series in summation notation

Teaching/Learning Strategies

- Discuss with the learners the special notation for representing sequences and series.

Sub-Topic 2: Arithmetic Progression

Competences	Content
The learner: • generates an Arithmetic progression (A.P). • determines the $n^{th}-$term of an A.P by using the formula. • finds the sum of the first n terms of an A.P. • determines the value of n given the sum of an A.P and common difference d or last term l of an A.P.	• $n^{th}-$term of an Arithmetic progression. • Sum of the first $n^{th}-$ terms of an Arithmetic progression.

Teaching/Learning Strategies

- Use the sum of a set of natural numbers and sum of squares of the natural numbers to guide the learners to express them using summation notation.
- Prepare different sets of numbers both positives and negatives to help the learners develop an A.P.
- Using the above sets, lead the learners to show how the formula for the nth term of an A.P, $U_n = a + (n-1)d$ is derived at.
- Derive the formulae, $S_n = \frac{n}{2}(2a + (n-1)\,d)$, $S_n = \frac{n}{2}(a + l)$ for finding the sum of the first n terms of an A.P and guide the learners to apply them to related questions.

Sub-topic 3: Geometric Progression

Competences	Content
The learner: • identifies the characteristics of a geometric progression. • uses the characteristics to generate a	• Geometric progression • n^{th} term of a geometric

geometric progression. • determines the nth term of a G.P by using the formula. • uses the formula to find the sum of the first n terms of a G.P. • finds the sum to infinity using the formula. • solves amount deposited in a bank using G.P.	progression • Sum of n terms of a geometric progression

Teaching/Learning Strategies

- Prepare different sets of numbers both positives and negatives to help the learners distinguish a G.P from an A.P.
- State the formula for the n^{th} term of a G.P $U_n = ar^{n-1}$ and guides the learners to apply it to related problems.
- Apply the formulae $S_n = \frac{a(r^n - 1)}{r-1}$ when r >1 and $S_n = \frac{a(1-r^n)}{1-r}$ when r<1) for finding the sum of the first n terms of a G.P and guide the learners to apply them to related questions.
- Through exposition, guide the learners to derive and apply the formula for the sum to infinity $S_\infty = \frac{a}{1-r}$.

Guidance to the Teacher

- Take the learners through this topic slowly so that they can internalise the competences to be developed.
- Let the learners identify situations where the knowledge achieved is applied.
- Use examples from both Science and Arts fields.

SECTION B

PROBABILITY AND STATISTICS

Topic 21: Random and Continuous Variables

Duration: 8 Periods

Background

Decision-making in many businesses, insurance companies, and other real-life situations is made possible by assigning probabilities to all possible outcomes to the situations and then evaluating the results. For example, a shopkeeper can compute the probability that he will make either 0, 1, 2, 3, 4, or more sales in a single day and will be able to compute the average number of sales he makes per day, per week, which will enable him to make better predictions over a period of time say, monthly.

When carrying out an experiment, *variables* are used to describe the event. A *variable* in this case can be defined as a characteristic that can assume different values. Letters of the alphabet such as X, Y, or Z can be used to represent variables. Since the variables are associated with probability, they are called *random variables*. Random variables may be either *discrete* or *continuous*. A *discrete random variable* is the variable that has values that can be counted. For example if a die is thrown, a letter such as X can be used to represent the possible outcomes, i.e. X is assigned values 1, 2, 3, 4, 5, or 6 corresponding to the outcomes. The relationship between the possible values of a random variable and the corresponding probabilities is term as the *probability distribution* of the random variable which may be specified in terms of a probability distribution function (p. d. f).

Learning Outcome

The learner should be able to identify the characteristics of a discrete or continuous random variable, compute its mean, variance and standard deviation.

Sub-Topic 1: Discrete Random Variables

Competences	Content
The learner: • identifies the: i) random variable. ii) discrete random variable. • identifies the properties of a p.d.f of a discrete random variable. • uses the properties of a p.d.f of a discrete random variable. • generates and constructs a probability distribution table. • calculates the expectation $E(x)$, Variance, $Var(x)$, and Standard deviation of a discrete random variable. • finds the mode and median of a discrete random variable.	• Concept of discrete random variable • Probability density function (p.d.f) of a discrete random variable • Properties of a p.d.f of a discrete random variable • Probability distribution table • Expectation $E(X)$, Variance, $Var(X)$, and Standard deviation of a discrete random variable • Mode, median

Teaching/Learning Strategies

- Use a die and ask the learners to list the possible outcomes.
- Relate the possible outcomes with a random variable.
- Define a discrete random variable.
- Through exposition, guide the learners to generate a probability distribution for throwing a die.
- Assign the learners to construct the probability distribution for a discrete random variable, "the number of heads obtained from tossing two coins.

- Engage the learners in tasks to find the sum of the probabilities of the distribution."
- Emphasise the two properties of the probability distribution of a discrete random variable.
- Lead the learners to represent the probability distribution in a columnar table.
- Involve the learners in calculating the expectation $E(X)$, variance, $Var(X)$, and standard deviation using the formulae.
- Help the learners to familiarise with the use of the table of distribution of probability while calculating the $E(X)$, $Var(X)$ and standard deviation using various exercises and assignments.

Sub-Topic 2: Continuous Random Variables

Competences	Content
The learner: • identifies the characteristics of a continuous random variable. • determines a p.d.f. of a continuous random variable. • applies properties of a continuous random variable. • finds the expectation, variance and standard deviation of a continuous random variable	• Concept of continuous random variable. • Continuous probability function • Properties of a Continuous random variable • Expectation, variance and standard deviation of a continuous random variable

Teaching/Learning Strategies

- Defines continuous random variable.
- Introduce to the learners the p.d.f of a continuous random variable as an area under the graph of the given function. i.e.

$$P(a \le X \le b) = \int_a^b f(x)\,dx$$

- Emphasise the two properties of the probability distribution of a continuous random variable.
- Involve the learners with various exercises and assignments in calculating the expectation $E(X)$, variance, $Var(X)$, and standard deviation using the formulae.

Guidance to the Teacher

- Help the learners to differentiate discrete and continuous random variables.
- Let the learners discuss and come up with situations where the competences in this topic can be applied.
- Take the students through this topic slowly.
- Give examples of benefits which students will get in this topic.

Topic 22: Binomial Distribution

Duration: 6 Periods

Background

There are some probability situations that may result into only two outcomes, or even be reduced to only two. Such situations may include:

i) when a baby is born, it may be either male or female

ii) in a final football match, a team either wins or loses.

Other situations that are reduced to only two possible outcomes may include:

i) a person taking a Pioneer bus may arrive either on time or not on time.

ii) a company producing items that are either defective or not defective

iii) a drug administered to a patient may be either effective or ineffectiveness.

All the above mentioned situations are called binomial or Bernoulli experiments. The outcomes of a binomial experiment are classified as *successes* or *failures.*

Therefore a ***binomial distribution*** is one that represents the outcomes of a binomial experiment and their corresponding probabilities.

Learning Outcome

The learner should be able to interpret a binomial distribution.

Sub-Topic: Binomial Distribution

Competences	Content
The learner: • identifies the characteristics of a binomial distribution. • applies properties of a binomial distribution.	• Concept of a binomial distribution • Properties of a binomial distribution

Competences	Content
• interprets the notation $B(n,\ p)$. • calculates the probability of event using formulae or tables. • finds the $E(x)$ and $Var(x)$ of binomial distribution	• Binomial notation • Binomial tables • Expectation, variance • Standard deviation

Teaching/Learning Strategies

- Introduce a binomial distribution as an example of a discrete random variable.
- Lead the learners into listing probability situations that may result into two outcomes of success or failure.
- Guide the learners through the properties of a binomial experiment.
- Define the binomial distribution and its notation.
- Help the learners to use the formula and the mathematical tables for binomial probabilities through various exercises and assignments.
- Involve the learners in calculating the expectation $E(x)$, variance, $Var(x)$, and standard deviation of a binomial distribution using the formula.

Guidance to the Teacher

- Let the learners understand the relationship of all parameters learnt in this topic.
- Give examples of situations in which this topic may be applied.
- Use examples which are drawn from both Arts and Science fields.

MECHANICS

Topic 23: Work, Power and Energy

Duration: 8 Periods

Background

The ***work done*** by a force is the product of the force and the distance moved in the direction of the force by its point of application. The SI unit of work is the joule (J).

The energy of a body is its ability for doing work. There are various forms of energy, however, for this course, we shall be confined to only forms of ***mechanical energy*** i.e. potential energy and kinetic energy. Potential energy is the energy a body possesses by virtue of its position. Kinetic energy is the energy possessed by a body by virtue of its motion.

The principle of conservation of mechanical energy states: "*If a particle is moving such that no external force other than gravity is doing work, then the total mechanical energy of the particle remains constant.*"

Power is the rate of doing work. The SI unit of power is the watt (W).

Learning Outcome

The learner should be able to find the work done, energy and power dissipated by a body.

Sub-Topic: Work, Energy and Power

Competences	Content
The learner: • defines work, energy and power. • works out problems involving work done against gravity, friction and by a constant force. • uses the principle of conservation of energy to solving problems. • relates work done to change in	• Work done against: - gravity by a constant force - friction • Principle of conservation of energy • Power

Competences	Content
energy. • solves power related problems.	

Teaching/Learning Strategies

- Discuss with the learners that once a mass of m kg is raised through a vertical distance, s, then work is done against gravity.
- Use a rough inclined plane to demonstrate how work is done both against gravity and frictional force.
- Through various exercises, lead the learners to calculate work done by a constant force, against gravity and friction.
- Explain that situations involving a moving body and there is no work done against friction and gravity is the only external force then the principle of conservation of energy is used.
- Ensure that the learners are exposed to various examples and exercises involving energy and power.
- Emphasise the use of SI units of work, energy and power every after a problem is worked

Guidance to the Teacher

- Do not make the impression that knowledge gained from this topic favours Science students only.
- Show the learners how the competences achieved from this topic are applied in their daily life.
- Use examples from both Science and Arts fields.
- Relate the concepts of work, energy and power.

SENIOR SIX TERM TWO

PURE MATHEMATICS

Topic 24: Matrices

Duration: 10 Periods

Background

A matrix is a rectangular array of numbers called elements or entries. Information can conveniently be presented as an array of rows and columns. The ***order*** of a matrix gives the format of how a matrix should be written. It is always in the form ***m x n*** where ***m*** is the number of rows and ***n*** is the number of columns in the matrix.

Learning Outcome

The learner should be able to carry out different operations on matrices of order 2 and order 3.

Sub-Topic: Matrices

Competences	Content
The learner: • states the order of a matrix. • carries out the operations on matrices. • calculates the determinant and inverse of a 2×2 matrix. • uses the inverse of a matrix to solve simultaneous equations. • uses the determinant method to solve simultaneous equations.	• Operations on matrices • Determinant of a 2×2 matrix • Inverse of a 2×2 matrix • Solution of simultaneous equations using matrices and determinant (Crammer's rule)

Teaching/Learning Strategies

- Lead the learners to formulate matrices of any order from real life situations.
- Involve learners in numerous exercises on addition, subtraction and multiplication of matrices.
- Emphasise to the learners about the commutative property $(AB \neq BA)$ in multiplication and compatibility of matrices through various exercises.
- Make revision on the determinant, inverse of a 2×2 matrix, and solving of simultaneous equations using matrices.
- Prepare various exercises on solving simultaneous equations using the determinant.

Guidance to the Teacher

- Let the learners identify situations where the competences developed can be applied.
- Let all learners know the purpose of learning matrices.
- Guide (especially the Arts) students to solve simultaneous equations using matrix method.

Topic 25: Normal Distribution

Duration: 8 Periods

Background

The normal distribution is a continuous, symmetric, bell-shaped distribution of a variable. It is the most important continuous probability distribution for both practical and theoretical statistics. The normal distribution provides a good probability model for many continuous variables whose values depend on the effect of a number of factors. Such variables may include the heights of people or objects and other measurements of biological importance, life time of batteries, electric bulbs, etc., weights of farm produce supplied, metal bars manufactured in a factory and packed in boxes, etc.

Learning Outcome

The learner should be able to identify the properties of a normal distribution and find probabilities for a normally distributed variable.

Sub-Topic: Normal Distribution

Competences	Content
The learner: • interprets the notation $N(\mu,\ \delta^2)$ of normal distribution. • identifies the properties of the normal distribution. • finds the area under the standard normal curve given various z values • writes the given problem in probability notation e.g. $P(X \leq 20)$ • standardises the random variable into the standard normal variable Z. • reads and uses the standard normal tables to find probabilities.	• Concept of normal distribution • Properties of normal distribution • Standardisation • Standard normal tables

Teaching/Learning Strategies

- Introduce a normal distribution as a continuous random variable.
- Guide the learners to outline the properties of the normal distribution.
- Help the learners to i) sketch the normal curve ii) shade the desired area iii) look up the z value in the table to get the area.
- Through various exercises, bring out the following, finding the area under the normal curve (required probability):

  i) between 0 and any z value.

  ii) from the z value to the end of the tail.

  iii) between two z values on the same side of the mean.

  iv) between two z values on opposite sides of the mean.

  v) Less than any z value to the right of the mean.

  vi) Greater than any z value to the left of the mean.

- Lead the learners to use the formula for a standard score to transform the normally distributed variable into the standard normal variable.
- Help the learners to use the mathematical tables for cumulative normal distribution through various exercises and assignments.
- Assign the learners in groups to determine the probabilities for a normally distributed variable by transforming it into a standard normal variable.

Guidance to the Teacher

- Take the learners through this topic so that they can understand the notation used.
- Give examples in which the competences developed in these topics are applied.
- Relate the concept of finding area under the curve using integration approach and finding the area under the standard normal curve in this topic.

SENIOR SIX TERM 3

PURE MATHEMATICS

Topic 26: Differential Equations

Duration: 8 Periods

Background

We learnt that the derivative is also an instantaneous rate of change i.e. we denoted the instantaneous rate of change of y with respect to time t as $\frac{dy}{dx}$.

For many growth processes, the rate of change of the amount of a substance with respect to time is proportional to the amount present. This can be represented by the equation $\frac{dy}{dt} = ky$ where k is a constant.

An equation of this type, where y is an unknown function of say x, is called a **differential equation** because it contains derivatives or differential coefficients.

There are several methods of solving differential equations. However, in our case, we are only going to use the method of separation of variables to solve these equations.

Differential equations are used to solve applied problems such as those involving carbon dating and radioactive decay; the amount of drug in an organ; mixtures; supply and demand; logistic growth and marginal productivity.

Learning Outcome

The learner should be able to use integration to find the general solution of a differential equation, and find particular solutions of differential equations in initial value problems.

Sub-Topic: Differential Equations

Competences	Content
The Learner :	
• identifies a differential equation.	• Concept of a differential equation
• states the order of a differential equation.	• Order of a differential equation
• finds the general solution and particular solution of a differential equation.	• General solution and particular solution of a differential equation
• solves problems involving separable differential equations.	• First order differential equations with separable variables
• solves problems involving separable differential equations related to natural occurrences.	• Formation of differential equations of natural occurrences

Teaching/Learning Strategies

- Introduce the topic using equations such as $y=x^2$ and involve them to find the first derivative.
- Relate the above derivatives to the definition of a differential equation.
- Define the order of a differential equation and prepares work for the learners to identify the different orders.
- Engage the learners in several tasks to use integration to determine the general and particular solution of first order differential equations.
- Prepare differential equations with separable variables and guides the learners to separate and integrate them.
- Provide word problems with separable variables for the learners to form and solve the differential equations.

Guidance to the Teacher

- Let the learners identify where the competences achieved/developed are applied in life subjects.
- Show the relevancy of this topic to their subject combination or their courses to be done after A level.

Appendix: Derivatives and Integrals of Trigonometric Functions

Trigonometric Function	Derivative	Integral
$y = \sin x$	$\frac{dy}{dx} = \cos x$	$\int \sin x \, dx = -\cos x + c$
$y = \cos x$	$\frac{dy}{dx} = -\sin x$	$\int \cos x \, dx = \sin x + c$
$y = \sin ax$	$\frac{dy}{dx} = a\cos ax$	$\int \sin ax \, dx = -\frac{1}{a}\cos ax + c$
$y = \cos bx$	$\frac{dy}{dx} = -b\sin bx$	$\int \cos bx \, dx = \frac{1}{b}\sin bx + c$
$y = \sin ax + b$	$\frac{dy}{dx} = a\cos ax$	
$y = \sin(bx + t)$	$\frac{dy}{dx} = b\cos(bx + t)$	

References

Backhouse, J.K. (2011), *Pure Mathematics,* Pearson Education Limited, International, Harlow

Chandler, L. (1996), *Mechanics and Statistics for Advanced Level*

Smedley, R. and Wiseman, G. (1998), *Introducing Pure Mathematics,* Oxford University Press.

Thorning, A. J. and D.W.S. (1996), *Understanding Mechanics,* Oxford University Press, United Kingdom

Thorning, A.J. and D.W.S. (1996), *Understanding Pure Mathematics,* Oxford University Press, United Kingdom